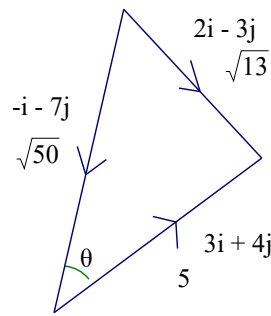
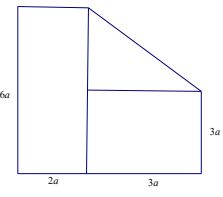
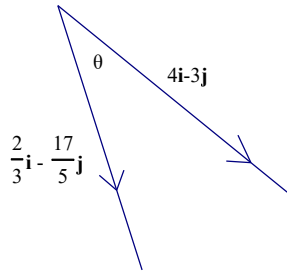


Question Number	Scheme	Marks	Notes
1(a)	Use of Impulse = change in momentum	M1	Dimensionally correct. Condone subtraction in the wrong order.
	$2((2\mathbf{i} - 3\mathbf{j}) - (3\mathbf{i} + 4\mathbf{j})) = \mathbf{I}$	A1	Correct unsimplified. Allow \pm
	$= -2\mathbf{i} - 14\mathbf{j}$	A1	Allow \pm
	Use of Pythagoras	M1	For <i>their</i> impulse
	$ \mathbf{I} = \sqrt{2^2 + 14^2} = \sqrt{200} = 10\sqrt{2}$ (Nm)	A1	14(Ns) or better. (14.142.....) From correct working (i.e. only from $\pm\mathbf{I}$)
		5	
1(b)	Correct use of trigonometry to find the required angle	M1	For $3\mathbf{i} + 4\mathbf{j}$ and <i>their</i> impulse
	$\tan^{-1} \frac{4}{3} + \frac{\pi}{2} + \tan^{-1} \frac{1}{7}$	A1	Correct unsimplified
	$\theta = 2.64$ (radians)	A1	or 150° or better (151.2602.....) Accept 209° or better: (208.739...) and 3.64 radians
		(3)	
Alt1 1(b)	Using scalar product to find the required angle:	M1	For $3\mathbf{i} + 4\mathbf{j}$ and <i>their</i> impulse
	$\cos \theta = \frac{-3 - 28}{5 \times \sqrt{50}}$	A1	Correct unsimplified equation in $\cos \theta$
	$\theta = 151^\circ$	A1	Accept 150° or better (151.2602.....), or 2.6(4) radians Accept 209° (208.739...) and 3.64 radians
		(3)	
Alt2 1(b)			
	Complete method to find the required angle e.g. use of correct cosine rule to find the obtuse angle	M1	For $3\mathbf{i} + 4\mathbf{j}$ and <i>their</i> impulse
	$\cos \theta = \frac{50 + 25 - 13}{2 \times 5 \times \sqrt{50}}$	A1	Correct unsimplified equation in $\cos \theta$
	required angle $(= 180^\circ - \theta) = 151^\circ$	A1	Accept 150° or better (151.2602.....), or 2.6(4) radians Accept 209° (208.739...) and 3.64 radians
		(3)	

Question Number	Scheme	Marks	Notes
	NB: If they are working with an incorrect impulse e.g. $2\mathbf{i} + 14\mathbf{j}$ from (a) then M1A0A0 can be scored in (b)		
		8	

2(a)	Use of $P = Fv$:	M1	$F = \frac{280}{2} (=140)(\text{N})$ Seen or implied
	Equation of motion: $F - 75g \sin \theta = R$	M1	Need all terms, condone trig confusion and sign errors Accept $\frac{P}{v} - 75g \sin \theta = R$
	$140 - 75g \sin \theta = R$ $\left(140 - 75 \times 9.8 \times \frac{1}{21} = R\right)$	A1	Correct substituted equation in R (and θ)
	$R = 105$ (or 110)	A1	3 sf or 2 sf only.
		(4)	
2(b)	Equation of motion:	M1	Need all terms, condone trig confusion and sign errors. Must be using $v = 3.5$ and $m = 75$ unless there is a clear consistent misread
	$75g \sin \theta + \frac{280}{3.5} - 60 = 75a$ $(75g \sin \theta + 80 - 60 = 75a)$	A1 A1	Unsimplified equation in g, θ and $\pm a$ with at most one error Correct unsimplified equation in g, θ and $\pm a$
	$a = 0.73$ (m s^{-2}) (0.733)	A1	a must be positive 2 sf or 3 sf only A0 for $\frac{11}{15}$
		(4)	
	SC: A candidate who uses $F = Pv$ can score M0M1A0A0 in (a) and 0/4 in (b)		
		8	

Q	Scheme				Marks	Notes
3(a)		square	triangle	template	B1 B1 B1	Correct area ratio Correct horizontal distances Correct vertical distances Be aware of other possible splits e.g.
	Area	$36a^2(6)$	$6a^2(1)$	$30a^2(5)$		
	From <i>OD</i>	$3a$	$\frac{14a}{3}$	\bar{x}		
	From <i>OA</i>	$3a$	$5a$	\bar{y}		
			Tall rectangle	triangle	Lower rectangle	template
		Area	$12a^2(2)$	$6a^2(1)$	$12a^2(2)$	$30a^2(5)$
		From <i>OD</i>	a	$\frac{10a}{3}$	$4a$	\bar{x}
		From <i>OA</i>	$3a$	$4a$	$\frac{3}{2}a$	\bar{y}
<p>If they do not give you a diagram allow their values to imply a correct table / split No diagram and incorrect values can score at most M1 for a dimensionally correct moments equation If they use a non-standard shape (e.g. trapezium) they must show full working for the centre of mass.</p>						
(i)	$6 \times 3a - 1 \times \frac{14a}{3} = 5\bar{x}$				M1	moments equation about <i>OD</i> or a parallel axis OR a moments equation about <i>OA</i> or a parallel axis. Moments equation must be dimensionally consistent. Condone sign errors. SC Allow M1 for work on a framework with 5 sections
	$\Rightarrow \bar{x} = \frac{8a}{3}$ ISW				A1	Allow $2.6a$ but not an approximate value e.g. $2.67a$
(ii)	$6 \times 3a - 1 \times 5a = 5\bar{y} \Rightarrow \bar{y} = \frac{13a}{5}$				A1	Accept $2.6a$
					(6)	
3(b)	Use of trig to find a relevant angle				M1	e.g. $\alpha, 180^\circ - \alpha$ or $90^\circ - \alpha$. Need correct pairing of angles i.e. do not accept $\frac{3}{4}$ vs $\frac{4}{3}$ confusion in their equation for α
	$\alpha = \arctan\left(\frac{6a - \bar{y}}{\bar{x} - 2a}\right) - \arctan\left(\frac{3}{4}\right)$				A1ft A1ft	Unsimplified expression with at most one error. For <i>their</i> \bar{x}, \bar{y} (need not be substituted)

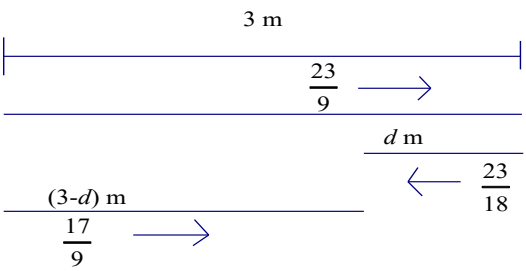
	Or equivalent e.g. $\tan^{-1} \frac{4}{3} - \tan^{-1} \frac{\bar{x} - 2a}{6a - \bar{y}} \left(= \tan^{-1} \frac{4}{3} - \tan^{-1} \frac{10}{51} \right)$		Correct unsimplified expression. For <i>their</i> \bar{x} , \bar{y} (need not be substituted)
	$= 42^\circ$	A1	Accept 138° , but not 42.0° The Q asks for the nearest degree
		(4)	
3b alt			
	Correct statement for scalar product to find a relevant angle	M1	Correct vector for their \bar{x} and \bar{y}
	$\cos \theta = \frac{4 \times \frac{2}{3} + 3 \times \frac{17}{5}}{5 \times \sqrt{\frac{2701}{225}}} (= 0.742)$	A1ft	Unsimplified expression with scalar product expanded and at most one error
		A1ft	Correct unsimplified expression with scalar product expanded
	$(\theta =) 42^\circ$	A1	Accept 138° but not 42.0° The Q asks for the nearest degree
		(4)	
		10	

Q	Scheme	Marks	Notes
4(a)	Accept working in column vectors throughout this question		
	Differentiate \mathbf{r} to obtain \mathbf{v}	M1	Powers going down by 1. M0 if clearly dividing by t Condone one slip. Seen or implied
	$\mathbf{v} = (3t^2 - 9t - 24)\mathbf{i} + (-3t^2 + 6t + 12)\mathbf{j}$	A1	Correct only. Seen or implied
	$3T^2 - 9T - 24 = -3T^2 + 6T + 12$ $\Rightarrow 6T^2 - 15T - 36 = 0$	M1	Equate coefficients of \mathbf{i} and \mathbf{j} from their \mathbf{v} to obtain a 3-term quadratic in t or T
	Solve for T : e.g. $3(2T+3)(T-4) = 0$	DM1	Ignore $T = -\frac{3}{2}$ if seen. Can be implied by correct roots of an incorrect quadratic. Allow if they solve a quadratic resulting from a slip in simplifying Dependent on the previous M1
	$(T =)4$	A1	Only
		(5)	
4(b)	Differentiate <i>their</i> \mathbf{v} to obtain \mathbf{a} :	M1	Powers going down by 1. M0 if clearly dividing by t Condone one slip. Seen or implied
	$\mathbf{a} = (6t - 9)\mathbf{i} + (-6t + 6)\mathbf{j}$	A1ft	Follow their \mathbf{v} . Seen or implied
	$\mathbf{a} = (6T - 9)\mathbf{i} + (-6T + 6)\mathbf{j} = 15\mathbf{i} - 18\mathbf{j}$	DM1	substitute <i>their</i> positive T Dependent on the previous M1 Seen or implied
	Use Pythagoras	DM1	Correct method for the magnitude: $ \mathbf{a} = \sqrt{15^2 + 18^2}$ Dependent on the previous DM1
	$ \mathbf{a} = \sqrt{549} = 23.4 \text{ (m s}^{-2}\text{)}$	A1	23 or better e.g. $3\sqrt{61}$
		(5)	
		10	

Q	Solution	Marks	Notes
5(a)	Moments about A :	M1	Or equivalent to obtain an equation for F e.g. moments about another point and vertical resolution. Requires all terms and dimensionally correct. Condone sign errors and sine / cosine confusion. Allow $\cos \theta = \frac{4}{5}$ used without comment.
	$F \times \frac{5a}{4} = mga \cos \theta + 2kmg a \cos \theta$	A1 A1	Unsimplified equation with at most one error. Consistent trig confusion counts as 1 error. g missing in both terms is a single error Correct unsimplified equation
	$F = \frac{4mg \cos \theta}{5} (1+2k) = \frac{16}{25} mg (1+2k) *$	A1*	Obtain given answer from full and correct working
			SC allow M1A1A1A0 if a missing throughout
		(4)	
5(b)	resolve horizontally: $H = F \sin \theta$	M1	Condone sine / cosine confusion
	$H = \frac{48}{125} mg (1+2k)$	A1	Or equivalent with trig substituted
	resolve vertically	M1	Requires all three terms. Condone sine/cosine confusion and sign errors.
	$\pm V = mg(1+k) - F \cos \theta$	A1	Correct unsimplified equation
	$= mg(1+k) - \frac{64}{125} mg(1+2k)$ $\left(= mg \left(\frac{61}{125} - \frac{3k}{125} \right) \right)$	A1	Or equivalent with trig substituted Allow for V acting downwards here ISW if they had a correct expression and make an error in simplifying it.
	Either of the above equations could be replaced with an alternative e.g. Resolve parallel to the rod: $\frac{4}{5}H + \frac{3}{5}V = \frac{3}{5}mg + \frac{3}{5}kmg$ Resolve perpendicular to the rod: $\frac{3}{5}H + \frac{4}{5}mg + \frac{4}{5}kmg = \frac{4}{5}V + F$ Moments about C : $\frac{5}{4}a \times \frac{4}{5}V + \frac{3}{4}a \times \frac{4}{5}kmg = \frac{5}{4}a \times \frac{3}{5}H + \frac{1}{4}a \times \frac{4}{5}mg$ NB: the names H and V are not given in the question so they might be using their own alternatives.		
	SC: If they have confused horizontal and vertical, obtain correct values for the two components but name them incorrectly then allow M1M1 (2/5)		
		(5)	

5(c)	Use $H = V$ to form equation in k	M1	For their $H, \pm V$
	Obtain $\frac{48}{125}mg(1+2k) = mg(1+k) - \frac{64}{125}mg(1+2k)$	A1ft	Must now be using V acting vertically upwards. Or equivalent e.g. $48(1+2k) = (61-128k)$ Follow their H and V upwards
	$k = \frac{13}{99}$	A1	Accept 0.13 or better
		(3)	
		[12]	

Q	Solution	Marks	Notes
6a	<p>Diagram description: A horizontal line represents a surface. Above it, two circles represent objects A and B. Object A is on the left, moving right with velocity 3 m/s. Object B is on the right, moving right with velocity 2 m/s. Below the surface, object A is moving right with velocity v, object B is moving right with velocity w, and the surface is moving left with velocity 1/2 w. A vertical line represents a wall on the right.</p>		
	Equation for CLM	M1	Need all terms. Dimensionally consistent. Condone sign errors.
	$3m + 4m = mv + 2mw$	A1	Or equivalent e.g. $7 = v + 2w$
	Equation for Impact Law	M1	Must be used the right way round. Condone sign errors
	$w - v = \frac{2}{3}(3 - 2)$	A1	Or equivalent e.g. $3w - 3v = 2$
	Solve for w or v e.g. $w - \frac{2}{3} + 2w = 7$	DM1	Dependent on both previous M marks Must have both equations. Not available if all they have is CLM and then work back from the given answer
	$3w = \frac{23}{3}, \quad w = \frac{23}{9} \quad *$	A1*	Obtain given answer with no incorrect working seen
	$\frac{23}{9} - v = \frac{6}{9}, \quad v = \frac{17}{9}$	A1	Correct only
		(7)	

6(b)			
	Velocity of B after hitting wall = $\frac{23}{18}$ (m s^{-1})	B1	Or equivalent: $\frac{1}{2} \times \frac{23}{9}$ (both values given) Allow +/-
	Time for B to get to the 2 nd collision	M1	Using time = $\frac{\text{distance}}{\text{speed}}$
	$= 3 \times \frac{9}{23} + d \times \frac{18}{23}$	A1	Or equivalent e.g. $\frac{27+18d}{23}$
	Time for A to get to the 2 nd collision $= (3-d) \frac{9}{17}$	B1ft	Follow their v : $\frac{3-d}{v}$
	Equation in d	M1	Using time between collisions for their "correct" times
	$17(27+18d) = 9 \times 23(3-d)$	A1	Or equivalent e.g. $\frac{27+18d}{23} = \frac{9}{17}(3-d)$
	$d = \frac{162}{513} \left(= \frac{6}{19} \right)$	A1	Accept 0.32 or better. ISW
		(7)	
6(b) Alt1	Velocity of B after hitting wall = $\frac{23}{18}$ (m s^{-1})	B1	Or equivalent: $\frac{1}{2} \times \frac{23}{9}$ (both values given) Allow +/-
	Time for B to reach wall $\frac{3}{w} = 3 \times \frac{9}{23} = \frac{27}{23}$ (s)	M1	
	Distance travelled by A in this time $= \frac{17}{9} \times \frac{27}{23} = \frac{51}{23}$ (m)	A1ft	Follow <i>their</i> v : $\frac{27}{23} \times \text{their } v$
	Distance between A and B now $= 3 - \frac{51}{23} = \frac{18}{23}$ (m)	A1	
	Gap closing at $\frac{23}{18} + \frac{17}{9} = \frac{19}{6}$ (m s^{-1})	M1	Relative velocity
	Time to meet = $\frac{18}{23} \div \frac{19}{6} = \frac{18}{23} \times \frac{6}{19} \left(= \frac{108}{437} \right)$	A1	Using time = $\frac{\text{distance}}{\text{speed}}$
	$d = \frac{18}{23} \times \frac{6}{19} \times \frac{23}{18} = \frac{6}{19}$	A1	Accept 0.32 or better ISW
		(7)	

6balt2	Velocity of B after hitting wall = $\frac{23}{18}$ (m s^{-1})	B1	Or equivalent: $\frac{1}{2} \times \frac{23}{9}$ (both values given) Allow +/-
	Time for B to reach wall $\frac{3}{w} = 3 \times \frac{9}{23} = \frac{27}{23}$ (s)	M1	
	Distance travelled by A in this time $= \frac{17}{9} \times \frac{27}{23} = \frac{51}{23}$ (m)	A1ft	Follow <i>their v</i> : $\frac{27}{23} \times \text{their } v$
	Total additional distance in time t between B hitting the wall and A & B colliding again $= \frac{23}{18}t + \frac{17}{9}t$	A1	
	$\Rightarrow \frac{17}{9} \times \frac{27}{23} + \frac{17}{9}t + \frac{23}{18}t = 3$	M1	Add these two distances to form an equation in t
	$t = \frac{108}{437}$	A1	Seen or implied
	$d = \frac{23}{18}t = \frac{6}{19}$	A1	ISW
		(7)	
		14	

7(c) Alt 2	Same height: $\frac{40}{g} = 9t - \frac{1}{2}gt^2$	M1	Form an equation in t only and solve for t
	Obtain $(gt - 10)(gt - 8) = 0$, $t = \frac{8}{g}$	A1	Accept 0.816 and $\frac{40}{49}$ Allow difference in final decimal place if working with 9.81 throughout and already penalised earlier. Must reject $\frac{10}{g}$ but can be implied by subsequent work
	$(\mathbf{r} =) \left(\frac{4}{5}k\right) \mathbf{i} + k\mathbf{j}$	A1	Accept decimal equivalent Accept column vector
	SC if they start with $k = 9t - \frac{1}{2}gt^2$ and stop they score 0/3, but if they solve the quadratic to get to $\frac{36 - 4\sqrt{81 - 19.6k}}{9.8} \mathbf{i} + k\mathbf{j}$ then allow 3/3		
	Allow an answer of $(\mathbf{r} =) \frac{32}{g} \mathbf{i} + k\mathbf{j}$ or $(\mathbf{r} =) \left(\frac{72}{g} - k\right) \mathbf{i} + k\mathbf{j}$ or $(\mathbf{r} =) 3.3\mathbf{i} + k\mathbf{j}$ or $(\mathbf{r} =) (7.35 - k) \mathbf{i} + k\mathbf{j}$ or equivalent Condone exact fractions and figures to more than 3 sf after substitution for g , but an answer of $(\mathbf{r} =) 3.3\mathbf{i} + 4.1\mathbf{j}$ or $(\mathbf{r} =) \frac{32}{g} \mathbf{i} + \frac{40}{g} \mathbf{j}$ scores M1A1A0 because k is not involved The Q asks for a vector. Should be in \mathbf{i} and \mathbf{j} or as a column vector. Coordinates can score M1A1A0		
		(3)	
7(d)	$4\mathbf{i} - n\mathbf{j}$ perpendicular to $4\mathbf{i} + 9\mathbf{j}$	M1	Use ratios or scalar product to find velocity
	$\frac{4}{n} = \frac{9}{4}$, $n = \frac{16}{9}$	A1	Accept 1.8 or better (1.7) Accept \pm
	$-\frac{16}{9} = 9 - gT$	dM1	Form an equation in T (or t) only Correct signs. Dependent on the previous M1
	$(T =) 1.1$ (1.10)	A1	2 sf or 3 sf $\frac{97}{9g}$ is A0 unless previously penalised
		(4)	
		[13]	