Pearson Edexcel IAL Further Mathematics
Further Mathematics 3
Past Paper Collection (from 2020)

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Last updated: July 1, 2024

Comments and suggestions to DrYuFromShanghai@QQ.com

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Thursday 15 October 2020

Morning (Time: 1 hour 30 minutes) | Paper Reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
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Leave blank

1. (a) Use the definition of sinh x in terms of exponentials to show that

$$\sinh 3x \equiv 4\sinh^3 x + 3\sinh x$$

(2)

(b) Hence determine the exact coordinates of the points of intersection of the curve with equation $y = \sinh 3x$ and the curve with equation $y = 19\sinh x$, giving your answers as simplified logarithms where necessary.

(5)

Leave blank

Question 1 continued

Q1

(Total 7 marks)

Leave blank

2. Determine

(i) $$\int \frac{1}{3x^2 + 12x + 24}\, dx$$

(4)

(ii) $$\int \frac{1}{\sqrt{27 - 6x - x^2}}\, dx$$

(4)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 8 marks)

Leave blank

3.

$$\mathbf{M} = \begin{pmatrix} 3 & -4 & k \\ 1 & -2 & k \\ 1 & -5 & 5 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

Given that 3 is an eigenvalue of **M**,

(a) find the value of *k*.

(3)

(b) Hence find the other two eigenvalues of **M**.

(3)

(c) Find a normalised eigenvector corresponding to the eigenvalue 3

(3)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 9 marks)

Leave blank

4. $$I_n = \int x^n \cos x \, dx$$

(a) Show that, for $n \geqslant 2$

$$I_n = x^n \sin x + nx^{n-1} \cos x - n(n-1)I_{n-2}$$

(4)

(b) Hence find the functions f(x) and g(x) such that

$$\int x^4 \cos x \, dx = f(x)\sin x + g(x)\cos x + c$$

where c is an arbitrary constant.

(5)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 9 marks)

Leave blank

5. The hyperbola H has equation $\frac{x^2}{25} - \frac{y^2}{4} = 1$

The line l has equation $y = mx + c$, where m and c are constants.

Given that l is a tangent to H,

(a) show that $25m^2 = 4 + c^2$

(4)

(b) Hence find the equations of the tangents to H that pass through the point (1, 2).

(5)

(c) Find the coordinates of the point of contact each of these tangents makes with H.

(3)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 12 marks)

Leave blank

6.

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & a \end{pmatrix} \quad a \neq 1$$

(a) Find $\mathbf{A}^{-1}$ in terms of a.

(4)

$$\mathbf{B} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix}$$

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix **B**.

The equation of l_2 is

$$\big(\mathbf{r} - (12\mathbf{i} + 4\mathbf{j} + 6\mathbf{k})\big) \times (-6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) = \mathbf{0}$$

(b) Find a vector equation for the line l_1

(4)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

7. The curve C has parametric equations

$$x = \cosh t + t, \quad y = \cosh t - t \qquad 0 \leqslant t \leqslant \ln 3$$

(a) Show that

$$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = 2\cosh^2 t$$

(3)

The curve C is rotated through 2π radians about the x-axis. The area of the curved surface generated is given by S.

(b) Show that

$$S = 2\pi\sqrt{2}\int_0^{\ln 3}\left(\cosh^2 t - t\cosh t\right)\mathrm{d}t$$

(2)

(c) Hence find the value of S, giving your answer in the form

$$\frac{\pi\sqrt{2}}{9}(a + b\ln 3)$$

where a and b are constants to be determined.

(7)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 12 marks)

Leave blank

8. The plane Π_1 has equation

$$x - 5y + 3z = 11$$

The plane Π_2 has equation

$$3x - 2y + 2z = 7$$

The planes Π_1 and Π_2 intersect in the line l.

(a) Find a vector equation for l, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a scalar parameter.

(5)

The point $P(2, 0, 3)$ lies on Π_1

The line m, which passes through P, is parallel to l.

The point $Q(3, 2, 1)$ lies on Π_2

The line n, which passes through Q, is also parallel to l.

(b) Find, in exact simplified form, the shortest distance between m and n.

(5)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Friday 22 January 2021

Afternoon (Time: 1 hour 30 minutes)	Paper Reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

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- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

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Leave blank

1. Relative to a fixed origin O, the points A, B, C and D have coordinates (0, 4, 1), (4, 0, 0), (3, 5, 2) and (2, 2, k) respectively, where k is a constant.

(a) Determine the exact area of triangle ABC.

(3)

(b) Determine in terms of k, the volume of the tetrahedron $ABCD$, simplifying your answer.

(3)

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Q1

(Total 6 marks)

Leave blank

2.

$$y = \ln(\tanh 2x) \qquad x > 0$$

(a) Show that

$$\frac{dy}{dx} = p \operatorname{cosech} 4x$$

where p is a constant to be determined.

(4)

(b) Hence determine, in simplest form, the exact value of x for which $\frac{dy}{dx} = 1$

(2)

Leave blank

Question 2 continued

(Total 6 marks)

Q2

Leave blank

3.

$$\mathbf{A} = \begin{pmatrix} 2 & k & 2 \\ 2 & 2 & k \\ 1 & 2 & 2 \end{pmatrix} \qquad \text{where } k \text{ is a constant}$$

(a) Determine the values of k for which $\mathbf{A}$ is singular.

(2)

Given that $\mathbf{A}$ is non-singular,

(b) find $\mathbf{A}^{-1}$, giving your answer in terms of k.

(4)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 6 marks)

Leave blank

4. Using the substitution $x = 4\cosh\theta$ show that

$$\int \frac{1}{(x^2 - 16)^{\frac{3}{2}}}\,\mathrm{d}x = \frac{ax}{\sqrt{x^2 - 16}} + c \qquad |x| > 4$$

where a is a constant to be determined and c is an arbitrary constant.

(6)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 6 marks)

Leave blank

5.

$$\mathbf{M} = \begin{pmatrix} 6 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{pmatrix}$$

Given that 8 is an eigenvalue of **M**

(a) determine an eigenvector corresponding to the eigenvalue 8

(2)

(b) Determine the other two eigenvalues of **M**.

(3)

(c) Hence find an orthogonal matrix **P** and a diagonal matrix **D** such that $\mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P} = \mathbf{D}$

(4)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 9 marks)

Leave blank

6.

$$I_n = \int \frac{x^n}{\sqrt{x^2 + 3}}\,dx \qquad n \in \mathbb{N}$$

(a) Show that

$$I_n = \frac{x^{n-1}}{n}\left(x^2 + 3\right)^{\frac{1}{2}} - \frac{3(n-1)}{n} I_{n-2} \qquad n \geqslant 3$$

(6)

(b) Hence show that

$$\int \frac{x^5}{\sqrt{x^2 + 3}}\,dx = \frac{1}{5}\left(x^2 + 3\right)^{\frac{1}{2}}\left(x^4 + px^2 + q\right) + k$$

where p and q are integers to be determined and k is an arbitrary constant.

(4)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 10 marks)

Leave blank

7. The point P has coordinates (1, 2, 1)

The line l has Cartesian equation

$$\frac{x-3}{5} = \frac{y+1}{3} = \frac{z+5}{-8}$$

The plane Π_1 contains the point P and the line l.

(a) Show that a Cartesian equation for Π_1 is

$$6x - 2y + 3z = 5$$

(5)

The point Q has coordinates $(2, k, -7)$, where k is a constant.

(b) Show that the shortest distance between Π_1 and Q is

$$\frac{2}{7}|k + 7|$$

(2)

The plane Π_2 has Cartesian equation $8x - 4y + z = -3$

Given that the shortest distance between Π_1 and Q is the same as the shortest distance between Π_2 and Q,

(c) determine the possible values of k.

(4)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 11 marks)

Leave blank

8. The curve C has equation

$$y = 2 + \ln(1 - x^2) \qquad \frac{1}{2} \leqslant x \leqslant \frac{3}{4}$$

(a) Show that the length of the curve C is given by

$$\int_{\frac{1}{2}}^{\frac{3}{4}} \left(\frac{1 + x^2}{1 - x^2}\right) \mathrm{d}x$$

(4)

(b) Hence, using algebraic integration, show that the length of the curve C is $p + \ln q$ where p and q are rational numbers to be determined.

(5)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 9 marks)

Leave blank

9. The ellipse E has equation

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

The point P lies on the ellipse and has coordinates $(5\cos\theta, 4\sin\theta)$ where $0 < \theta < \frac{\pi}{2}$

The line l is the normal to the ellipse at the point P.

(a) Show that an equation for l is

$$5x\sin\theta - 4y\cos\theta = 9\sin\theta\cos\theta$$

(5)

The point F is the focus of E that lies on the positive x-axis.

(b) Determine the coordinates of F.

(2)

The line l crosses the x-axis at the point Q.

(c) Show that

$$\frac{|QF|}{|PF|} = e$$

where e is the eccentricity of E.

(5)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hours 30 minutes

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

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- Good luck with your examination.

Leave blank

1. (a) Using the definitions of hyperbolic functions in terms of exponentials, show that

$$1 - \tanh^2 x \equiv \operatorname{sech}^2 x$$

(3)

(b) Solve the equation

$$2\operatorname{sech}^2 x + 3\tanh x = 3$$

giving your answer as an exact logarithm.

(3)

Leave blank

Question 1 continued

Q1

(Total 6 marks)

Leave blank

2. A curve has equation

$$y = \sqrt{9 - x^2} \qquad 0 \leqslant x \leqslant 3$$

(a) Using calculus, show that the length of the curve is $\dfrac{3\pi}{2}$

(4)

The curve is rotated through 2π radians about the x-axis.

(b) Using calculus, find the exact area of the surface generated.

(3)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 7 marks)

Leave blank

3.

$$\mathbf{M} = \begin{pmatrix} 3 & 1 & p \\ 1 & 1 & 2 \\ -1 & p & 2 \end{pmatrix} \quad \text{where } p \text{ is a real constant}$$

(a) Find the exact values of p for which **M** has no inverse.

(4)

Given that **M** does have an inverse,

(b) find $\mathbf{M}^{-1}$ in terms of p.

(5)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 9 marks)

Leave blank

4. (i) $$f(x) = x \arccos x \qquad -1 \leqslant x \leqslant 1$$

Find the exact value of $f'(0.5)$.

(3)

(ii) $$g(x) = \arctan(e^{2x})$$

Show that

$$g''(x) = k \operatorname{sech}(2x) \tanh(2x)$$

where k is a constant to be found.

(5)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 8 marks)

Leave blank

5.

$$I_n = \int \sec^n x \, dx \qquad n \geqslant 0$$

(a) Prove that for $n \geqslant 2$

$$(n-1)I_n = \tan x \sec^{n-2} x + (n-2)I_{n-2}$$

(6)

(b) Hence, showing each step of your working, find the exact value of

$$\int_0^{\frac{\pi}{4}} \sec^6 x \, dx$$

(4)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 10 marks)

Leave blank

6. The line l_1 has equation

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{k})$$

and the line l_2 has equation

$$\text{r} = 2\mathbf{i} + s\mathbf{j} + \mu(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

where s is a constant and λ and μ are scalar parameters.

Given that l_1 and l_2 both lie in a common plane Π_1

(a) show that an equation for Π_1 is $3x + y - z = 3$

(4)

(b) find the value of s.

(1)

The plane Π_2 has equation $\mathbf{r}.(\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 3$

(c) Find an equation for the line of intersection of Π_1 and Π_2

(4)

(d) Find the acute angle between Π_1 and Π_2 giving your answer in degrees to 3 significant figures.

(4)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 13 marks)

Leave blank

7. Using calculus, find the exact values of

(i) $$\int_1^2 \frac{1}{x^2 - 4x + 5}\,\mathrm{d}x$$

(3)

(ii) $$\int_{\sqrt{3}}^3 \frac{\sqrt{x^2 - 3}}{x^2}\,\mathrm{d}x$$

(5)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 8 marks)

Leave blank

8. The hyperbola H has equation

$$4x^2 - y^2 = 4$$

(a) Write down the equations of the asymptotes of H.

(1)

(b) Find the coordinates of the foci of H.

(2)

The point $P(\sec\theta, 2\tan\theta)$ lies on H.

(c) Using calculus, show that the equation of the tangent to H at the point P is

$$y\tan\theta = 2x\sec\theta - 2$$

(4)

The point $V(-1, 0)$ and the point $W(1, 0)$ both lie on H.

The point $Q(\sec\theta, -2\tan\theta)$ also lies on H.

Given that P, Q, V and W are distinct points on H and that the lines VP and WQ intersect at the point S,

(d) show that, as θ varies, S lies on an ellipse with equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where a and b are integers to be found.

(7)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

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Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F3

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Leave blank

1. The curve C has equation

$$y = \frac{1}{2}\operatorname{arcosh}(2x) \qquad \frac{7}{2} \leqslant x \leqslant 13$$

Using calculus, determine the exact length of the curve C.

Give your answer in the form $p\sqrt{q}$, where p and q are constants to be found.

(6)

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Q1

(Total 6 marks)

Leave blank

2. Given that

$$\cosh y = x \qquad \text{and} \qquad y < 0$$

use the definition of $\cosh y$ in terms of exponential functions to prove that

$$y = \ln\left(x - \sqrt{x^2 - 1}\right)$$

(6)

Leave blank

Question 2 continued

Q2

(Total 6 marks)

Leave blank

3. The ellipse E has equation

$$\frac{x^2}{64} + \frac{y^2}{36} = 1$$

The line l is the normal to E at the point $P(8\cos\theta, 6\sin\theta)$.

(a) Using calculus, show that an equation for l is

$$4x\sin\theta - 3y\cos\theta = 14\sin\theta\cos\theta$$

(4)

The line l meets the x-axis at the point A and meets the y-axis at the point B.

The point M is the midpoint of AB.

(b) Determine a Cartesian equation for the locus of M as θ varies, giving your answer in the form $ax^2 + by^2 = c$ where a, b and c are integers.

(5)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 9 marks)

Leave blank

4. The matrix **M** is given by

$$\begin{pmatrix} 2 & 0 & -1 \\ k & 3 & 2 \\ -2 & 1 & k \end{pmatrix}$$

(a) Show that $\det \mathbf{M} = 5k - 10$

(2)

Given that $k \neq 2$

(b) find $\mathbf{M}^{-1}$ in terms of k.

(4)

The points $O(0, 0, 0)$, $A(4, -8, 3)$, $B(-2, 5, -4)$ and $C(4, -6, 8)$ are the vertices of a tetrahedron T.

The transformation represented by matrix **M** transforms T to a tetrahedron with volume 50

(c) Determine the possible values of k.

(5)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 11 marks)

Leave blank

5. The skew lines l_1 and l_2 have equations

$$l_1 : \mathbf{r} = (\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}) + \lambda(5\mathbf{i} + \mathbf{j})$$

and

$$l_2 : \mathbf{r} = (2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}) + \mu(8\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$$

where λ and μ are scalar parameters.

(a) Determine a vector that is perpendicular to both l_1 and l_2

(2)

(b) Determine an equation of the plane parallel to l_1 that contains l_2

(i) in the form $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

(1)

(ii) in the form $\mathbf{r}.\mathbf{n} = p$

(2)

(c) Determine the shortest distance between l_1 and l_2

Give your answer in simplest form.

(5)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 10 marks)

Leave blank

6.

$$I_n = \int_0^{\sqrt{\frac{\pi}{2}}} x^n \cos(x^2)\,dx \qquad n \geqslant 1$$

(a) Prove that, for $n \geqslant 5$

$$I_n = \frac{1}{2}\left(\frac{\pi}{2}\right)^{\frac{n-1}{2}} - \frac{1}{4}(n-1)(n-3)I_{n-4}$$

(6)

(b) Hence, determine the exact value of I_5, giving your answer in its simplest form.

(3)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 9 marks)

Leave blank

7. A hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{25} = 1$$

where a is a positive constant.

The eccentricity of H is e.

(a) Determine an expression for e^2 in terms of a.

(1)

The line l is the directrix of H for which $x > 0$

The points A and A' are the points of intersection of l with the asymptotes of H.

(b) Determine, in terms of e, the length of the line segment AA'.

(3)

The point F is the focus of H for which $x < 0$

Given that the area of triangle AFA' is $\frac{164}{3}$

(c) show that a is a solution of the equation

$$30a^3 - 164a^2 + 375a - 4100 = 0$$

(4)

(d) Hence, using algebra and making your reasoning clear, show that the only possible value of a is $\frac{20}{3}$

(3)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 11 marks)

Leave blank

8.

$$y = \arccos\left(2\sqrt{x}\right)$$

(a) Determine $\dfrac{dy}{dx}$

(3)

(b) Show that

$$\int y\,dx = x\arccos\left(2\sqrt{x}\right) + \int \frac{\sqrt{x}}{\sqrt{1-4x}}\,dx$$

(2)

(c) Use the substitution $\sqrt{x} = \dfrac{1}{2}\cos\theta$ to show that

$$\int_0^{\frac{1}{8}} \frac{\sqrt{x}}{\sqrt{1-4x}}\,dx = \frac{1}{4}\int_a^b \cos^2\theta\,d\theta$$

where a and b are limits to be determined.

(4)

(d) Hence, determine the exact value of

$$\int_0^{\frac{1}{8}} \arccos\left(2\sqrt{x}\right)dx$$

(4)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | | | | | | Candidate Number | | | | |

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Leave blank

1 (a) Use the definitions of hyperbolic functions in terms of exponentials to prove that

$$8\cosh^4 x = \cosh 4x + p\cosh 2x + q$$

where p and q are constants to be determined.

(3)

(b) Hence, or otherwise, solve the equation

$$\cosh 4x - 17\cosh 2x + 9 = 0$$

giving your answers in exact simplified form in terms of natural logarithms.

(5)

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Q1

(Total 8 marks)

Leave blank

2.

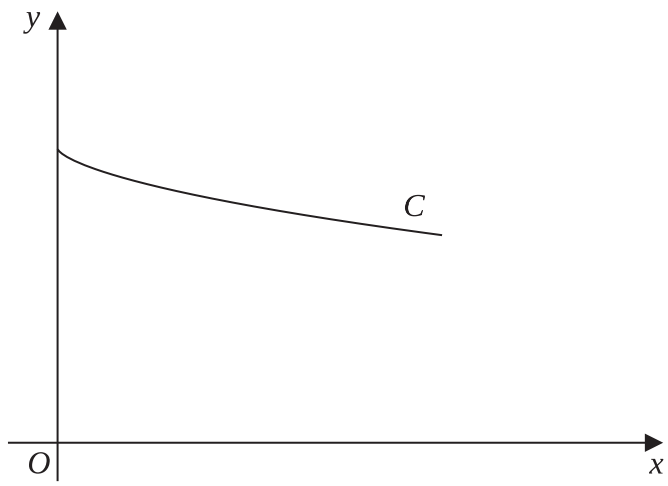

Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = \ln(\sec\theta + \tan\theta) - \sin\theta \qquad y = \cos\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{4}$$

The curve C is rotated through 2π radians about the x-axis and is used to form a solid of revolution S.

Using calculus, show that the **total** surface area of S is given by

$$\frac{\pi}{2}\left(p + q\sqrt{2}\right)$$

where p and q are integers to be determined.

(8)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 8 marks)

Leave blank

3. (a) Given that $y = \text{arsech}\left(\frac{x}{2}\right)$, where $0 < x \leqslant 2$, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{p}{x\sqrt{q - x^2}}$$

where p and q are constants to be determined.

(4)

In part (b) solutions based entirely on calculator technology are not acceptable.

$$\mathrm{f}(x) = \text{artanh}(x) + \text{arsech}\left(\frac{x}{2}\right) \qquad 0 < x \leqslant 1$$

(b) Determine, in simplest form, the exact value of x for which $\mathrm{f}'(x) = 0$

(5)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 9 marks)

Leave blank

4. $$\mathbf{M} = \begin{pmatrix} 6 & k & 2 \\ k & 5 & 0 \\ 2 & 0 & 7 \end{pmatrix}$$

where k is a constant.

Given that 3 is an eigenvalue of **M**,

(a) determine the possible values of k.

(3)

Given that $k < 0$

(b) determine the other eigenvalues of **M**.

(3)

(c) Determine a normalised eigenvector corresponding to the eigenvalue 3

(3)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 9 marks)

Leave blank

5. Determine

(i) $$\int \frac{1}{\sqrt{x^2 - 3x + 5}}\,\mathrm{d}x$$

(3)

(ii) $$\int \frac{1}{\sqrt{63 + 4x - 4x^2}}\,\mathrm{d}x$$

(4)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 7 marks)

Leave blank

6. $$I_n = \int e^x \sin^n x \, dx \qquad n \in \mathbb{Z} \qquad n \geqslant 0$$

(a) Show that

$$I_n = \frac{e^x \sin^{n-1} x}{n^2 + 1}(\sin x - n\cos x) + \frac{n(n-1)}{n^2+1} I_{n-2} \qquad n \geqslant 2$$

(6)

(b) Hence find the exact value of

$$\int_0^{\frac{\pi}{2}} e^x \sin^4 x \, dx$$

giving your answer in the form $Ae^{\frac{\pi}{2}} + B$ where A and B are rational numbers to be determined.

(4)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 10 marks)

Leave blank

7. The line l_1 has equation

$$\frac{x-3}{4} = \frac{y-5}{-2} = \frac{z-4}{7}$$

The plane Π has equation

$$2x + 4y - z = 1$$

The line l_1 intersects the plane Π at the point P

(a) Determine the coordinates of P

(3)

The acute angle between l_1 and Π is θ degrees.

(b) Determine, to one decimal place, the value of θ

(3)

The line l_2 lies in Π and passes through P

Given that the acute angle between l_1 and l_2 is also θ degrees,

(c) determine a vector equation for l_2

(5)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 11 marks)

Leave blank

8. The ellipse E has equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

(a) Determine the eccentricity of E

(2)

(b) Hence, for this ellipse, determine

(i) the coordinates of the foci,

(ii) the equations of the directrices.

(2)

The point P lies on E and has coordinates $(3\cos\theta, 2\sin\theta)$.

The line l_1 is the tangent to E at the point P

(c) Using calculus, show that an equation for l_1 is

$$2x\cos\theta + 3y\sin\theta = 6$$

(3)

The line l_2 passes through the origin and is perpendicular to l_1

The line l_1 intersects the line l_2 at the point Q

(d) Determine the coordinates of Q

(3)

(e) Show that, as θ varies, the point Q lies on the curve with equation

$$(x^2 + y^2)^2 = \alpha x^2 + \beta y^2$$

where α and β are constants to be determined.

(3)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
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- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. (a) Use the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials to show that

$$\cosh A \cosh B + \sinh A \sinh B \equiv \cosh(A + B)$$

(2)

(b) Hence find the value of x for which

$$\cosh(x + \ln 2) = 5 \sinh x$$

giving your answer in the form $\frac{1}{2} \ln k$, where k is a rational number to be determined.

(5)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 7 marks)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Determine

$$\int \frac{1}{\sqrt{5 + 4x - x^2}}\,\mathrm{d}x$$

(3)

(ii) Use the substitution $x = 3\sec\theta$ to determine the exact value of

$$\int_{2\sqrt{3}}^{6} \frac{18}{(x^2 - 9)^{\frac{3}{2}}}\,\mathrm{d}x$$

Give your answer in the form $A + B\sqrt{3}$ where A and B are constants to be found.

(6)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3.

$$\mathbf{M} = \begin{pmatrix} -2 & 5 & 0 \\ 5 & 1 & -3 \\ 0 & -3 & 6 \end{pmatrix}$$

Given that $\mathbf{i} + \mathbf{j} + \mathbf{k}$ is an eigenvector of **M**,

(a) determine the corresponding eigenvalue.

(1)

Given that 8 is an eigenvalue of **M**,

(b) determine a corresponding eigenvector.

(2)

(c) Determine a diagonal matrix **D** and an orthogonal matrix **P** such that

$$\mathbf{D} = \mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P}$$

(5)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 8 marks)

4.

$$y = \text{artanh}\left(\frac{\cos x + a}{\cos x - a}\right)$$

where a is a non-zero constant.

Show that

$$\frac{dy}{dx} = k \tan x$$

where k is a constant to be determined.

(4)

Question 4 continued

(Total for Question 4 is 4 marks)

5. A curve has parametric equations

$$x = 4e^{\frac{1}{2}t} \qquad y = e^{t} - t \qquad 0 \leqslant t \leqslant 4$$

The curve is rotated through 2π radians about the x-axis.

Show that the area of the curved surface generated is

$$\pi(e^{8} + Ae^{4} + B)$$

where A and B are constants to be determined.

(7)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 7 marks)

6.

$$\mathbf{A} = \begin{pmatrix} x & 1 & 3 \\ 2 & 4 & x \\ -4 & -2 & -1 \end{pmatrix}$$

(a) Show that **A** is non-singular for all real values of x.

(4)

(b) Determine, in terms of x, $\mathbf{A}^{-1}$

(4)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 8 marks)

7. $$I_n = \int \frac{x^n}{\sqrt{10 - x^2}}\,\mathrm{d}x \qquad n \in \mathbb{N} \qquad |x| < \sqrt{10}$$

(a) Show that

$$nI_n = 10(n-1)I_{n-2} - x^{n-1}(10 - x^2)^{\frac{1}{2}} \qquad n \geqslant 2$$

(6)

(b) Hence find the exact value of

$$\int_0^1 \frac{x^5}{\sqrt{10 - x^2}}\,\mathrm{d}x$$

giving your answer in the form $\frac{1}{15}\left(p\sqrt{10} + q\right)$ where p and q are integers to be determined.

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

8. The plane Π has equation

$$3x + 4y - z = 17$$

The line l_1 is perpendicular to Π and passes through the point $P(-4, -5, 3)$

The line l_1 intersects Π at the point Q

(a) Determine the coordinates of Q

(4)

Given that the point $R(-1, 6, 4)$ lies on Π

(b) determine a Cartesian equation of the plane containing PQR

(4)

The line l_2 passes through P and R

The line l_3 is the reflection of l_2 in Π

(c) Determine a vector equation for l_3

(4)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 12 marks)

9. The ellipse E has equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

The line l has equation $y = kx - 3$, where k is a constant.

Given that E and l meet at 2 distinct points P and Q

(a) show that the x coordinates of P and Q are solutions of the equation

$$(9k^2 + 4)x^2 - 54kx + 45 = 0$$

(2)

The point M is the midpoint of PQ

(b) Determine, in simplest form in terms of k, the coordinates of M

(3)

(c) Hence show that, as k varies, M lies on the curve with equation

$$x^2 + py^2 = qy$$

where p and q are constants to be determined.

(5)

Question 9 continued

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

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- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Given that

$$y = 3x \arcsin 2x \qquad 0 \leqslant x \leqslant \frac{1}{2}$$

(a) determine an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$

(2)

(b) Hence determine the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = \frac{1}{4}$, giving your answer in the form $a\pi + b$ where a and b are fully simplified constants to be found.

(1)

Question 1 continued

(Total for Question 1 is 3 marks)

2. A hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{5} = 1 \qquad \text{where } a \text{ is a positive constant}$$

The line with equation $x = \frac{4}{3}$ is a directrix of H

(a) Write down an equation of the other directrix.

(1)

(b) Determine

(i) the value of a

(ii) the coordinates of each of the foci of H

(5)

Question 2 continued

(Total for Question 2 is 6 marks)

3. Solve the equation

$$4\tanh x - \operatorname{sech} x = 1$$

giving your answer in the form $x = \ln k$ where k is a fully simplified rational number.

(6)

Question 3 continued

(Total for Question 3 is 6 marks)

4. (a) Determine

$$\int \frac{1}{\sqrt{9x^2 + 16}}\, \mathrm{d}x$$

(2)

(b) Hence determine the exact value of

$$\int_{-2}^{2} \frac{1}{\sqrt{9x^2 + 16}}\, \mathrm{d}x$$

Give your answer in the form $a \ln(b + c\sqrt{13})$, where a, b and c are rational numbers.

(3)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 5 marks)

5.

$$\mathbf{A} = \begin{pmatrix} a & a & 1 \\ -a & 4 & 0 \\ 4 & a & 5 \end{pmatrix} \quad \text{where } a \text{ is a positive constant}$$

(a) Determine the exact value of a for which the matrix $\mathbf{A}$ is singular.

(2)

Given that 2 is an eigenvalue of $\mathbf{A}$

(b) determine

(i) the value of a

(ii) the other two eigenvalues of $\mathbf{A}$

(5)

A normalised eigenvector for the eigenvalue 2 is $\begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \end{pmatrix}$

(c) Determine a normalised eigenvector for each of the other eigenvalues of $\mathbf{A}$

(5)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

6. A curve has parametric equations

$$x = a(\theta - \sin\theta)$$
$$y = a(1 - \cos\theta)$$

where a is a positive constant.

(a) Show that

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = ka^2 \sin^2 \frac{\theta}{2}$$

where k is a constant to be determined.

(4)

The part of the curve from $\theta = 0$ to $\theta = 2\pi$ is rotated through 2π radians about the x-axis.

(b) Determine the area of the surface generated, giving your answer in terms of π and a.

[*Solutions relying on calculator technology are not acceptable.*]

(5)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Determine a vector perpendicular to Π

(2)

The line l meets Π at the point (1, 2, 3) and passes through the point (1, 0, 1)

(b) Determine the size of the acute angle between Π and l

Give your answer to the nearest degree.

(4)

(c) Determine the shortest distance between Π and the point (6, −3, −6)

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

8.

$$I_n = \int \cos^n x \, \mathrm{d}x \qquad n \geqslant 0$$

(a) Prove that for $n \geqslant 2$

$$I_n = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$$

(4)

(b) Show that for positive even integers n

$$\int_0^{\frac{\pi}{2}} \cos^n x \, \mathrm{d}x = \frac{(n-1)(n-3)\ldots 5 \times 3 \times 1}{n(n-2)(n-4)\ldots 6 \times 4 \times 2} \times \frac{\pi}{2}$$

(4)

(c) Hence determine the exact value of

$$\int_0^{\frac{\pi}{2}} \cos^6 x \sin^2 x \, \mathrm{d}x$$

(3)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 11 marks)

9. The ellipse E has equation

$$x^2 + 9y^2 = 9$$

The foci of E are F_1 and F_2

(a) (i) Determine the coordinates of F_1 and the coordinates of F_2

(ii) Write down the equation of each of the directrices of E

(4)

The point P lies on the ellipse.

(b) Show that $|PF_1| + |PF_2| = 6$

(3)

The straight line through P with equation $y = 2x + c$ meets E again at the point Q

The point M is the midpoint of PQ

(c) Show that as P varies the locus of M is a straight line passing through the origin.

(6)

Question 9 continued

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 15 June 2023

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
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- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Solve the equation

$$7\cosh x + 3\sinh x = 2e^{x} + 7$$

Give your answers as simplified natural logarithms.

(5)

Question 1 continued

(Total for Question 1 is 5 marks)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$\mathbf{M} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 4 \\ 3 & -2 & -3 \end{pmatrix}$$

(a) Determine $\mathbf{M}^{-1}$

(3)

The transformation represented by **M** maps the plane Π_1 to the plane Π_2

The point (x, y, z) on Π_1 maps to the point (u, v, w) on Π_2

(b) Determine x, y and z in terms of u, v and w as appropriate.

(3)

The plane Π_1 has equation

$$3x - 7y + 2z = -3$$

(c) Find a Cartesian equation for Π_2

Give your answer in the form $au + bv + cw = d$ where a, b, c and d are integers to be determined.

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 8 marks)

3.

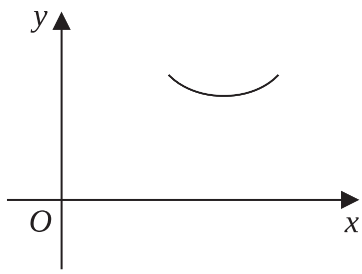

Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{1}{2}\left(\tan x + \cot x\right) \qquad \frac{\pi}{6} \leqslant x \leqslant \frac{\pi}{3}$$

(a) Show that the length of C is given by

$$\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\tan^2 x + \cot^2 x\right) \mathrm{d}x$$

(6)

(b) Hence determine the exact length of C, giving your answer in simplest form.

(5)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 11 marks)

4. The plane Π_1 contains the point $A(2, 4, -5)$ and is normal to the vector $\begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix}$

The plane Π_2 contains the point $B(3, 6, -2)$ and is normal to the vector $\begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix}$

The line l is the line of intersection of Π_1 and Π_2

(a) Determine a vector equation for l.

(7)

The points C and D both lie on l.

Given that C and D are 5 units apart,

(b) determine the exact volume of the tetrahedron *ABCD*.

(5)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 12 marks)

5.

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & k \\ -1 & -3 & 4 \\ 2 & 6 & -8 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

Given that **M** has a repeated eigenvalue, determine

(i) the possible values of k,

(ii) all corresponding eigenvalues of **M** for each value of k.

(7)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 7 marks)

6. The ellipse E has equation $\frac{x^2}{16} + \frac{y^2}{9} = 1$

The point $P(4\cos\theta, 3\sin\theta)$ lies on E.

(a) Use calculus to show that an equation of the tangent to E at P is

$$3x\cos\theta + 4y\sin\theta = 12$$

(5)

(b) Determine an equation for the normal to E at P.

(2)

The tangent to E at P meets the x-axis at the point A.
The normal to E at P meets the y-axis at the point B.

(c) Show that the locus of the midpoint of A and B as θ varies has equation

$$x^2(p - qy^2) = r$$

where p, q and r are integers to be determined.

(6)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 13 marks)

7. $$I_n = \int \cosh^n 2x \, dx \qquad n \geqslant 0$$

(a) Show that, for $n \geqslant 2$

$$I_n = \frac{\cosh^{n-1} 2x \sinh 2x}{2n} + \frac{n-1}{n} I_{n-2}$$

(5)

(b) Hence determine

$$\int (1 + \cosh 2x)^3 \, dx$$

collecting any like terms in your answer.

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)

8. (a) Differentiate $x \operatorname{arcosh} 5x$ with respect to x

(2)

(b) Hence, or otherwise, show that

$$\int_{\frac{1}{4}}^{\frac{3}{5}} \operatorname{arcosh} 5x \, dx = \frac{3}{20} - \frac{2\sqrt{2}}{5} + \ln\left(p + q\sqrt{2}\right)^k - \frac{1}{4}\ln r$$

where p, q, r and k are rational numbers to be determined.

(8)

Question 8 continued

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Monday 22 January 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Show that

$$\int_{4}^{4\sqrt{3}} \frac{8}{16 + x^2}\,\mathrm{d}x = p\pi$$

where p is a rational number to be determined.

(3)

(ii) Determine the exact value of k for which

$$\int_{\frac{3}{4}}^{k} \frac{2}{\sqrt{9 - 4x^2}}\,\mathrm{d}x = \frac{\pi}{12}$$

(4)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 7 marks)

2.

$$\mathbf{T} = \begin{pmatrix} 2 & 3 & 7 \\ 3 & 2 & 6 \\ a & 4 & b \end{pmatrix} \qquad \mathbf{U} = \begin{pmatrix} 6 & -1 & -4 \\ 15 & c & -9 \\ -8 & a & 5 \end{pmatrix}$$

where a, b and c are constants.

Given that $\mathbf{TU} = \mathbf{I}$

(a) determine the value of a, the value of b and the value of c

(4)

The transformation represented by the matrix $\mathbf{T}$ transforms the line l_1 to the line l_2

Given that l_2 has equation

$$\frac{x-1}{3} = \frac{y}{-4} = z + 2$$

(b) determine a Cartesian equation for l_1

(4)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 8 marks)

3. The ellipse E has equation

$$\frac{x^2}{49} + \frac{y^2}{b^2} = 1$$

where b is a constant and $0 < b < 7$

The eccentricity of the ellipse is e

(a) Write down, in terms of e only,

(i) the coordinates of the foci of E

(ii) the equations of the directrices of E

(2)

Given that

- the point $P(x, y)$ lies on E where $x > 0$
- the point S is the focus of E on the positive x-axis
- the line l is the directrix of E which crosses the positive x-axis
- the point M lies on l such that the line through P and M is parallel to the x-axis

(b) determine an expression for

(i) PS^2 in terms of e, x and y

(ii) PM^2 in terms of e and x

(2)

(c) Hence show that

$$b^2 = 49\left(1 - e^2\right)$$

(2)

Given that E crosses the y-axis at the points with coordinates $\left(0, \pm 4\sqrt{3}\right)$

(d) determine the value of e

(2)

Given that the x coordinate of P is $\frac{7}{2}$

(e) determine the area of triangle OPM, where O is the origin.

(3)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 11 marks)

4.

$$\mathbf{M} = \begin{pmatrix} 0 & -1 & 3 \\ -1 & 4 & -1 \\ 3 & -1 & 0 \end{pmatrix}$$

Given that $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ is an eigenvector of **M**

(a) determine its corresponding eigenvalue.

(2)

Given that –3 is an eigenvalue of **M**

(b) determine a corresponding eigenvector.

(2)

Hence, given that $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is also an eigenvector of **M**

(c) determine a diagonal matrix **D** and an orthogonal matrix **P** such that $\mathbf{D} = \mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P}$

(4)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

5. (a) Use the definitions of hyperbolic functions in terms of exponentials to prove that

$$1 - \operatorname{sech}^2 x \equiv \tanh^2 x$$

(3)

$$I_n = \int_0^{\frac{1}{3}\ln 2} \tanh^n 3x \, dx \qquad n \in \mathbb{Z} \qquad n \geqslant 0$$

(b) Show that

$$I_n = I_{n-2} - \frac{p^{n-1}}{3(n-1)} \qquad n \geqslant 2$$

where p is a rational number to be determined.

(4)

(c) Hence determine the exact value of

$$\int_0^{\frac{1}{3}\ln 2} \tanh^5 3x \, dx$$

giving your answer in the form $a \ln b + c$ where a, b and c are rational numbers to be found.

(4)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 11 marks)

6. The points A, B and C have coordinates (3, 2, 2), (–1, 1, 3) and (–2, 4, 2) respectively.

The plane Π_1 contains the points A, B and C

(a) Determine a Cartesian equation of Π_1

(4)

Given that

- point D has coordinates (–1, 1, –2)
- line l passes through D and is perpendicular to Π_1
- plane Π_2 has equation $\mathbf{r}.(14\mathbf{i} - \mathbf{j} - 17\mathbf{k}) = -66$
- l meets Π_2 at the point E

(b) show that $DE = p\sqrt{22}$ where p is a rational number to be determined.

(5)

The point F has coordinates (4, 3, q) where q is a constant.

Given that A, B, C and F are the vertices of a tetrahedron of volume 12

(c) determine the possible values of q

(3)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 12 marks)

7.

$$y = \arccos(\operatorname{sech} x) \qquad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \operatorname{sech} x$$

(3)

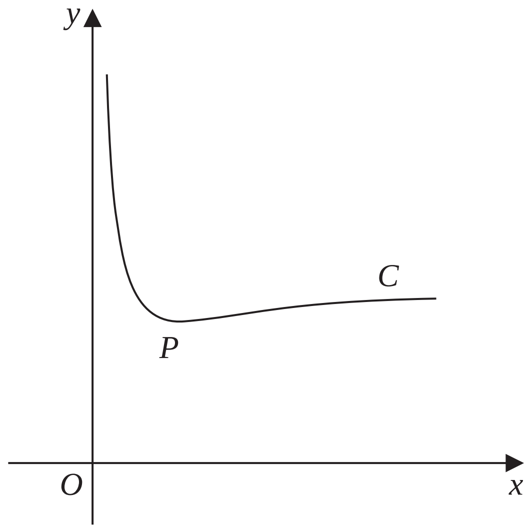

Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = \arccos(\operatorname{sech} x) + \coth x \qquad x > 0$$

The point P is a minimum turning point of C

(b) Show that the x coordinate of P is $\ln\left(q + \sqrt{q}\right)$ where $q = \frac{1}{2}\left(1 + \sqrt{k}\right)$ and k is an integer to be determined.

(6)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

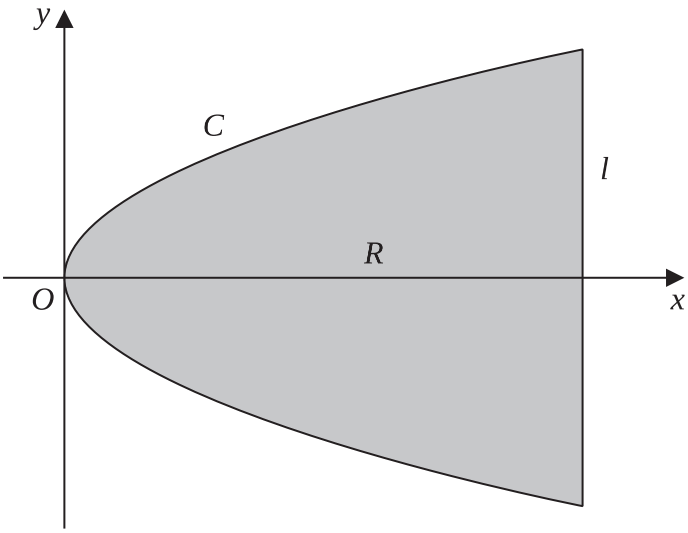

Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y^2 = 8x$ and part of the line l with equation $x = 18$

The region R, shown shaded in Figure 2, is bounded by C and l

(a) Show that the perimeter of R is given by

$$\alpha + 2\int_0^{\beta} \sqrt{1 + \frac{y^2}{16}}\, dy$$

where α and β are positive constants to be determined.

(3)

(b) Use the substitution $y = 4\sinh u$ and algebraic integration to determine the exact perimeter of R, giving your answer in simplest form.

(6)

Question 8 continued

Question 8 continued

(Total for Question 8 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 11 June 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

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- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. The hyperbola H has

- foci with coordinates $\left(\pm\frac{13}{2}, 0\right)$
- directrices with equations $x = \pm\frac{72}{13}$
- eccentricity e

Determine

(a) the value of e

(3)

(b) an equation for H, giving your answer in the form $px^2 - qy^2 = r$, where p, q and r are integers.

(3)

Question 1 continued

(Total for Question 1 is 6 marks)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$\mathbf{M} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & -4 & -3 \\ 0 & -4 & 0 \end{pmatrix}$$

Given that **M** has exactly two distinct eigenvalues λ_1 and λ_2 where $\lambda_1 < \lambda_2$

(a) determine a normalised eigenvector corresponding to the eigenvalue λ_1

(6)

The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$, where μ is a scalar parameter.

The transformation T is represented by **M**.

The line l_1 is transformed by T to the line l_2

(b) Determine a vector equation for l_2, giving your answer in the form $\mathbf{r} \times \mathbf{b} = \mathbf{c}$ where **b** and **c** are constant vectors.

(3)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3. $$y = \text{arsinh}\left(\sqrt{x^2 - 1}\right) \qquad x > 1$$

(a) Prove that $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{x^2 - 1}}$

(3)

$$f(x) = \frac{1}{3}\text{arsinh}\left(\sqrt{x^2 - 1}\right) - \arctan x \qquad x > 1$$

(b) Determine the exact values of x for which $f'(x) = 0$

(4)

Question 3 continued

(Total for Question 3 is 7 marks)

4. (a) Use the definitions of hyperbolic functions in terms of exponentials to show that

$$\sinh(A + B) \equiv \sinh A \cosh B + \cosh A \sinh B$$

(3)

(b) Hence express $10\sinh x + 8\cosh x$ in the form $R\sinh(x + \alpha)$ where $R > 0$, giving α in the form $\ln p$ where p is an integer.

(4)

(c) Hence solve the equation

$$10\sinh x + 8\cosh x = 18\sqrt{7}$$

giving your answer in the form $\ln\left(\sqrt{7} + q\right)$ where q is a rational number to be determined.

(2)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. $$4x^2 + 4x + 17 \equiv (2x + p)^2 + q$$

where p and q are integers.

(a) Determine the value of p and the value of q

(2)

Given that

$$\frac{8x+5}{\sqrt{4x^2+4x+17}} \equiv \frac{1}{\sqrt{4x^2+4x+17}} + \frac{Ax+B}{\sqrt{4x^2+4x+17}}$$

where A and B are integers,

(b) write down the value of A and the value of B

(1)

(c) Hence use algebraic integration to show that

$$\int_{\frac{1}{3}}^{1} \frac{8x+5}{\sqrt{4x^2+4x+17}}\,dx = k + \frac{1}{2}\ln k$$

where k is a rational number to be determined.

(5)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 8 marks)

6. The ellipse E has equation

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

The line l is the normal to E at the point $P(5\cos\theta, 3\sin\theta)$ where $0 < \theta < \frac{\pi}{2}$

(a) Using calculus, show that an equation for l is

$$5x\sin\theta - 3y\cos\theta = 16\sin\theta\cos\theta$$

(4)

Given that

- l intersects the y-axis at the point Q
- the midpoint of the line segment PQ is M

(b) determine the exact maximum area of triangle OMP as θ varies, where O is the origin.

You must justify your answer.

(5)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

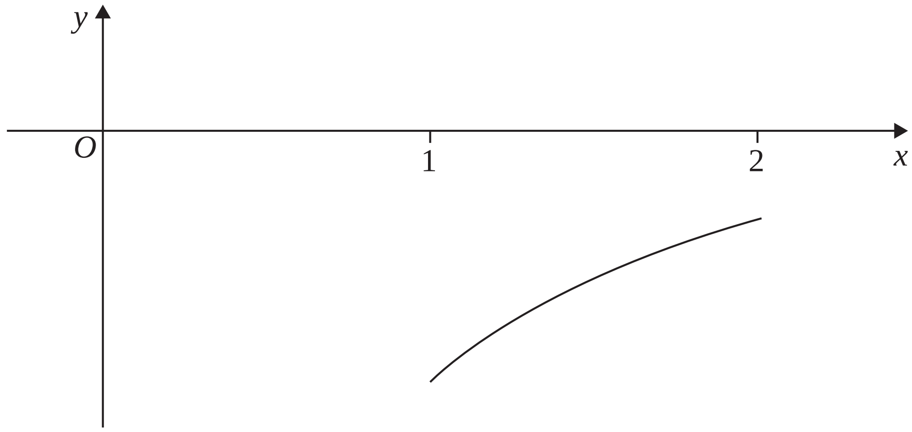

Figure 1

Figure 1 shows the curve with equation

$$y = \ln\left(\tanh\frac{x}{2}\right) \qquad 1 \leqslant x \leqslant 2$$

(a) Show that the length, s, of the curve is given by

$$s = \int_1^2 \coth x \, dx$$

(4)

(b) Hence show that

$$s = \ln\left(e + \frac{1}{e}\right)$$

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

8. $$I_n = \int_0^k x^n (k - x)^{\frac{1}{2}}\, dx \qquad n \geqslant 0$$

where k is a positive constant.

(a) Show that

$$I_n = \frac{2kn}{3 + 2n} I_{n-1} \qquad n \geqslant 1$$

(5)

Given that

$$\int_0^k x^2 (k - x)^{\frac{1}{2}}\, dx = \frac{9\sqrt{3}}{280}$$

(b) use the result in part (a) to determine the exact value of k.

(4)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 9 marks)

9. The plane Π_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix} + s\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

where s and t are scalar parameters.

(a) Determine a Cartesian equation for Π_1

(3)

The plane Π_2 has vector equation $\mathbf{r}.\begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} = 1$

(b) Determine a vector equation for the line of intersection of Π_1 and Π_2

Give your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where **a** and **b** are constant vectors and λ is a scalar parameter.

(4)

The plane Π_3 has Cartesian equation $4x - 3y - z = 0$

(c) Use the answer to part (b) to determine the coordinates of the point of intersection of Π_1, Π_2 and Π_3

(3)

Question 9 continued

Question 9 continued

(Total for Question 9 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS