Question Number	Scheme	Notes	Marks
1	**Notes:** • Some candidates omit brackets around their matrices and vectors. This is condoned for all marks. • If a correct answer comes from no working in part (a) or part (b), please send to review. $\mathbf{M}=\begin{pmatrix}-5 & 3\\ 3 & 3\end{pmatrix}$		
(a)	$\{\det(\mathbf{M}-\lambda\mathbf{I})=\}\quad (-5-\lambda)(3-\lambda)-(3)(3)$	Completes an attempt at $\det(\mathbf{M}-\lambda\mathbf{I})$ with one product correct	M1
	$\Rightarrow \lambda^2+2\lambda-24=0 \Rightarrow (\lambda+6)(\lambda-4)=0 \Rightarrow \lambda=-6,\ 4$	M1: Forms and solves 3TQ (usual rules) A1: Both correct eigenvalues	M1 A1
			(3)
(b)	$\begin{pmatrix}-5 & 3\\ 3 & 3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}="-6"\begin{pmatrix}x\\ y\end{pmatrix}\Rightarrow \begin{matrix}-5x+3y="-6"x\\ 3x+3y="-6"y\end{matrix}\Rightarrow x=\ldots,\ y=\ldots$; or $\begin{pmatrix}-5-"-6" & 3\\ 3 & 3-"-6"\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}\Rightarrow x=\ldots,\ y=\ldots$ Uses $\mathbf{Mx}=\lambda\mathbf{x}$ or $(\mathbf{M}-\lambda\mathbf{I})\mathbf{x}=0$ with one of their eigenvalues leading to either $x=\ldots, y=\ldots$ or an eigenvector. Accept any x and y values following their matrix equation. Do not be concerned with the mechanics of their solution.		M1
	$\lambda=-6\{\Rightarrow x=-3y\}\Rightarrow\begin{pmatrix}-3\\ 1\end{pmatrix}$, or $\lambda=4\{\Rightarrow 3x=y\}\Rightarrow\begin{pmatrix}1\\ 3\end{pmatrix}$	One correct eigenvector (any equivalents). Allow if a general form is given, e.g. $k\begin{pmatrix}1\\ 3\end{pmatrix}$, or if only normalised eigenvectors are given.	A1
	$\dfrac{1}{\sqrt{10}}\begin{pmatrix}-3\\ 1\end{pmatrix}$ and $\dfrac{1}{\sqrt{10}}\begin{pmatrix}1\\ 3\end{pmatrix}$ oe e.g. $\begin{pmatrix}\frac{3\sqrt{10}}{10}\\ -\frac{\sqrt{10}}{10}\end{pmatrix}$ and $\begin{pmatrix}-\frac{\sqrt{10}}{10}\\ -\frac{3\sqrt{10}}{10}\end{pmatrix}$	Both correct normalised eigenvectors (either direction). Must be numerical. The normalised eigenvectors must be seen in part (b). Do not allow for correct columns in their **P** in part (c).	A1
			(3)
(c)	$\{\mathbf{D}=\}\begin{pmatrix}"-6" & 0\\ 0 & "4"\end{pmatrix}$	Correct **D** ft their eigenvalues	B1ft
	$\{\mathbf{P}=\}\ \dfrac{1}{\sqrt{10}}\begin{pmatrix}-3 & 1\\ 1 & 3\end{pmatrix}$ or e.g., $\begin{pmatrix}\frac{3\sqrt{10}}{10} & -\frac{\sqrt{10}}{10}\\ -\frac{\sqrt{10}}{10} & -\frac{3\sqrt{10}}{10}\end{pmatrix}$	Correct **P** and **D**. The position of the two columns of **P** must correspond correctly with the position of their eigenvalues in **D**.	B1
			Total 8

Question Number	Scheme	Notes	Marks
2	$\dfrac{x^2}{64}+\dfrac{y^2}{16}=1$		
(a)	$a=8,\ b=4$ $b^2=a^2\left(1-e^2\right)\Rightarrow 16=64\left(1-e^2\right)\Rightarrow e=...$	Uses any correct eccentricity formula and solves for e	M1
	$e=\dfrac{\sqrt{3}}{2}$	Correct value. $\pm\dfrac{\sqrt{3}}{2}$ is A0	A1
			(2)
(b)	$\dfrac{a}{e}-ae=\dfrac{"8"}{"\frac{\sqrt{3}}{2}"}-"8"\times"\dfrac{\sqrt{3}}{2}"$	Uses $\dfrac{a}{e}-ae$ (or equivalent work) to obtain a numerical value. Their values of $\dfrac{a}{e}$ and ae must be seen separately. Subtraction may be implied by answer. Condone subtraction the wrong way round.	M1
	$=\dfrac{16\sqrt{3}}{3}-4\sqrt{3}=\dfrac{4\sqrt{3}}{3}$ or $\dfrac{4}{\sqrt{3}}$	Correct distance in simplest form. Mark can be given if subtraction was the wrong way round and answer subsequently made positive.	A1
			(2)
(c)	P is $(8\cos\theta,\ 4\sin\theta)$	Correct coordinates. May be implied.	B1
	M is $\left(\dfrac{8\cos\theta+14}{2},\ 4\sin\theta\right)$	Correct coordinates for M (following through on their P) in terms of θ	B1ft
	$x=4\cos\theta+7\Rightarrow\cos\theta=\dfrac{x-7}{4}$ $y=4\sin\theta\Rightarrow\sin\theta=\dfrac{y}{4}$ $\cos^2\theta+\sin^2\theta=1\Rightarrow\left(\dfrac{x-7}{4}\right)^2+\left(\dfrac{y}{4}\right)^2=1$ $\left\{(x-7)^2+y^2=16\right\}$	M1: Using their M, whose x and y coordinates are linear functions of $\cos\theta$ and $\sin\theta$ respectively, uses a valid algebraic method to eliminate θ to obtain an equation in x and y only. Usually this will involve use of $\cos^2\theta+\sin^2\theta=1$, but there are other valid ways to eliminate θ, so check working carefully. A1: Correct equation of circle with the same denominator for the x and y terms	M1A1
Alt	Consider $M(X, Y)$ $X=\dfrac{14+x}{2}, Y=y$ (where (x,y) are coordinates of P) $x=2X-14, y=Y$ Since P is on the ellipse: $\dfrac{(2X-14)^2}{64}+\dfrac{Y^2}{16}=1\Rightarrow\dfrac{4(X-7)^2}{64}+\dfrac{Y^2}{16}=1$ $\Rightarrow\dfrac{(X-7)^2}{16}+\dfrac{Y^2}{16}=1$ $\left\{(X-7)^2+Y^2=16\right\}$	B1: Correct coordinates of M in terms of the Cartesian coordinates of P. May be implied. B1ft: Finds x and y (the coordinates of P) in terms of X and Y (the coordinates of M). Follow through on their equations for X and Y. M1: Substitutes their expressions for x and y (in terms of the coordinates of M) into the equation of the ellipse. A1: Correct equation of circle with the same denominator for the x and y terms	B1 B1ft M1A1
			(4)

(d)	radius $= 4$, centre $(7, 0)$	**Note:** For both marks, follow through on their equation of a circle, which must have come from a genuine attempt at part (c). If this is unclear, please send to review. B1ft: Correct radius or centre B1ft: Correct radius and centre	B1ft B1ft
			(2)
			Total 10

Question Number	Scheme	Notes	Marks
3(a)	**Note:** The question requires differentiation of the exponential form of $\operatorname{sech} x$. Responses using hyperbolic differentiation score no marks in part (a).		
	$\operatorname{sech} x = \dfrac{2}{e^x + e^{-x}}$ $\left\{= 2\left(e^x + e^{-x}\right)^{-1}\right\}$	States or uses a correct exponential definition for $\operatorname{sech} x$	M1
	$\dfrac{d}{dx}(\operatorname{sech} x) = -2\left(e^x + e^{-x}\right)^{-2}\left(e^x - e^{-x}\right)$	Attempts to differentiate an expression of the form $k\left(e^x \pm e^{-x}\right)^{-1}$ o.e. correctly using the product rule or quotient rule. Look for $\pm p\left(e^x \pm e^{-x}\right)^{-2}\left(e^x \mp e^{-x}\right)$ o.e.	M1
	$= -\dfrac{2}{e^x + e^{-x}} \times \dfrac{e^x - e^{-x}}{e^x + e^{-x}} = -\operatorname{sech} x \tanh x$ *	Obtains given answer with intermediate line and no errors. Must split the expression into separate fractions for the exponential forms of $\operatorname{sech} x$ and $\tanh x$ before reaching final answer.	A1*
	Note: A "meet in the middle" approach is possible. First M1 as above. For second M1: any acceptable differentiation of $\operatorname{sech} x$ (see above). Then, award A1* if $-\operatorname{sech} x \tanh x$ is expanded using exponential definitions to reach an identical expression, AND a conclusion is made, e.g. $\ldots = \dfrac{d}{dx}(\operatorname{sech} x)$, or "=LHS", "QED", etc. If this approach is used, LHS and RHS can be written in any correct exponential form via correct algebra, e.g. $\dfrac{2e^{-x} - 2e^x}{e^{2x} + 2 + e^{-2x}}$		
			(3)
(b)	$y = k\arctan\left(e^x\right) + \operatorname{sech} 2x \Rightarrow$ $\dfrac{dy}{dx} = \dfrac{ke^x}{1+\left(e^x\right)^2} - 2\operatorname{sech} 2x \tanh 2x$ o.e. (the second term may be in exponential form)	M1: Either term differentiated correctly A1: Fully correct differentiation. Note: it is possible to revert to the exponential form for $\operatorname{sech} x$ and differentiate the second term as in part (a).	M1 A1
	$x = \ln 3 \Rightarrow \dfrac{3k}{1+9} - 2 \times \dfrac{2}{9+\frac{1}{9}} \times \dfrac{9-\frac{1}{9}}{9+\frac{1}{9}} = 0 \Rightarrow k = \ldots$	Makes some attempt to substitute $x = \ln 3$ into their $\dfrac{dy}{dx}$, sets $= 0$ and proceeds to a value for k	M1
	$\dfrac{3k}{10} = \dfrac{720}{1681} \Rightarrow k = \dfrac{2400}{1681}$ or $1\dfrac{719}{1681}$	Correct exact value in simplest form	A1
			(4)
			Total 7

Question Number	Scheme	Notes	Marks
4	$x=4\cos^3\theta \quad y=4\sin^3\theta \quad 0 \leqslant \theta \leqslant k \quad 0<k<\dfrac{\pi}{2}$		
(a)	$\dfrac{dx}{d\theta}=-12\cos^2\theta\sin\theta$ $\dfrac{dy}{d\theta}=12\sin^2\theta\cos\theta$	M1: Both derivatives of the correct form. Look for $\dfrac{dx}{d\theta}=p\cos^2\theta\sin\theta$ and $\dfrac{dy}{d\theta}=q\sin^2\theta\cos\theta$ o.e. A1: Both correct derivatives. Be aware of alternative ways to differentiate, e.g. first writing $x=4\left(\cos^2\theta\right)\cos\theta$ and $y=4\left(\sin^2\theta\right)\sin\theta$. M1A1 can be given for correct derivatives, but it is unlikely further progress will be made.	M1 A1
	$S=2\pi\displaystyle\int_{\{0\}}^{\{k\}} y\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,\{d\theta\}$ $=2\pi\displaystyle\int_{\{0\}}^{\{k\}} 4\sin^3\theta\sqrt{144\cos^4\theta\sin^2\theta+144\sin^4\cos^2\theta}\,\{d\theta\}$	Uses a correct formula with their derivatives	M1
	$=96\pi\displaystyle\int_{\{0\}}^{\{k\}}\sin^3\theta\sqrt{\cos^2\theta\sin^2\theta\left(\cos^2\theta+\sin^2\theta\right)}\,\{d\theta\}$ $=96\pi\displaystyle\int_{\{0\}}^{\{k\}}\sin^3\theta\sqrt{\cos^2\theta\sin^2\theta}\,\{d\theta\}$ $\left\{=96\pi\displaystyle\int_{\{0\}}^{\{k\}}\sin^3\theta\cos\theta\sin\theta\,d\theta\right\}$	Uses $\cos^2\theta+\sin^2\theta=1$ oe at some point in the working to process the integrand (may be implied: see the note for the A* mark below)	M1
	$=96\pi\displaystyle\int_0^k\sin^4\theta\cos\theta\,d\theta$ *	Obtains the given answer with no errors. Must be at least one intermediate step following $96\pi\displaystyle\int_0^k\sin^3\theta\sqrt{\cos^2\theta\sin^2\theta\left(\cos^2\theta+\sin^2\theta\right)}\,d\theta$, possibly in correct side working. Proceeding from this expression directly to the given answer is M1 A0*. Condone the limits and/or $d\theta$ appearing for the first time in the final line. Do not condone invisible brackets at any stage, even if recovered, but do condone a missing trailing bracket.	A1*
			(5)

(b)	$S=\frac{96\pi}{5}\left[\sin^5\theta\right]_0^k$	Integrates to obtain $p\sin^5\theta$	M1
	$\frac{96\pi}{5}\left(\sin^5 k\right)=\frac{27\sqrt{3}}{5}\pi$ $\Rightarrow \sin^5 k=\frac{9\sqrt{3}}{32}$ $\Rightarrow \sin k=\ldots$	Substitutes k, sets equal to $\frac{27\sqrt{3}}{5}\pi$ and solves for sin k. **Note:** - Condone no working with the lower limit - Some candidates do not write down a value for $\sin k$, but go from $\sin^5 k$ to k. This scores M1 if their k is consistent with their $\sin^5 k$ - $k=\frac{\sqrt{3}}{2}$ o.e. is M0, unless there is recovery - This mark is not dependent on the first M1	M1
	$\sin k=\frac{\sqrt{3}}{2}$ o.e. or $0.866\ldots\Rightarrow k=\frac{\pi}{3}$ or awrt 1.05	Correct k	A1
			(3)
(c)	$s=\int_{\{0\}}^{\{\frac{\pi}{3}\}}\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,d\theta=\ldots$ $=12\int_{\{0\}}^{\{\frac{\pi}{3}\}}\cos\theta\sin\theta\,d\theta$	Attempts to use a correct formula with their derivatives. If no formula is quoted, condone an incorrect or missing multiplying constant, which should be 12. Ignore the limits for this mark.	M1
	$=-6\left[\cos^2\theta\right]_{\{0\}}^{\{\frac{\pi}{3}\}}$ or $6\left[\sin^2\theta\right]_{\{0\}}^{\{\frac{\pi}{3}\}}$ or $=6\int_{\{0\}}^{\{\frac{\pi}{3}\}}\sin 2\theta\,d\theta=-3\left[\cos 2\theta\right]_{\{0\}}^{\{\frac{\pi}{3}\}}$	Integrates to $\ldots\cos^2\theta$ or $\ldots\sin^2\theta$ or $\ldots\cos 2\theta$	M1
	$=-6\left(\frac{1}{4}-1\right)$ or $6\left(\frac{3}{4}-0\right)$ or $-3\left(-\frac{1}{2}-1\right)=\frac{9}{2}$	Correct exact value $\frac{9}{2}$, 4.5 or $4\frac{1}{2}$	A1
			(3)
			Total 11

Question Number	Scheme	Notes	Marks
5	$\mathbf{S}=\begin{pmatrix}1&0&p\\-1&2&3\\2&-2&0\end{pmatrix}$ $\mathbf{T}=\begin{pmatrix}2&1&-3\\0&-3&-1\\q&0&1\end{pmatrix}$		
(a)	$\det\mathbf{S}=1\left(0-(-2)(3)\right)-0+p\left((-1)(-2)-(2)(2)\right)$ $\det\mathbf{T}=2\left((-3)(1)-0\right)-1\left(0-(q)(-1)\right)-3\left(0-(q)(-3)\right)$	Recognisable attempt at either det **S** or det **T**	M1
	e.g. $\det\mathbf{S}=6+p(-2)$, $\det\mathbf{T}=2(-3)-1(q)-3(3q)$ $\det\mathbf{S}=6-2p$, $\det\mathbf{T}=-6-q-9q$	One determinant correct with minors simplified	A1
	$6-2p=-6-10q\Rightarrow p=5q+6$*	Correctly shown with no errors	A1*
			(3)
(b)	$\mathbf{U}=\mathbf{TS}=\begin{pmatrix}2&1&-3\\0&-3&-1\\q&0&1\end{pmatrix}\begin{pmatrix}1&0&p\\-1&2&3\\2&-2&0\end{pmatrix}=\begin{pmatrix}-5&8&2p+3\\1&-4&-9\\q+2&-2&pq\end{pmatrix}$ or $\begin{pmatrix}2&1&-3\\0&-3&-1\\q&0&1\end{pmatrix}\begin{pmatrix}1&0&5q+6\\-1&2&3\\2&-2&0\end{pmatrix}=\begin{pmatrix}-5&8&10q+15\\1&-4&-9\\q+2&-2&5q^2+6q\end{pmatrix}$ M1: Attempts **TS** in terms of p and q or just q A1: Six correct elements in their **TS** in terms of p and q or just q A1: Fully correct in terms of q		M1 A1 A1
			(3)
	Mark (c) and (d) together. **Note:** Full marks are possible in parts (c) and (d) if the relevant rows of their matrix **U** are correct.		
(c)	$\begin{pmatrix}-5&8&10q+15\\1&-4&-9\\q+2&-2&5q^2+6q\end{pmatrix}\begin{pmatrix}2\\-3\\1\end{pmatrix}=\begin{pmatrix}r\\5\\14\end{pmatrix}$ $\Rightarrow 2q+4+6+5q^2+6q=14$	Sets up the correct matrix equation with their **U** and multiplies out to obtain a quadratic equation in q. Attempts involving finding the inverse of **U** please send to review.	M1
	$5q^2+8q-4=0\Rightarrow(5q-2)(q+2)=0\Rightarrow q=\frac{2}{5},\ -2$	M1: Solves their 3TQ to find two values of q (usual rules including calculator solutions) A1: Both correct values	M1 A1
			(3)
(d)	$r=(-5)(2)+8(-3)+10\left(\frac{2}{5}\right)+15$ $=-15$	M1: Uses the first row of their matrix equation to obtain a numerical expression for r. For the substitution of q, they must use the positive solution of their 3TQ that has two solutions of different sign. Allow slips if the intention is clear. A1: Correct value	M1 A1
			(2)
			Total 11

Question Number	Scheme	Notes	Marks
6(a)	$16x^2-48x+40$ $=16\left(x^2-3x+\frac{5}{2}\right)$ $=16\left(\left(x-\frac{3}{2}\right)^2+\frac{1}{4}\right)$ $=16\left(x-\frac{3}{2}\right)^2+4$	B1: Two of $a=16$, $b=-\frac{3}{2}$ and $c=4$ either listed or embedded within their expression B1: Fully correct expression or correct values given as their final answer **Note:** neither $4\left((2x-3)^2+1\right)$ nor $1\left((4x-6)^2+4\right)$ are in the required form and score B0 B0, but marks in parts (b) and (c) may be available.	B1 B1
			(2)
(b)	**Notes:** • **In this part, if there is no hyperbolic substitution, the response should be scored B0 M0 dM0 A0.** • If the candidate has a negative c value from part (a), the integration in (b) may require a different hyperbolic substitution, e.g. using cosh rather than sinh. Please send any such responses to review.		
	$2x-3=\sinh u \Rightarrow x=\frac{1}{2}\sinh u+\frac{3}{2} \Rightarrow \frac{dx}{du}=\frac{1}{2}\cosh u$	Correct differentiation of $x=d\sinh u+e,\quad d,\ e\neq 0$ $\Rightarrow \frac{dx}{du}=d\cosh u$	B1
	$\int \frac{dx}{\sqrt{16x^2-48x+40}}=\frac{1}{4}\int \frac{dx}{\sqrt{\left(x-\frac{3}{2}\right)^2+\frac{1}{4}}}=\frac{1}{4}\int \frac{\frac{1}{2}\cosh u\ du}{\sqrt{\left(\frac{1}{2}\sinh u\right)^2+\frac{1}{4}}}$ Complete attempt at substitution using $x=d\sinh u+e$ $d,e\neq 0$ and $\frac{dx}{du}=d\cosh u$		M1
	$=\frac{1}{4}\int \frac{\cosh u\,du}{\sqrt{\sinh^2 u+1}}=\frac{1}{4}\int du$	Reaches $k\int du$ following a substitution of the required form. Do not be too concerned about errors in the processing to reach this point.	dM1
	$=\frac{1}{4}u\ (+c)=\frac{1}{4}\text{arsinh}(2x-3)\ \{+c\}$ or $\frac{1}{4}\ln\left(2x-3+\sqrt{(2x-3)^2+1}\right)\ \{+c\}$ or $\frac{1}{4}\ln\left(x-\frac{3}{2}+\sqrt{\left(x-\frac{3}{2}\right)^2+\frac{1}{4}}\right)\{+c\}$	Correct answer in terms of arsinh or using a correct logarithmic form.	A1
			(4)

(c)	$\displaystyle\int_{\frac{41}{24}}^{\frac{15}{8}} \frac{dx}{\sqrt{16x^2-48x+40}} = \frac{1}{4}\left(\text{arsinh}\frac{3}{4} - \text{arsinh}\frac{5}{12}\right)$	Substitutes given limits into an arsinh function (or a valid logarithmic form) and subtracts either way round. If their final answer to (b) is in terms of u, accept an attempt to calculate and substitute the corresponding u values.	M1
	$= \dfrac{1}{4}\left(\ln\left(\dfrac{3}{4}+\sqrt{\left(\dfrac{3}{4}\right)^2+1}\right) - \ln\left(\dfrac{5}{12}+\sqrt{\left(\dfrac{5}{12}\right)^2+1}\right)\right)$	One correct use of a logarithmic form of arsinh. Not dependent on first M1.	M1
	$= \dfrac{1}{4}\left(\ln 2 - \ln\dfrac{3}{2}\right) = \dfrac{1}{4}\ln\dfrac{4}{3}$ or $-\dfrac{1}{4}\ln\left(\dfrac{3}{4}\right)$	Correct answer in required form	A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
7	$P(6, 1, -2)$ $\quad \mathbf{a} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ $\quad \mathbf{b} = \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}$ $\quad Q(0, 9, 32)$		
(a)	$\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \times \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ -8 \\ -14 \end{pmatrix}$ Alternatively, if the normal vector is $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$, then: $\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \Rightarrow 2a + 3b - c = 0$; and $\begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \Rightarrow 4a - b + 2c$ With solutions, e.g. $a = 5, b = -8, c = -14$	Attempts vector product with two components correct. Alternatively, attempts to find the three components a, b and c of the normal vector using scalar products with the two vectors in the plane, forming two equations in a, b and c and solving.	M1
	$\mathbf{r}.\begin{pmatrix} 5 \\ -8 \\ -14 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -8 \\ -14 \end{pmatrix}$	Applies **r.n = a.n** If $\begin{pmatrix} 6 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -8 \\ -14 \end{pmatrix}$ is not shown, then it can be implied by 50. If no working and an incorrect value, then M0.	M1
	$5x - 8y - 14z = 50$	Any correct Cartesian equation	A1
Alt (Using sim equations)	$x = 6 + 2s + 4t \quad (1)$ $y = 1 + 3s - t \quad (2)$ $z = -2 - s + 2t \quad (3)$ e.g. (1) and (3) $\Rightarrow x + 2z = 2 + 8t \quad (4)$	Forms three equations in x, y, z, s and t, and eliminates one of s or t	M1
	e.g. (2) and (3) $\Rightarrow y + 3z = -5 + 5t \quad (5)$ e.g. (4) and (5) $\Rightarrow 5x - 8y - 14z = 50$	Eliminates both s and t Correct equation of plane	M1 A1
			(3)

(b)	Line from Q to Π in the direction of **n**: $\{\mathbf{r}=\}\begin{pmatrix}5t\\9-8t\\32-14t\end{pmatrix}$ or $\begin{pmatrix}0\\9\\32\end{pmatrix}+\begin{pmatrix}5t\\-8t\\-14t\end{pmatrix}$	Attempts parametric form with two components correct for their normal	M1
	$5(5t)-8(9-8t)-14(32-14t)=50$ $\Rightarrow 285t-520=50\Rightarrow t=2$ **Alt 1**: Use distance of Q from plane: Distance from Q(0, 9, 32) to plane is $\frac{\lvert 0(5)+9(-8)+32(-14)-50\rvert}{\sqrt{5^2+8^2+14^2}}=\frac{570}{\sqrt{285}}$ Distance from $R(5t,9-8t,32-14t)$ to plane is $\frac{\lvert 5t(5)+(9-8t)(-8)+(32-14t)(-14)-50\rvert}{\sqrt{5^2+8^2+14^2}}=\frac{\lvert 285t-570\rvert}{\sqrt{285}}$ $\frac{\lvert 285t-570\rvert}{\sqrt{285}}=\frac{570}{\sqrt{285}}\Rightarrow t=0,4$, so $t=4$ at R **Alt 2**: Construct a parallel plane through Q: $\mathbf{r}.\begin{pmatrix}5\\-8\\-14\end{pmatrix}=\begin{pmatrix}0\\9\\32\end{pmatrix}.\begin{pmatrix}5\\-8\\-14\end{pmatrix}=-520$, or $\mathbf{r}.\hat{\mathbf{n}}=\frac{-520}{\sqrt{285}}$. Equation of Π is $\mathbf{r}.\hat{\mathbf{n}}=\frac{50}{\sqrt{285}}$. Distance between planes is $\frac{50}{\sqrt{285}}-\frac{-520}{\sqrt{285}}=\frac{570}{\sqrt{285}}$. Then consider R as above.	Substitutes their parametric form into their plane equation and solves for their parameter. **Alt 1:** Finds the distance from Q to plane; finds distance from $R(5t,9-8t,32-14t)$ to plane; equates and solves to find t **Alt 2:** Constructs a parallel plane through Q and calculates distance between the two planes.	M1
	$t=4\Rightarrow R(20,-23,-24)$ **or** $t=2\Rightarrow X(10,-7,4)$ on the plane. Then $\frac{Q+R}{2}=X\Rightarrow R(20,-23,-24)$	M1: Substitutes the appropriate value of t into their equation of line to obtain a point; **or** finds the point X on the plane where the line meets the plane and uses a correct midpoint calculation to find R. A1: Correct point. Condone vector form.	M1 A1
			(4)

(c) (Using vector product)	$\overrightarrow{PQ}=\begin{pmatrix}-6\\8\\34\end{pmatrix}$, $\overrightarrow{PR}=\begin{pmatrix}14\\-24\\-22\end{pmatrix}$, $\overrightarrow{QR}=\begin{pmatrix}20\\-32\\-56\end{pmatrix}$ e.g. $\overrightarrow{PQ}\times\overrightarrow{PR}=\ldots$	Uses subtraction to obtain two relevant vectors and attempts a vector product. You do not need to check their answer.	M1
	Area $=\frac{1}{2}\left\|\begin{pmatrix}640\\344\\32\end{pmatrix}\right\|=\frac{1}{2}\sqrt{640^2+344^2+32^2}=\ldots$	Obtains a value from a correct method for the area of the triangle	M1
	$=\frac{1}{2}\sqrt{528960}=4\sqrt{8265}$	Correct exact value in required form	A1
Alt (Using right-angled triangles)	e.g. QR intersects Π at $X(10, -7, 4)$ e.g., $\overrightarrow{PX}=\begin{pmatrix}4\\-8\\6\end{pmatrix}$, $\overrightarrow{QX}=\begin{pmatrix}10\\-16\\-28\end{pmatrix}$ $PX=\sqrt{4^2+8^2+6^2}$ $QX=\sqrt{10^2+16^2+28^2}$	Attempts base and perpendicular height of a relevant triangle	M1
	Area $=2\times$ Area $\Delta PQX=2\times\frac{1}{2}\times2\sqrt{29}\times2\sqrt{285}=\ldots$	Obtains a value from a correct method for the area of the triangle	M1
	$=4\sqrt{8265}$	Correct exact value in required form	A1
Alt (Using area of triangle formula)	$\overrightarrow{PQ}=\begin{pmatrix}-6\\8\\34\end{pmatrix}$, $\overrightarrow{PR}=\begin{pmatrix}14\\-24\\-22\end{pmatrix}$, $\overrightarrow{QR}=\begin{pmatrix}20\\-32\\-56\end{pmatrix}$ e.g. $\|PQ\|=2\sqrt{314}$, $\|PR\|=2\sqrt{314}$, $\|QR\|=4\sqrt{285}$ By cosine rule, $\cos QPR=-\frac{128}{157}\Rightarrow\sin QPR=0.579\ldots$	Uses a correct method to find one of the angles of triangle PQR or its sine.	M1
	Area $=\frac{1}{2}\left(2\sqrt{314}\right)\left(2\sqrt{314}\right)(0.579\ldots)$	Uses a correct area of triangle formula	M1
	$=4\sqrt{8265}$	Correct exact value in required form. Note that methods involving an angle are unlikely to result in an exact value for the area, so probably score A0	A1
	Note: There are many variations on the above. If the right-angled triangle PQX is considered, angles may be calculated using SOHCAHTOA and side lengths using Pythagoras' Theorem.		
			(3)
			Total 10

Question Number	Scheme	Notes	Marks
8	$I_n = \int_0^{\frac{1}{2}} x^n e^{-2x}\,\mathrm{d}x \quad J_n = \int_0^{\frac{1}{2}} x^n e^{2x}\,\mathrm{d}x \quad n \geqslant 0$		
(a)	$I_n = -\frac{1}{2}\left[x^n e^{-2x}\right]_0^{\frac{1}{2}} + \frac{n}{2}\int_0^{\frac{1}{2}} x^{n-1} e^{-2x}\,\mathrm{d}x$ o.e.	M1: Applies integration by parts and obtains an expression of the correct form, allowing for slips with the numerical coefficients. Limits and dx are not required for this mark. A1: A correct expression. Condone missing dx. This mark requires correct limits, but condone invisible limits on either term if implied by subsequent work.	M1 A1
	$= -\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^n\left(\frac{1}{e}\right) + \frac{n}{2} I_{n-1}$ $= \frac{n}{2} I_{n-1} - \frac{1}{2^{n+1} e}$ *	Obtains given answer with an intermediate step and no errors. Following integration by parts, the limits and dx must be seen on the integral term at some point in the working for this mark.	A1*
			(3)
(b)	$\int_0^{\frac{1}{2}} x^n \sinh 2x\,\mathrm{d}x = \int_0^{\frac{1}{2}} x^n \left(\frac{e^{2x} - e^{-2x}}{2}\right)\mathrm{d}x = \frac{J_n - I_n}{2}$ *	Correct proof. If working from left to right, it is not necessary to see the starting expression $\int_0^{\frac{1}{2}} x^n \sinh 2x\,\mathrm{d}x$, but correct limits must be seen at least once. If working from right to left, dx and the correct limits must be seen in the final answer.	B1*
			(1)

(c)	**Note:** For the first two method marks, condone slips. The marks can be awarded for an attempt, using the reduction formulae, to find formula(e) allowing them to find J_3 from J_0 and I_3 from I_0.		
	$J_3 = \frac{e}{16} - \frac{3}{2}\left(\frac{e}{8} - \left(\frac{e}{4} - \frac{1}{2}J_0\right)\right)$ $I_3 = -\frac{1}{16e} + \frac{3}{2}\left(-\frac{1}{8e} - \frac{1}{4e} + \frac{1}{2}I_0\right)$	M1: Uses reduction formulae to attempt to obtain J_3 in terms of J_0 **or** I_3 in terms of I_0 M1: Uses reduction formulae to attempt to obtain J_3 in terms of J_0 **and** I_3 in terms of I_0	M1 M1
	Alt for M1 M1 $J_3 = \frac{1}{16}e - \frac{3}{2}J_2$ $J_2 = \frac{1}{8}e - J_1$ $J_1 = \frac{1}{4}e - \frac{1}{2}J_0$ $I_3 = -\frac{1}{16}e^{-1} + \frac{3}{2}I_2$ $I_2 = -\frac{1}{8}e^{-1} + I_1$ $I_1 = -\frac{1}{4}e^{-1} + \frac{1}{2}I_0$	M1: Attempts to find J_3 in terms of J_2, J_2 in terms of J_1 and J_1 in terms of J_0 **or** I_3 in terms of I_2, I_2 in terms of I_1 and I_1 in terms of I_0 M1: Attempts to find J_3 in terms of J_2, J_2 in terms of J_1 and J_1 in terms of J_0 **and** I_3 in terms of I_2, I_2 in terms of I_1 and I_1 in terms of I_0	M1 M1 (Alt)
	$J_0 = \int_0^{\frac{1}{2}} e^{2x}\,dx = \frac{1}{2}\left[e^{2x}\right]_0^{\frac{1}{2}} = \frac{1}{2}e - \frac{1}{2}$ $I_0 = \int_0^{\frac{1}{2}} e^{-2x}\,dx = -\frac{1}{2}\left[e^{-2x}\right]_0^{\frac{1}{2}} = -\frac{1}{2e} + \frac{1}{2}$	For either J_0 or I_0 correct in terms of e.	B1
	For reference $J_1 = \frac{1}{4}$ $J_2 = \frac{e}{8} - \frac{1}{4}$ $J_3 = \frac{3}{8} - \frac{e}{8}$ $I_1 = \frac{1}{4} - \frac{1}{2e}$ $I_2 = \frac{1}{4} - \frac{5}{8e}$ $I_3 = \frac{3}{8} - \frac{1}{e}$		
	$\int_0^{\frac{1}{2}} x^3 \sinh 2x\,dx = \frac{1}{2}\left(\frac{e}{8} - \left(-\frac{1}{e}\right)\right)$	Attempts $\frac{J_3 - I_3}{2}$. J_3 and I_3 must be in terms of e and/or J_0 or I_0 respectively. **Requires both previous M marks.**	**dd**M1
	$= \frac{1}{2e} - \frac{e}{16} = \frac{1}{16}\left(\frac{8}{e} - e\right)$	Correct answer in the required form	A1

(c) Alt for M1 M1 B1	Some candidates are finding J_3 and I_3 in terms of J_1 and I_1, then finding J_1 and I_1 using integration by parts.		
	$J_3=\frac{e}{16}-\frac{3}{2}\left(\frac{e}{8}-J_1\right)$ $I_3=-\frac{1}{16e}+\frac{3}{2}\left(I_1-\frac{1}{8e}\right)$ **or** $J_3=\frac{1}{16}e-\frac{3}{2}J_2,\ J_2=\frac{1}{8}e-J_1$ $I_3=-\frac{1}{16}e^{-1}+\frac{3}{2}I_2,\ I_2=-\frac{1}{8}e^{-1}+I_1$	M1: Uses reduction formulae to obtain J_3 in terms of J_1 **or** I_3 in terms of I_1 M1: Uses reduction formulae to obtain both J_3 in terms of J_1 **and** I_3 in terms of I_1	M1 M1
	$J_1=\int_0^{\frac{1}{2}}x\mathrm{e}^{2x}\mathrm{d}x=\frac{1}{2}\left[x\mathrm{e}^{2x}\right]_0^{\frac{1}{2}}-\frac{1}{2}\int_0^{\frac{1}{2}}\mathrm{e}^{2x}\mathrm{d}x=\frac{1}{4}\mathrm{e}-\frac{1}{4}(\mathrm{e}-1)=\frac{1}{4}$ $I_1=\int_0^{\frac{1}{2}}x\mathrm{e}^{-2x}\mathrm{d}x=-\frac{1}{2}\left[x\mathrm{e}^{-2x}\right]_0^{\frac{1}{2}}+\frac{1}{2}\int_0^{\frac{1}{2}}\mathrm{e}^{-2x}\mathrm{d}x=-\frac{1}{4\mathrm{e}}-\frac{1}{4}\left(\frac{1}{\mathrm{e}}-1\right)=\frac{1}{4}-\frac{1}{2\mathrm{e}}$ For either J_1 or I_1 correct in terms of e. If working is shown, it must be a correct application of integration by parts. $\left\{\Rightarrow J_3=\frac{3}{8}-\frac{\mathrm{e}}{8},\ I_3=\frac{3}{8}-\frac{1}{\mathrm{e}}\right\}$		B1
	Then ddM1 A1 as above		
			(5)
			Total 9
			PAPER TOTAL: 75