Pearson Edexcel IAL Further Mathematics

Further Mathematics 2

Past Paper Collection (from 2020)

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Last updated: July 1, 2024

Comments and suggestions to DrYuFromShanghai@QQ.com

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International Advanced Level

Centre Number	Candidate Number

Friday 09 October 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WFM02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
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Leave blank

1. $$\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} = 2\cos x$$

(a) Express $\frac{d^3y}{dx^3}$ in terms of x, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

(3)

At $x = 0, y = 2$ and $\frac{dy}{dx} = 5$

(b) Determine the value of $\frac{d^3y}{dx^3}$ at $x = 0$

(1)

(c) Express y as a series in ascending powers of x, up to and including the term in x^3

(3)

Leave blank

Question 1 continued

Q1

(Total 7 marks)

Leave blank

2. (a) Write $\dfrac{3r+1}{r(r-1)(r+1)}$ in partial fractions.

(2)

(b) Hence find

$$\sum_{r=2}^{n} \frac{3r+1}{r(r-1)(r+1)} \qquad n \geqslant 2$$

giving your answer in the form

$$\frac{an^2 + bn + c}{2n(n+1)}$$

where a, b and c are integers to be determined.

(5)

(c) Hence determine the exact value of

$$\sum_{r=15}^{20} \frac{3r+1}{r(r-1)(r+1)}$$

(2)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 9 marks)

Leave blank

3. Use algebra to obtain the set of values of x for which

$$\left|\frac{x^2 + 3x + 10}{x + 2}\right| < 7 - x$$

(9)

Leave blank

Question 3 continued

Q3

(Total 9 marks)

Leave blank

4. (a) Express the complex number $18\sqrt{3} - 18\text{i}$ in the form

$$r(\cos\theta + \text{i}\sin\theta) \qquad -\pi < \theta \leqslant \pi$$

(3)

(b) Solve the equation

$$z^4 = 18\sqrt{3} - 18\text{i}$$

giving your answers in the form $r\text{e}^{\text{i}\theta}$ where $-\pi < \theta \leqslant \pi$

(5)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 8 marks)

Leave blank

5. The transformation T from the z-plane to the w-plane is given by

$$w = \frac{z - 3\text{i}}{z + 2\text{i}} \qquad z \neq -2\text{i}$$

The circle with equation $|z| = 1$ in the z-plane is mapped by T onto the circle C in the w-plane.

Determine

(i) the centre of C,

(ii) the radius of C.

(7)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 7 marks)

Leave blank

6. Obtain the general solution of the equation

$$x^2 \frac{\mathrm{d}y}{\mathrm{d}x} + (x\cot x + 2)xy = 4\sin x \qquad 0 < x < \pi$$

Give your answer in the form $y = \mathrm{f}(x)$

(8)

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

7.

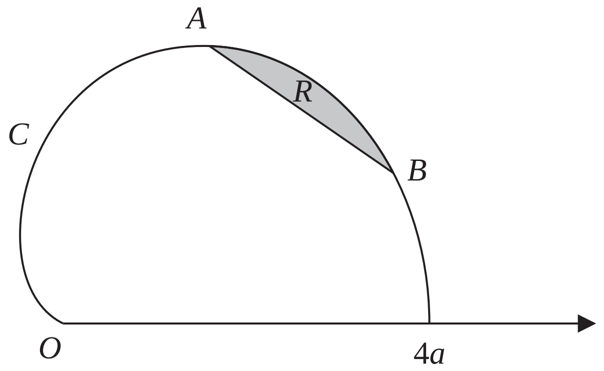

Figure 1

The curve C, shown in Figure 1, has polar equation

$$r = 2a(1 + \cos\theta) \qquad 0 \leqslant \theta \leqslant \pi$$

where a is a positive constant.

The tangent to C at the point A is parallel to the initial line.

(a) Determine the polar coordinates of A.

(6)

The point B on the curve has polar coordinates $\left(a\left(2+\sqrt{3}\right), \frac{\pi}{6}\right)$

The finite region R, shown shaded in Figure 1, is bounded by the curve C and the line AB.

(b) Use calculus to determine the exact area of the shaded region R.

Give your answer in the form

$$\frac{a^2}{4}\left(d\pi - e + f\sqrt{3}\right)$$

where d, e and f are integers.

(7)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 13 marks)

Leave blank

8. (a) Show that the transformation $x = e^u$ transforms the differential equation

$$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 8y = 4\ln x \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{du^2} + 2\frac{dy}{du} - 8y = 4u \qquad \text{(II)}$$

(6)

(b) Determine the general solution of differential equation (II), expressing y as a function of u.

(7)

(c) Hence obtain the general solution of differential equation (I).

(1)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Friday 15 January 2021

Morning (Time: 1 hour 30 minutes) | Paper Reference **WFM02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

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Information

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- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
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Advice

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- Try to answer every question.
- Check your answers if you have time at the end.
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Leave blank

1. The transformation T from the z-plane, where $z = x + \mathrm{i}y$, to the w-plane, where $w = u + \mathrm{i}v$, is given by

$$w = \frac{z + p\mathrm{i}}{\mathrm{i}z + 3} \qquad z \neq 3\mathrm{i} \quad p \in \mathbb{Z}$$

The point representing $\mathrm{i}\left(1 + \sqrt{3}\right)$ is invariant under T.

Determine the value of p.

(3)

Leave blank

Question 1 continued

Q1

(Total 3 marks)

Leave blank

2. (a) Show that, for $r > 0$

$$\frac{r+2}{r(r+1)} - \frac{r+3}{(r+1)(r+2)} = \frac{r+4}{r(r+1)(r+2)}$$

(2)

(b) Hence show that

$$\sum_{r=1}^{n} \frac{r+4}{r(r+1)(r+2)} = \frac{n(an+b)}{c(n+1)(n+2)}$$

where a, b and c are integers to be determined.

(4)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 6 marks)

Leave blank

3. Use algebra to obtain the set of values of x for which

$$|x^2 + x - 2| < \frac{1}{2}(x + 5)$$

(7)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 7 marks)

Leave blank

4. (a) Show that the substitution $y^2 = \dfrac{1}{z}$ transforms the differential equation

$$\frac{dy}{dx} + 2y = 3xy^3 \qquad y \neq 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - 4z = -6x \qquad \text{(II)}$$

(3)

(b) Obtain the general solution of differential equation (II).

(5)

(c) Hence obtain the general solution of differential equation (I), giving your answer in the form $y^2 = f(x)$

(1)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 9 marks)

Leave blank

5. Given that

$$(2 - x^2)\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 = 3y$$

(a) show that

$$\frac{d^3y}{dx^3} = \frac{1}{(2 - x^2)}\left(2x\frac{d^2y}{dx^2}\left(1 - 5\frac{dy}{dx}\right) - 5\left(\frac{dy}{dx}\right)^2 + 3\frac{dy}{dx}\right)$$

(5)

Given also that $y = 3$ and $\frac{dy}{dx} = \frac{1}{4}$ at $x = 0$

(b) obtain a series solution for y in ascending powers of x with simplified coefficients, up to and including the term in x^3

(4)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 9 marks)

Leave blank

6. (a) Determine the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 6\cos x$$

(7)

(b) Find the particular solution for which $y = 0$ and $\frac{dy}{dx} = 0$ at $x = 0$

(5)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 12 marks)

Leave blank

7.

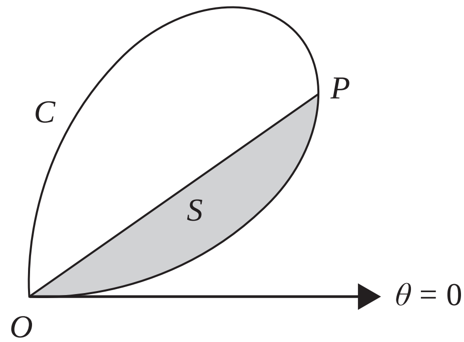

Figure 1

Figure 1 shows a sketch of curve C with polar equation

$$r = 3\sin 2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

The point P on C has polar coordinates (R, ϕ). The tangent to C at P is perpendicular to the initial line.

(a) Show that $\tan\phi = \dfrac{1}{\sqrt{2}}$

(4)

(b) Determine the exact value of R.

(2)

The region S, shown shaded in Figure 1, is bounded by C and the line OP, where O is the pole.

(c) Use calculus to show that the exact area of S is

$$p\arctan\frac{1}{\sqrt{2}} + q\sqrt{2}$$

where p and q are constants to be determined.

Solutions relying entirely on calculator technology are not acceptable.

(7)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 13 marks)

Leave blank

8. Given that $z = e^{i\theta}$

(a) show that $z^n + \frac{1}{z^n} = 2\cos n\theta$

where n is a positive integer.

(2)

(b) Show that

$$\cos^6\theta = \frac{1}{32}\left(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10\right)$$

(5)

(c) Hence solve the equation

$$\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta = 0 \qquad 0 \leqslant \theta \leqslant \pi$$

Give your answers to 3 significant figures.

(4)

(d) Use calculus to determine the exact value of

$$\int_0^{\frac{\pi}{3}} \left(32\cos^6\theta - 4\cos^2\theta\right)\mathrm{d}\theta$$

Solutions relying entirely on calculator technology are not acceptable.

(5)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

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Instructions

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  Answers without working may not gain full credit.
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Information

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Advice

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- Try to answer every question.
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- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Leave blank

1. (a) Express $\dfrac{2}{r(r^2-1)}$ in partial fractions.

(3)

(b) Hence find, in terms of n,

$$\sum_{r=2}^{n} \frac{1}{r(r^2-1)}$$

Give your answer in the form

$$\frac{n^2 + An + B}{Cn(n+1)}$$

where A, B and C are constants to be found.

(5)

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Q1

(Total 8 marks)

Leave blank

2. The transformation T from the z-plane, where $z = x + \mathrm{i}y$, to the w-plane, where $w = u + \mathrm{i}v$, is given by

$$w = \frac{z+2}{z-\mathrm{i}} \quad z \neq \mathrm{i}$$

The transformation T maps the circle $|z| = 2$ in the z-plane onto a circle C in the w-plane.

Find (i) the centre of C,

(ii) the radius of C.

(8)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 8 marks)

Leave blank

3. The curve C, with pole O, has polar equation

$$r = 1 + \cos\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point A on C, the tangent to C is parallel to the initial line.

(a) Find the polar coordinates of A.

(4)

(b) Find the finite area enclosed by the initial line, the line OA and the curve C, giving your answer in the form $a\pi + b\sqrt{3}$, where a and b are rational constants to be found.

(6)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 10 marks)

Leave blank

4. Given that

$$y\frac{d^2y}{dx^2} - 4\left(\frac{dy}{dx}\right)^2 + 3y = 0$$

(a) show that

$$\frac{d^3y}{dx^3} = \frac{28}{y^2}\left(\frac{dy}{dx}\right)^3 - \frac{24}{y}\left(\frac{dy}{dx}\right)$$

(5)

Given also that $y = 8$ and $\frac{dy}{dx} = 1$ at $x = 0$

(b) find a series solution for y in ascending powers of x, up to and including the term in x^3, simplifying the coefficients where possible.

(4)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 9 marks)

Leave blank

5. Use algebra to find the set of values of x for which

$$\left|2x^2 + x - 3\right| > 3(1 - x)$$

(7)

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 7 marks)

Leave blank

6. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 2x^2 + x$$

(8)

(b) Find the particular solution of this differential equation for which $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$

(5)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 13 marks)

Leave blank

7. (a) Use de Moivre's theorem to show that

$$\tan 4\theta \equiv \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$$

(6)

(b) Use the identity given in part (a) to find the 2 positive roots of

$$x^4 + 2x^3 - 6x^2 - 2x + 1 = 0$$

giving your answers to 3 significant figures.

(3)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 9 marks)

Leave blank

8. (a) Show that the substitution $v = y^{-2}$ transforms the differential equation

$$\frac{dy}{dx} + 6xy = 3xe^{x^2}y^3 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{dv}{dx} - 12vx = -6xe^{x^2} \qquad x > 0 \qquad \text{(II)}$$

(5)

(b) Hence find the general solution of the differential equation (I), giving your answer in the form $y^2 = f(x)$.

(6)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F2

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Total Marks

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Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
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- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Leave blank

1. Solve the equation

$$z^5 - 32\text{i} = 0$$

giving each answer in the form $r\text{e}^{\text{i}\theta}$ where $0 < \theta < 2\pi$

(4)

Leave blank

Question 1 continued

Q1

(Total 4 marks)

Leave blank

2. Use algebra to determine the set of values of x for which

$$\frac{x}{2-x} \leqslant \frac{x+3}{x}$$

(*Solutions relying entirely on graphical methods are not acceptable.*)

(8)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 8 marks)

Leave blank

3. A transformation maps points from the z-plane, where $z = x + \mathrm{i}y$, to the w-plane, where $w = u + \mathrm{i}v$. The transformation is given by

$$w = \frac{(2+\mathrm{i})z + 4}{z - \mathrm{i}} \qquad z \neq \mathrm{i}$$

The transformation maps the imaginary axis in the z-plane onto the line l in the w-plane.

Determine a Cartesian equation of l, giving your answer in the form $au + bv + c = 0$ where a, b and c are integers to be found.

(6)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 6 marks)

Leave blank

4. (a) Determine the general solution of the differential equation

$$(x+1)\frac{\mathrm{d}y}{\mathrm{d}x} - xy = \mathrm{e}^{3x} \qquad x > -1$$

giving your answer in the form $y = \mathrm{f}(x)$.

(7)

(b) Determine the particular solution of the differential equation for which $y = 5$ when $x = 0$

(2)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 9 marks)

Leave blank

5. Given that $y = \tan^2 x$

(a) show that

$$\frac{d^3y}{dx^3} = 8\tan x \sec^2 x\left(p\sec^2 x + q\right)$$

where p and q are integers to be determined.

(5)

(b) Hence determine the Taylor series expansion about $\frac{\pi}{3}$ of $\tan^2 x$ in ascending powers of $\left(x - \frac{\pi}{3}\right)$ up to and including the term in $\left(x - \frac{\pi}{3}\right)^3$, giving each coefficient in simplest form.

(3)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 8 marks)

Leave blank

6. The complex number z on an Argand diagram is represented by the point P where

$$|z + 1 - 13\mathrm{i}| = 3|z - 7 - 5\mathrm{i}|$$

Given that the locus of P is a circle,

(a) determine the centre and radius of this circle.

(5)

The complex number w, on the same Argand diagram, is represented by the point Q, where

$$\arg(w - 8 - 6\mathrm{i}) = -\frac{3\pi}{4}$$

Given that the locus of P intersects the locus of Q at the point R,

(b) determine the complex number representing R.

(4)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 9 marks)

Leave blank

7. (a) Show that the transformation $x = t^2$ transforms the differential equation

$$4x\frac{d^2y}{dx^2} + 2\left(1 + 2\sqrt{x}\right)\frac{dy}{dx} - 15y = 15x \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 15y = 15t^2 \qquad \text{(II)}$$

(5)

(b) Solve differential equation (II) to determine y in terms of t.

(5)

(c) Hence determine the general solution of differential equation (I).

(1)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 11 marks)

Leave blank

8.

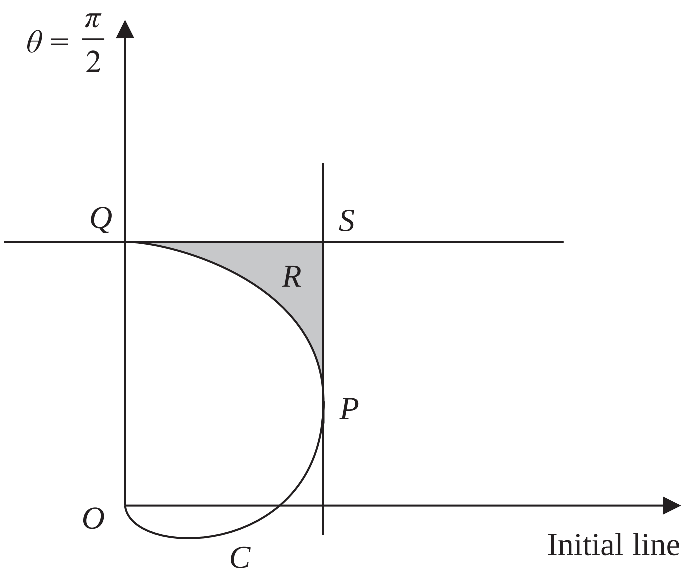

Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 1 + \sin\theta \qquad -\frac{\pi}{2} < \theta \leqslant \frac{\pi}{2}$$

The point P lies on C such that the tangent to C at P is perpendicular to the initial line.

(a) Use calculus to determine the polar coordinates of P.

(5)

The tangent to C at the point Q where $\theta = \frac{\pi}{2}$ is parallel to the initial line.

The tangent to C at Q meets the tangent to C at P at the point S, as shown in Figure 1.

The finite region R, shown shaded in Figure 1, is bounded by the line segments QS, SP and the curve C.

(b) Use algebraic integration to show that the area of R is

$$\frac{1}{32}\left(a\sqrt{3} + b\pi\right)$$

where a and b are integers to be determined.

(6)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 11 marks)

Leave blank

9. (a) Show that

$$n^5 - (n-1)^5 \equiv 5n^4 - 10n^3 + 10n^2 - 5n + 1$$

(2)

(b) Hence, using the method of differences, show that for all integer values of n,

$$\sum_{r=1}^{n} r^4 = \frac{1}{30}n(n+1)(2n+1)(an^2 + bn + c)$$

where a, b and c are integers to be determined.

(7)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
  Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Leave blank

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Express the complex number

$$-4 - 4\sqrt{3}\text{i}$$

in the form $r(\cos\theta + \text{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$

(3)

(b) Solve the equation

$$z^3 + 4 + 4\sqrt{3}\text{i} = 0$$

giving your answers in the form $r\text{e}^{\text{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$

(4)

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Q1

(Total 7 marks)

Leave blank

2. Determine the general solution of the differential equation

$$2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 3y = 2e^{3x}$$

(6)

Leave blank

Question 2 continued

Q2

(Total 6 marks)

Leave blank

3.

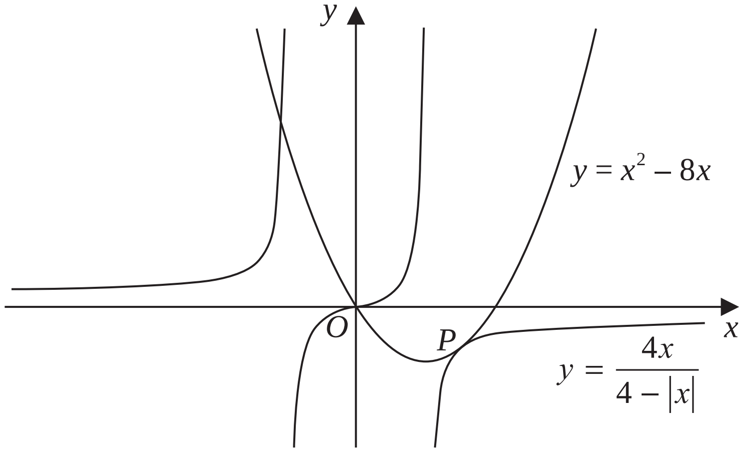


Figure 1

Figure 1 shows a sketch of the curve C_1 with equation

$$y = \frac{4x}{4 - |x|}$$

and the curve C_2 with equation

$$y = x^2 - 8x$$

For $x > 0$, C_1 has equation $y = \dfrac{4x}{4 - x}$

(a) Use algebra to show that C_1 touches C_2 at a point P, stating the coordinates of P

(5)

(b) Hence or otherwise, using algebra, solve the inequality

$$x^2 - 8x > \frac{4x}{4 - |x|}$$

(6)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 11 marks)

Leave blank

4.

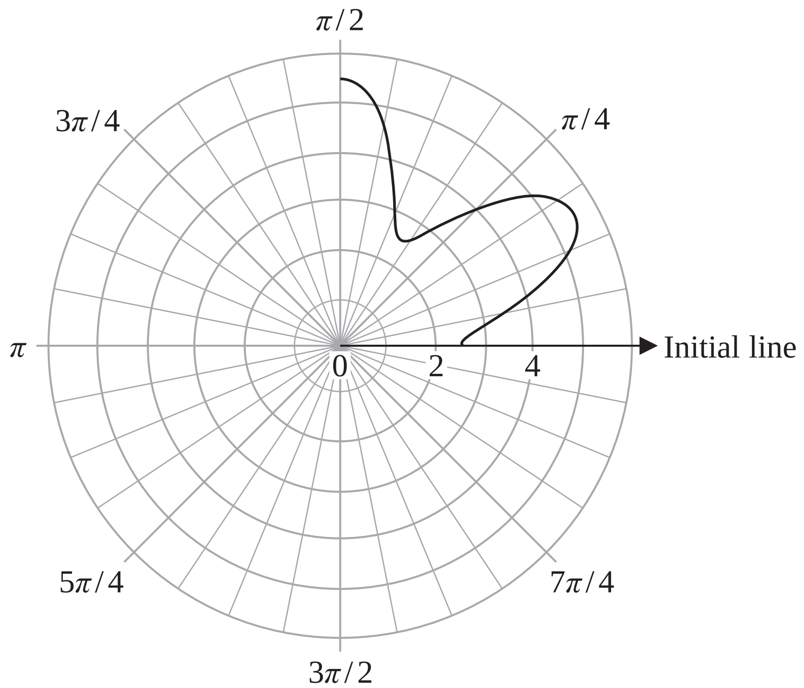


Figure 2

Figure 2 shows part of the curve with polar equation

$$r = 4 - \frac{3}{2}\cos 6\theta \qquad 0 \leqslant \theta < 2\pi$$

(a) Sketch, on the polar grid in Figure 2,

(i) the rest of the curve with equation $\quad r = 4 - \frac{3}{2}\cos 6\theta \qquad 0 \leqslant \theta < 2\pi$

(ii) the polar curve with equation $\quad r = 1 \qquad 0 \leqslant \theta < 2\pi$

A spare copy of the grid is given on page 15.

(3)

In part (b) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(b) Determine the exact area enclosed between the two curves defined in part (a).

(7)

Leave blank

Question 4 continued

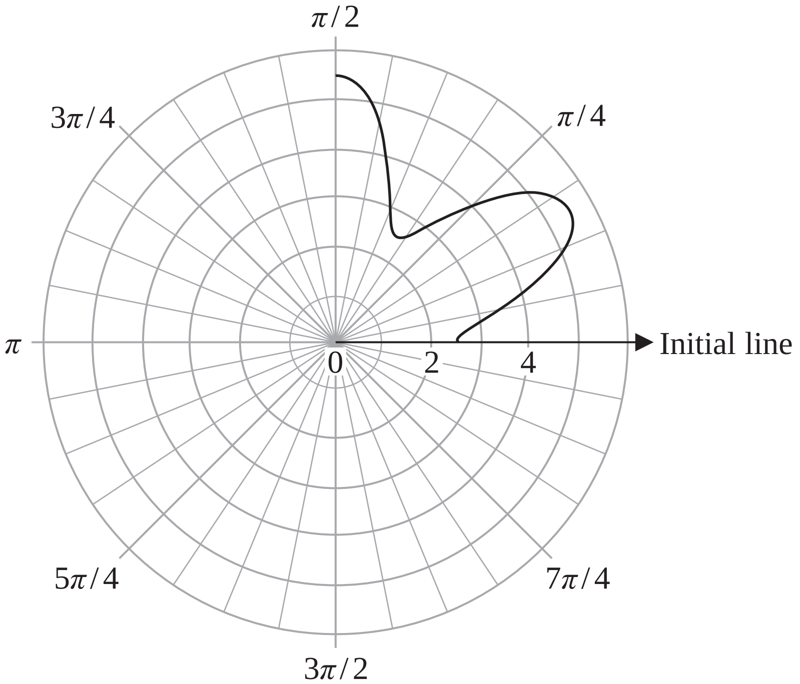
Leave blank

Question 4 continued

Leave blank

Question 4 continued

Only use this grid if you need to redraw your answer to part (a)



Copy of Figure 2

Q4

(Total 10 marks)

Leave blank

5. $$y = \sqrt{4 + \ln x} \qquad x > \frac{1}{2}$$

(a) Show that

$$\frac{d^2y}{dx^2} = -\frac{9 + 2\ln x}{4x^2(4 + \ln x)^{\frac{3}{2}}}$$

(5)

(b) Hence, or otherwise, determine the Taylor series expansion about $x = 1$ for y, in ascending powers of $(x - 1)$, up to and including the term in $(x - 1)^2$, giving each coefficient in simplest form.

(3)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 8 marks)

Leave blank

6. Given that $A > B > 0$, by letting $x = \arctan A$ and $y = \arctan B$

(a) prove that

$$\arctan A - \arctan B = \arctan\left(\frac{A - B}{1 + AB}\right)$$

(3)

(b) Show that when $A = r + 2$ and $B = r$

$$\frac{A - B}{1 + AB} = \frac{2}{(1 + r)^2}$$

(2)

(c) Hence, using the method of differences, show that

$$\sum_{r=1}^{n} \arctan\left(\frac{2}{(1 + r)^2}\right) = \arctan(n + p) + \arctan(n + q) - \arctan 2 - \frac{\pi}{4}$$

where p and q are integers to be determined.

(4)

(d) Hence, making your reasoning clear, determine

$$\sum_{r=1}^{\infty} \arctan\left(\frac{2}{(1 + r)^2}\right)$$

giving the answer in the form $k\pi - \arctan 2$, where k is a constant.

(2)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 11 marks)

Leave blank

7. A transformation from the z-plane to the w-plane is given by

$$w = \frac{(1+\mathrm{i})z + 2(1-\mathrm{i})}{z-\mathrm{i}} \qquad z \neq \mathrm{i}$$

The transformation maps points on the imaginary axis in the z-plane onto a line in the w-plane.

(a) Find an equation for this line.

(2)

The transformation maps points on the real axis in the z-plane onto a circle in the w-plane.

(b) Find the centre and radius of this circle.

(6)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 8 marks)

Leave blank

8. (a) Show that the transformation $v = y - 2x$ transforms the differential equation

$$\frac{dy}{dx} + 2yx(y - 4x) = 2 - 8x^3 \qquad \text{(I)}$$

into the differential equation

$$\frac{dv}{dx} = -2xv^2 \qquad \text{(II)}$$

(4)

(b) Solve the differential equation (II) to determine v as a function of x

(4)

(c) Hence obtain the general solution of the differential equation (I).

(1)

(d) Sketch the solution curve that passes through the point (–1, –1).

On your sketch show clearly the equation of any horizontal or vertical asymptotes.

You do **not** need to find the coordinates of any intercepts with the coordinate axes or the coordinates of any stationary points.

(5)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Given that

$$\frac{2n+1}{n^2(n+1)^2} \equiv \frac{A}{n^2} + \frac{B}{(n+1)^2}$$

(a) determine the value of A and the value of B

(1)

(b) Hence show that, for $n \geqslant 5$

$$\sum_{r=5}^{n} \frac{2r+1}{r^2(r+1)^2} = \frac{n^2+an+b}{c(n+1)^2}$$

where a, b and c are integers to be determined.

(4)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 5 marks)

2. (a) Use algebra to determine the set of values of x for which

$$x - 5 < \frac{9}{x + 3}$$

(6)

(b) Hence, or otherwise, determine the set of values of x for which

$$x - 5 < \frac{9}{|x + 3|}$$

(2)

Question 2 continued

(Total for Question 2 is 8 marks)

3. The transformation T from the z-plane to the w-plane is given by

$$w = \frac{z}{z + 4\text{i}} \qquad z \neq -4\text{i}$$

The circle with equation $|z| = 3$ is mapped by T onto the circle C

Determine

(i) a Cartesian equation of C

(ii) the centre and radius of C

(8)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 8 marks)

4. (a) Determine the general solution of the differential equation

$$\frac{dy}{dx} - 3y\tan x = e^{4x}\sec^3 x$$

giving your answer in the form $y = f(x)$

(5)

(b) Determine the particular solution for which $y = 4$ at $x = 0$

(2)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 7 marks)

5. Given that

$$y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 - 2y = 0 \qquad y > 0$$

(a) determine $\frac{d^3y}{dx^3}$ in terms of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ and y

(4)

Given that $y = 2$ and $\frac{dy}{dx} = 1$ at $x = 0$

(b) determine a series solution for y in ascending powers of x, up to and including the term in x^3, giving each coefficient in its simplest form.

(4)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 8 marks)

6.

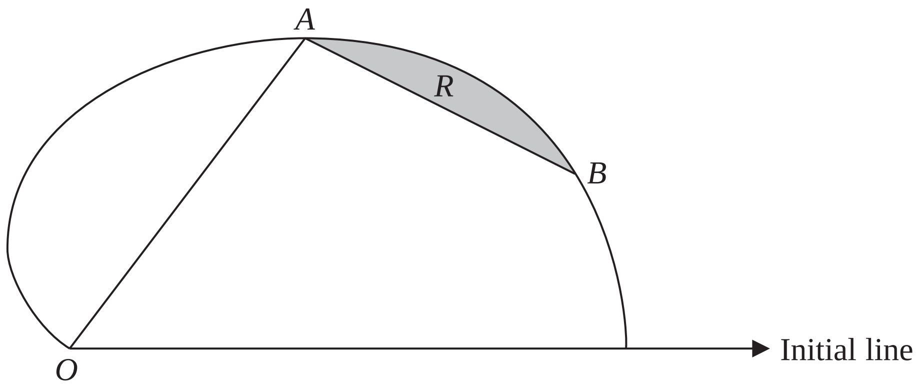

Figure 1

The curve shown in Figure 1 has polar equation

$$r = 4a(1 + \cos\theta) \qquad 0 \leqslant \theta < \pi$$

where a is a positive constant.

The tangent to the curve at the point A is parallel to the initial line.

(a) Show that the polar coordinates of A are $\left(6a, \frac{\pi}{3}\right)$

(6)

The point B lies on the curve such that angle $AOB = \frac{\pi}{6}$

The finite region R, shown shaded in Figure 1, is bounded by the line AB and the curve.

(b) Use calculus to determine the area of the shaded region R, giving your answer in the form $a^2\left(n\pi + p\sqrt{3} + q\right)$, where n, p and q are integers.

(7)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 13 marks)

7. (a) Show that the transformation $y = xv$ transforms the equation

$$3\frac{d^2y}{dx^2} - \frac{6}{x}\frac{dy}{dx} + \frac{6y}{x^2} + 3y = x^2 \qquad x \neq 0 \qquad \text{(I)}$$

into the equation

$$3\frac{d^2v}{dx^2} + 3v = x \qquad \text{(II)}$$

(6)

(b) Hence obtain the general solution of the differential equation (I), giving your answer in the form $y = f(x)$

(6)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 12 marks)

8. (a) Use de Moivre's theorem to show that

$$\sin 5\theta \equiv 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$

(5)

(b) Hence determine the five distinct solutions of the equation

$$16x^5 - 20x^3 + 5x + \frac{1}{5} = 0$$

giving your answers to 3 decimal places.

(5)

(c) Use the identity given in part (a) to show that

$$\int_0^{\frac{\pi}{4}} \left(4\sin^5\theta - 5\sin^3\theta - 6\sin\theta\right)\mathrm{d}\theta = a\sqrt{2} + b$$

where a and b are rational numbers to be determined.

(4)

Question 8 continued

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

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- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Given that $y = \ln(5 + 3x)$

(a) determine, in simplest form, $\dfrac{d^3y}{dx^3}$

(3)

(b) Hence determine the Maclaurin series expansion of $\ln(5 + 3x)$, in ascending powers of x up to and including the term in x^3, giving each coefficient in simplest form.

(2)

(c) Hence write down the Maclaurin series expansion of $\ln(5 - 3x)$, in ascending powers of x up to and including the term in x^3, giving each coefficient in simplest form.

(1)

(d) Use the answers to parts (b) and (c) to determine the first 2 non-zero terms, in ascending powers of x, of the Maclaurin series expansion of

$$\ln\left(\frac{5 + 3x}{5 - 3x}\right)$$

(2)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2. (a) Express

$$\frac{1}{(2n-1)(2n+1)(2n+3)}$$

in partial fractions.

(2)

(b) Hence, using the method of differences, show that for all integer values of n,

$$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)(2r+3)} = \frac{n(n+2)}{a(2n+b)(2n+c)}$$

where a, b and c are integers to be determined.

(4)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 6 marks)

3. (a) Show that the transformation $y = \frac{1}{z}$ transforms the differential equation

$$x^2 \frac{dy}{dx} + xy = 2y^2 \qquad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - \frac{z}{x} = -\frac{2}{x^2} \qquad \text{(II)}$$

(3)

(b) Solve differential equation (II) to determine z in terms of x.

(4)

(c) Hence determine the particular solution of differential equation (I) for which $y = -\frac{3}{8}$ at $x = 3$

Give your answer in the form $y = f(x)$.

(2)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4.

$$\frac{dy}{dx} = y^2 - x$$

(a) Show that

$$\frac{d^4y}{dx^4} = Ay\frac{d^3y}{dx^3} + B\frac{dy}{dx}\frac{d^2y}{dx^2}$$

where A and B are integers to be determined.

(4)

Given that $y = 1$ at $x = -1$

(b) determine the Taylor series solution for y, in ascending powers of $(x + 1)$ up to and including the term in $(x + 1)^4$, giving each coefficient in simplest form.

(3)

Question 4 continued

(Total for Question 4 is 7 marks)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Use algebra to determine the set of values of x for which

$$\frac{x^2 - 9}{|x + 8|} > 6 - 2x$$

(6)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 6 marks)

6. A complex number z is represented by the point P in an Argand diagram.

Given that

$$|z - 2\text{i}| = |z - 3|$$

(a) sketch the locus of P. You do **not** need to find the coordinates of any intercepts.

(2)

The transformation T from the z-plane to the w-plane is given by

$$w = \frac{\text{i}z}{z - 2\text{i}} \qquad z \neq 2\text{i}$$

Given that T maps $|z - 2\text{i}| = |z - 3|$ to a circle C in the w-plane,

(b) find the equation of C, giving your answer in the form

$$|w - (p + q\text{i})| = r$$

where p, q and r are real numbers to be determined.

(6)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 8 marks)

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Use de Moivre's theorem to show that

$$\cos 5x \equiv \cos x\left(a\sin^4 x + b\sin^2 x + c\right)$$

where a, b and c are integers to be determined.

(4)

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$

$$\cos 5\theta = \sin 2\theta \sin\theta - \cos\theta$$

giving your answers to 3 decimal places.

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

8.

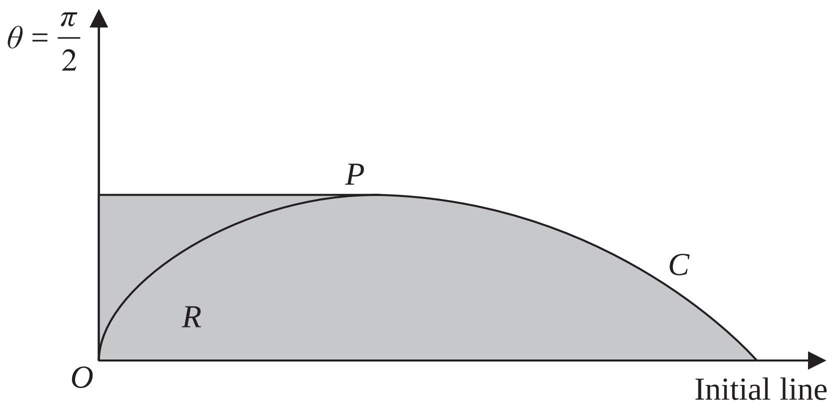

Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 1 - \sin\theta \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point P lies on C, such that the tangent to C at P is parallel to the initial line.

(a) Use calculus to determine the polar coordinates of P

(4)

The finite region R, shown shaded in Figure 1, is bounded by

- the line with equation $\theta = \frac{\pi}{2}$
- the tangent to C at P
- part of the curve C
- the initial line

(b) Use algebraic integration to show that the area of R is

$$\frac{1}{32}\left(a\pi + b\sqrt{3} + c\right)$$

where a, b and c are integers to be determined.

(6)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9. (a) Given that $x = t^{\frac{1}{2}}$, determine, in terms of y and t,

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(5)

(b) Hence show that the transformation $x = t^{\frac{1}{2}}$, where $t > 0$, transforms the differential equation

$$x\frac{d^2y}{dx^2} - \left(6x^2 + 1\right)\frac{dy}{dx} + 9x^3y = x^5 \qquad \text{(I)}$$

into the differential equation

$$4\frac{d^2y}{dt^2} - 12\frac{dy}{dt} + 9y = t \qquad \text{(II)}$$

(2)

(c) Solve differential equation (II) to determine a general solution for y in terms of t.

(5)

(d) Hence determine the general solution of differential equation (I).

(1)

Question 9 continued

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 8 June 2023

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

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Instructions

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Information

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Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Show that, for $r \geqslant 2$

$$\frac{2}{\sqrt{r}+\sqrt{r-2}} = \sqrt{r} - \sqrt{r-2}$$

(2)

(b) Hence use the method of differences to determine

$$\sum_{r=2}^{n} \frac{2}{\sqrt{r}+\sqrt{r-2}}$$

giving your answer in simplest form.

(3)

(c) Hence show that

$$\sum_{r=4}^{50} \frac{2}{\sqrt{r}+\sqrt{r-2}} = A + B\sqrt{2} + C\sqrt{3}$$

where A, B and C are integers to be determined.

(2)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 7 marks)

2. The complex number z_1 is defined as

$$z_1 = \frac{\left(\cos\frac{5\pi}{12} + \mathrm{i}\sin\frac{5\pi}{12}\right)^4}{\left(\cos\frac{\pi}{3} - \mathrm{i}\sin\frac{\pi}{3}\right)^3}$$

(a) Without using your calculator show that

$$z_1 = \cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}$$

(4)

(b) Shade, on a single Argand diagram, the region R defined by

$$|z - z_1| \leqslant 1 \qquad \textbf{and} \qquad 0 \leqslant \arg(z - z_1) \leqslant \frac{3\pi}{4}$$

(4)

Given that the complex number z lies in R

(c) determine the smallest possible positive value of $\arg z$

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 10 marks)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given that

$$\frac{x+2}{x+4} \leqslant \frac{x}{k(x-1)}$$

where k is a positive constant,

(a) show that

$$(x+4)(x-1)(px^2+qx+r) \leqslant 0$$

where p, q and r are expressions in terms of k to be determined.

(3)

(b) Hence, or otherwise, determine the values for x for which

$$\frac{x+2}{x+4} \leqslant \frac{x}{3(x-1)}$$

(4)

Question 3 continued

(Total for Question 3 is 7 marks)

4. (a) Determine the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 48x^2 - 34$$

(5)

Given that $y = 4$ and $\frac{dy}{dx} = 21$ at $x = 0$

(b) determine the particular solution of the differential equation.

(4)

(c) Hence find the value of y at $x = -2$, giving your answer in the form $pe^q + r$ where p, q and r are integers to be determined.

(2)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 11 marks)

5. The transformation T from the z-plane, where $z = x + \mathrm{i}y$, to the w-plane, where $w = u + \mathrm{i}v$ is given by

$$w = \frac{z+1}{z-3} \qquad z \neq 3$$

The straight line in the z-plane with equation $y = 4x$ is mapped by T onto the circle C in the w-plane.

(a) Show that C has equation

$$3u^2 + 3v^2 - 2u + v + k = 0$$

where k is a constant to be determined.

(5)

(b) Hence determine

(i) the coordinates of the centre of C

(ii) the radius of C

(2)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 7 marks)

6. Given that $y = \sec x$

(a) show that

$$\frac{d^3y}{dx^3} = \sec x \tan x \left(p\sec^2 x + q\right)$$

where p and q are integers to be determined.

(4)

(b) Hence determine the Taylor series expansion about $\frac{\pi}{3}$ of $\sec x$ in ascending powers of $\left(x - \frac{\pi}{3}\right)$, up to and including the term in $\left(x - \frac{\pi}{3}\right)^3$, giving each coefficient in simplest form.

(3)

(c) Use the answer to part (b) to determine, to four significant figures, an approximate value of $\sec\left(\frac{7\pi}{24}\right)$

(2)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. (a) Show that the substitution $z = y^{-2}$ transforms the differential equation

$$x\frac{dy}{dx} + y + 4x^2y^3\ln x = 0 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - \frac{2z}{x} = 8x\ln x \qquad x > 0 \qquad \text{(II)}$$

(5)

(b) By solving differential equation (II), determine the general solution of differential equation (I), giving your answer in the form $y^2 = f(x)$

(6)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8.

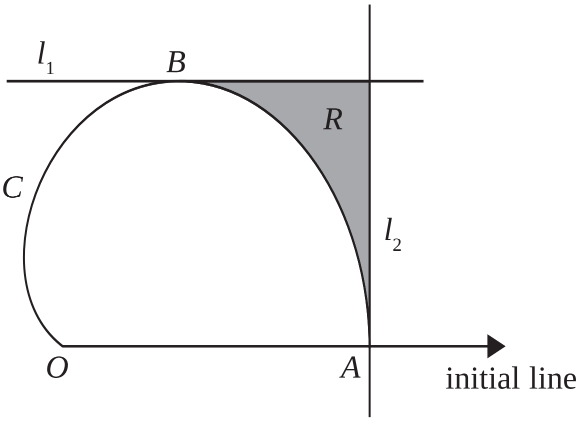

Figure 1

Figure 1 shows a sketch of the curve C with equation

$$r = 6(1 + \cos\theta) \qquad 0 \leqslant \theta \leqslant \pi$$

Given that C meets the initial line at the point A, as shown in Figure 1,

(a) write down the polar coordinates of A.

(1)

The line l_1, also shown in Figure 1, is the tangent to C at the point B and is parallel to the initial line.

(b) Use calculus to determine the polar coordinates of B.

(4)

The line l_2, also shown in Figure 1, is the tangent to C at A and is perpendicular to the initial line.

The region R, shown shaded in Figure 1, is bounded by C, l_1 and l_2

(c) Use algebraic integration to find the exact area of R, giving your answer in the form $p\sqrt{3} + q\pi$ where p and q are constants to be determined.

(8)

Question 8 continued

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Monday 15 January 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Using algebra, solve the inequality

$$\frac{1}{x+2} > 2x+3$$

(5)

Question 1 continued

(Total for Question 1 is 5 marks)

2.

$$z = 6 - 6\sqrt{3}\,\mathrm{i}$$

(a) (i) Determine the modulus of z

(ii) Show that the argument of z is $-\frac{\pi}{3}$

(3)

Using de Moivre's theorem, and making your method clear,

(b) determine, in simplest form, z^4

(2)

(c) Determine the values of w such that $w^2 = z$, giving your answers in the form $a + \mathrm{i}b$, where a and b are real numbers.

(3)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 8 marks)

3. (a) Show that for $r \geqslant 1$

$$\frac{r}{\sqrt{r(r+1)}+\sqrt{r(r-1)}} \equiv A\left(\sqrt{r(r+1)}-\sqrt{r(r-1)}\right)$$

where A is a constant to be determined.

(2)

(b) Hence use the method of differences to determine a simplified expression for

$$\sum_{r=1}^{n} \frac{r}{\sqrt{r(r+1)}+\sqrt{r(r-1)}}$$

(3)

(c) Determine, as a surd in simplest form, the constant k such that

$$\sum_{r=1}^{n} \frac{kr}{\sqrt{r(r+1)}+\sqrt{r(r-1)}} = \sqrt{\sum_{r=1}^{n} r}$$

(2)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 7 marks)

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Determine, in ascending powers of $\left(x-\frac{\pi}{6}\right)$ up to and including the term in $\left(x-\frac{\pi}{6}\right)^3$, the Taylor series expansion about $\frac{\pi}{6}$ of

$$y = \tan\left(\frac{3x}{2}\right)$$

giving each coefficient in simplest form.

(7)

(b) Hence show that

$$\tan\frac{3\pi}{8} \approx 1 + \frac{\pi}{4} + \frac{\pi^2}{A} + \frac{\pi^3}{B}$$

where A and B are integers to be determined.

(2)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5.

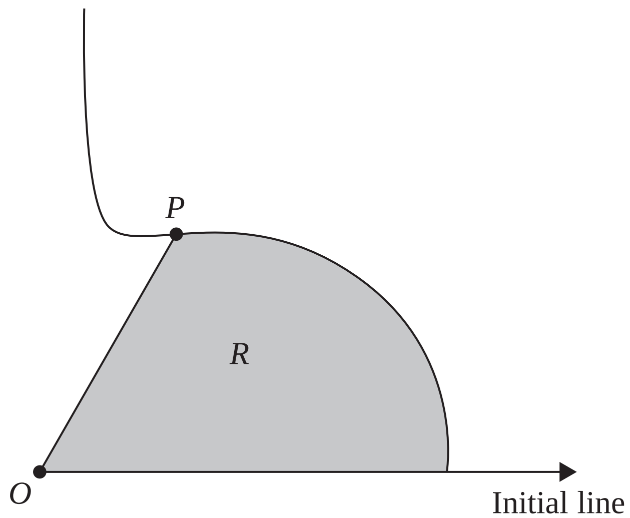

Figure 1

Figure 1 shows a sketch of the curve with polar equation

$$r = 10\cos\theta + \tan\theta \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point P lies on the curve where $\theta = \frac{\pi}{3}$

The region R, shown shaded in Figure 1, is bounded by the initial line, the curve and the line OP, where O is the pole.

Use algebraic integration to show that the exact area of R is

$$\frac{1}{12}\left(a\pi + b\sqrt{3} + c\right)$$

where a, b and c are integers to be determined.

(9)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6. The differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 13x = 8e^{-3t} \qquad t \geqslant 0$$

describes the motion of a particle along the x-axis.

(a) Determine the general solution of this differential equation.

(6)

Given that the motion of the particle satisfies $x = \frac{1}{2}$ and $\frac{dx}{dt} = \frac{1}{2}$ when $t = 0$

(b) determine the particular solution for the motion of the particle.

(4)

On the graph of the particular solution found in part (b), the first turning point for $t > 0$ occurs at $x = a$.

(c) Determine, to 3 significant figures, the value of a.

[*Solutions relying entirely on calculator technology are not acceptable.*]

(4)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 14 marks)

7. A transformation T from the z-plane, where $z = x + \mathrm{i}y$, to the w-plane, where $w = u + \mathrm{i}v$ is given by

$$w = \frac{z-3}{2\mathrm{i}-z} \qquad z \neq 2\mathrm{i}$$

The line in the z-plane with equation $y = x + 3$ is mapped by T onto a circle C in the w-plane.

(a) Determine

(i) the coordinates of the centre of C

(ii) the exact radius of C

(8)

The region $y > x + 3$ in the z-plane is mapped by T onto the region R in the w-plane.

(b) On a single Argand diagram

(i) sketch the circle C

(ii) shade and label the region R

(2)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

8. (a) For all the values of x where the identity is defined, prove that

$$\cot 2x + \tan x \equiv \operatorname{cosec} 2x$$

(3)

(b) Show that the substitution $y^2 = w\sin 2x$, where w is a function of x, transforms the differential equation

$$y\frac{\mathrm{d}y}{\mathrm{d}x} + y^2 \tan x = \sin x \qquad 0 < x < \frac{\pi}{2} \qquad \text{(I)}$$

into the differential equation

$$\frac{\mathrm{d}w}{\mathrm{d}x} + 2w\operatorname{cosec} 2x = \sec x \qquad 0 < x < \frac{\pi}{2} \qquad \text{(II)}$$

(4)

(c) By solving differential equation (II), determine a general solution of differential equation (I) in the form $y^2 = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is a function in terms of $\cos x$

$$\left[\text{You may use without proof } \int \operatorname{cosec} 2x \,\mathrm{d}x = \frac{1}{2}\ln|\tan x| \;\; (+\text{constant})\right]$$

(6)

Question 8 continued

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 4 June 2024

Morning (Time: 1 hour 30 minutes)

Paper reference **WFM02/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F2

You must have:
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. The complex number $z = x + \mathrm{i}y$ satisfies the equation

$$|z - 3 - 4\mathrm{i}| = |z + 1 + \mathrm{i}|$$

(a) Determine an equation for the locus of z giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(3)

(b) Shade, on an Argand diagram, the region defined by

$$|z - 3 - 4\mathrm{i}| \leqslant |z + 1 + \mathrm{i}|$$

You do **not** need to determine the coordinates of any intercepts on the coordinate axes.

(1)

Question 1 continued

(Total for Question 1 is 4 marks)

2.

$$x\frac{dy}{dx} - y^3 = 4$$

(a) Show that

$$x\frac{d^3y}{dx^3} = ay\left(\frac{dy}{dx}\right)^2 + \left(by^2 + c\right)\frac{d^2y}{dx^2}$$

where a, b and c are integers to be determined.

(4)

Given that $y = 1$ at $x = 2$

(b) determine the Taylor series expansion for y in ascending powers of $(x - 2)$, up to and including the term in $(x - 2)^3$, giving each coefficient in simplest form.

(3)

Question 2 continued

(Total for Question 2 is 7 marks)

3. (a) Express

$$\frac{1}{(n+3)(n+5)}$$

in partial fractions.

(2)

(b) Hence, using the method of differences, show that for all positive integer values of n,

$$\sum_{r=1}^{n} \frac{1}{(r+3)(r+5)} = \frac{n(pn+q)}{40(n+4)(n+5)}$$

where p and q are integers to be determined.

(4)

(c) Use the answer to part (b) to determine, as a simplified fraction, the value of

$$\frac{1}{9\times 11} + \frac{1}{10\times 12} + \ldots + \frac{1}{24\times 26}$$

(2)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 8 marks)

4. (a) Show that the substitution $y^2 = \frac{1}{t}$ transforms the differential equation

$$\frac{dy}{dx} + y = xy^3 \qquad \text{(I)}$$

into the differential equation

$$\frac{dt}{dx} - 2t = -2x \qquad \text{(II)}$$

(3)

(b) Solve differential equation (II) and determine y^2 in terms of x.

(6)

Question 4 continued

(Total for Question 4 is 9 marks)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Use algebra to determine the values of x for which

$$\frac{x+1}{(x-3)(x+2)} \leqslant 1-\frac{2}{x-3}$$

(6)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 6 marks)

6. The transformation T from the z-plane to the w-plane is given by

$$w = \frac{z - \mathrm{i}}{z + 1} \qquad z \neq -1$$

Given that T maps the imaginary axis in the z-plane to the circle C in the w-plane, determine

(i) the coordinates of the centre of C

(ii) the radius of C

(7)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 7 marks)

7. Given that $y = e^x \sin x$

(a) show that

$$\frac{d^6 y}{dx^6} = k \frac{d^2 y}{dx^2}$$

where k is a constant to be determined.

(4)

(b) Hence determine the first 5 non-zero terms in the Maclaurin series expansion for y, giving each coefficient in simplest form.

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 7 marks)

8. (a) Given that $t = \ln x$, where $x > 0$, show that

$$\frac{d^2y}{dx^2} = e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$

(3)

(b) Hence show that the transformation $t = \ln x$, where $x > 0$, transforms the differential equation

$$x^2\frac{d^2y}{dx^2} - 2y = 1 + 4\ln x - 2(\ln x)^2 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 1 + 4t - 2t^2 \qquad \text{(II)}$$

(1)

(c) Solve differential equation (II) to determine y in terms of t.

(5)

(d) Hence determine the general solution of differential equation (I).

(1)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Use De Moivre's theorem to show that

$$\cos 6\theta \equiv 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$$

(4)

(b) Hence determine the smallest positive root of the equation

$$48x^6 - 72x^4 + 27x^2 - 1 = 0$$

giving your answer to 3 decimal places.

(4)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 8 marks)

10.

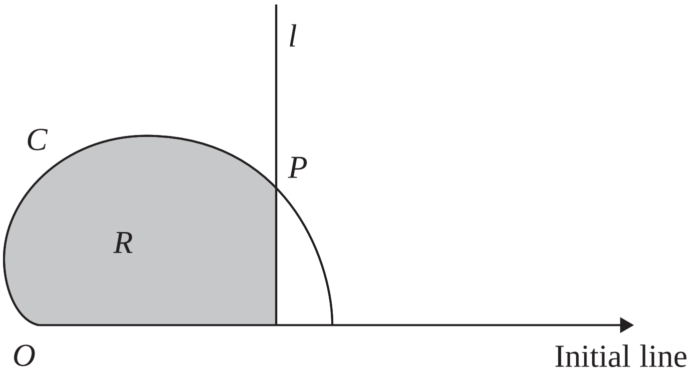

Figure 1

Figure 1 shows a sketch of the curve C with polar equation

$$r = 1 + \cos\theta \qquad 0 \leqslant \theta \leqslant \pi$$

and the line l with polar equation

$$r = k\sec\theta \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

where k is a positive constant.

Given that

- C and l intersect at the point P
- $OP = 1 + \frac{\sqrt{3}}{2}$

(a) determine the exact value of k.

(2)

The finite region R, shown shaded in Figure 1, is bounded by C, the initial line and l.

(b) Use algebraic integration to show that the area of R is

$$p\pi + q\sqrt{3} + r$$

where p, q and r are simplified rational numbers to be determined.

(7)

Question 10 continued

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS