Question Number	Scheme	Marks
1	$\dfrac{2}{x+3} < 1 - \dfrac{3}{x-2}$	
	CV's $x=-3, x=2$ seen anywhere (perhaps on a diagram)	B1
	$2(x+3)(x-2)^2 < (x+3)^2(x-2)^2 - 3(x+3)^2(x-2)$ **OR** $\dfrac{2}{x+3} < \dfrac{x-5}{x-2} \Rightarrow 2(x+3)(x-2)^2 < (x-5)(x+3)^2(x-2)$ **OR** $\dfrac{2}{x+3} < \dfrac{x-5}{x-2} \Rightarrow \dfrac{2}{x+3} - \dfrac{x-5}{x-2} \Rightarrow \dfrac{\mathrm{f}(x)}{(x+3)(x-2)} < 0$	M1
	$(x+3)(x-2)[2(x-2)-(x+3)(x-2)+3(x+3)]$ $\Rightarrow (x+3)(x-2)(-x^2+4x+11)\ (<0)$ **OR** $(x+3)(x-2)[2(x-2)-(x-5)(x+3)]$ $\Rightarrow (x+3)(x-2)(-x^2+4x+11)\ (<0)$ **OR** $\dfrac{2(x-2)-(x+3)(x-5)}{(x+3)(x-2)} \Rightarrow \dfrac{-x^2+4x+11}{(x+3)(x-2)}\ (<0)$ **OR** $x^4-3x^3-21x^2+13x+66\ (<0)$	M1
	$-x^2+4x+11=0 \Rightarrow x = \ldots$.(usual rules)	M1
	$x = 2 \pm \sqrt{15}$	A1
	$x<-3,\ \ 2-\sqrt{15}<x<2,\ x>2+\sqrt{15}$ **OR** $x \in (-\infty,-3) \cup (2-\sqrt{15},2) \cup (2+\sqrt{15},\infty)$	M1A1
		(7)
		Total 7

Notes
B1: CV's -3 **and** 2 seen anywhere, on a diagram or stated **M1:** Attempts to multiply both sides by $(x+3)^2(x-2)^2$ - allow if one side correct **OR** combine terms on RHS with common denominator and multiply by $(x+3)^2(x-2)^2$ allow if one side correct **OR** combine fractions to a single fraction with common denominator. Allow with >, <, = etc between the terms. **M1:** Factorises out $(x+3)(x-2)$ and makes progress towards a 3-term quadratic **OR** combines terms on the numerator and makes progress towards a 3-term quadratic **OR** multiplies out and combines all terms to obtain a 4-term quartic (terms must be collected) Allow sign errors only if they progress to a quartic. Allow with >, <, = etc between the terms. **M1**: Correct attempt at solving **their** quadratic factor (usual rules) or their quartic- may be solved on a calculator, but if incorrect solutions for their equation, then method needed to gain this mark. Allow with >, <, = etc between the terms. **A1:** Correct solutions obtained from correct quadratic/quartic and condone decimal equivalents for this mark (awrt 5.87 and awrt -1.87) **M1:** Obtains the correct form for the solution using their CV's (condoning strict/non-strict inequalities) $x < a$, $b < x < c$, $x > d$ with $a < b < c < d$ **A1:** All three correct in exact form (not decimals) and no extras and must have all inequality signs correct. Allow equivalent notations e.g. soft brackets for intervals. Ignore any word between the regions but withhold this last mark if $\cap$ used with set notation. **Allow non-strict inequalities to score for all but the final mark**. **Allow recovery from non-strict to strict if the final solution set is fully correct.** **Multiplication by $(x+3)(x-2)$ only can gain max B1M0M0M1A1M1A0**

Question Number	Scheme	Marks
2(a)	$\frac{1}{(r+1)!}-\frac{1}{(r+2)!}=\frac{(r+2)!-(r+1)!}{(r+1)!(r+2)!}=\left\{\frac{(r+1)!((r+2)-1)}{(r+1)!(r+2)!}\right\}$ OR $\frac{1}{(r+1)!}-\frac{1}{(r+2)!}=\frac{r+2-1}{(r+2)!}$	M1
	$\frac{\cancel{(r+1)!}(r+1)}{\cancel{(r+1)!}(r+2)!}=\frac{r+1}{(r+2)!}$	A1*
		(2)
(b)	$r=1: \quad \left(\frac{1}{2!}-\cancel{\frac{1}{3!}}\right)$. $r=2: \quad \left(\cancel{\frac{1}{3!}}-\cancel{\frac{1}{4!}}\right)$ ⋮ ⋮ $r=n-1: \quad \left(\cancel{\frac{1}{n!}}-\cancel{\frac{1}{(n+1)!}}\right)$ $r=n: \quad \left(\cancel{\frac{1}{(n+1)!}}-\frac{1}{(n+2)!}\right)$	M1
	$\left[\sum_{r=1}^{n}\frac{r+1}{(r+2)!}=\right]\frac{1}{2}-\frac{1}{(n+2)!}=\frac{(n+2)!-2}{2(n+2)!}$	A1A1
		(3)
(c)	$\left[\sum_{r=4}^{10}\frac{3r+3}{(r+2)!}=\right]3\left[\mathrm{f}(10)-\mathrm{f}(3)\right]=3\left[\left(\frac{1}{2}-\frac{1}{12!}\right)-\left(\frac{1}{2}-\frac{1}{5!}\right)\right]$ $=3\left[\frac{(12)!-2}{2(12)!}-\frac{(5)!-2}{2(5)!}\right]$	M1
	$=3\left[\frac{1}{5!}-\frac{1}{12!}\right]=0.024999\ldots=0.0250$	A1
		(2)
		Total 7

Notes
(a) **M1:** Indicates intention to combine over a common denominator, with only sign errors condoned on the numerator. This mark may be awarded by sight of two separate correct terms with a common denominator. May work in reverse from right hand side to left. This mark can be scored with two separate fractions with the same denominator **A1*:** Fully correct proof. Must see evidence of cancellation (which can be implied by a correct factorised form) or one line of correct intermediate working before the stated printed result. (b) **M1:** Correct process of differences evidenced in their work, e.g. attempts first two and last or first and last two etc. There should be enough evidence of at least one pair of cancelling terms. Ignore any attempts at $r = 0$ if they start earlier than $r = 1$. **A1:** Correct non-cancelling terms. Accept 2 or 2! Sight of the correct non-cancelling terms is M1A1 **A1:** Correct simplified expression obtained. Accept 2 or 2! Note that A0A1 is not possible. (c) **M1:** Attempts $3\left[\mathrm{f}(10)-\mathrm{f}(3)\right]$ using **their answer to (b)**. Must be indication of subtraction and the 3 must be included. The '$-\frac{1}{2}$'s may be omitted and allow a slip in signs/brackets. They may use (a) again to obtain $\mathrm{f}(3)$ but their answer to (b) must be used for $\mathrm{f}(10)$. Allow from attempts starting at $r = 0$ in their summations. $\mathrm{f}(10)-\mathrm{f}(4)$ is M0 and subtraction of decimals is M0 unless supported by evidence of using (b) **A1:** Correct answer only. Note: 0.025 or 1/40 is A0

Question Number	Scheme	Marks
3(a)	$y=(1-2x)^2\ln(1-2x)$ $\Rightarrow \dfrac{dy}{dx}=-4(1-2x)\ln(1-2x)+(1-2x)^2\left(\dfrac{-2}{1-2x}\right)$ $=-2(1-2x)\left(2\ln(1-2x)+1\right)$ o.e.	M1A1
	$\Rightarrow \dfrac{d^2y}{dx^2}=4\left(2\ln(1-2x)+1\right)-2(1-2x)\left(\dfrac{-4}{1-2x}\right)$ oe $\left\{=8\ln(1-2x)+12\right\}$	**d**M1 A1
	$\dfrac{d^3y}{dx^3}=\dfrac{-16}{1-2x}$	A1
		(5)
(b)	$\dfrac{d^4y}{dx^4}=16(1-2x)^{-2}\times -2=-32(1-2x)^{-2}$	M1 A1ft
	$y_0=0 \quad y_0'=-2 \quad y_0''=12 \quad y_0'''=-16 \quad y_0^{(4)}=-32$	M1
	$y=y_0+y_0'x+y_0''\dfrac{x^2}{2!}+y_0'''\dfrac{x^3}{3!}+y_0^{(4)}\dfrac{x^4}{4!}\ldots$ $\Rightarrow y=-2x+12\dfrac{x^2}{2!}-16\dfrac{x^3}{3!}-32\dfrac{x^4}{4!}\ldots$	M1
	$y=-2x+6x^2-\dfrac{8}{3}x^3-\dfrac{4}{3}x^4\ldots$	A1
		(5)
		Total 10

Notes
(a) **M1:** Differentiates using product rule, allowing for sign/coefficient errors only **A1:** Fully correct first derivative in any equivalent form. Award when first seen correct and ISW if they simplify incorrectly. **dM1: Dependent on the previous M mark.** Uses product rule again to obtain an expression for the 2nd derivative, allowing for sign/coefficient errors only **A1:** Fully correct expression in any form. Award when first seen correct and ISW if they simplify incorrectly. **A1:** Fully correct expression for the third derivative, which must come from correct work (b) **M1:** Correct form for the 4th derivative $\frac{\mathrm{d}^4y}{\mathrm{d}x^4}=\alpha(1-2x)^{-2}$ **A1ft:** Follow through their 3rd derivative. ISW if they then simplify a correct expression Incorrectly- award when first seen. **M1:** Attempt at calculating values for y and their 1st, 2nd 3rd **and** 4th derivatives at $x=0$ No need to check all values are correct for their derivatives, but award if the intention to evaluate all at $x=0$ is clear and values are actually found for each. **M1:** Correct method for finding the Maclaurin expansion **with all five** of their values (condone the missing 0 at the start). May have to check that their series follows from their values if formula not quoted. Series must be correct for their values but allow slips if a correct formula is quoted. **A1:** Correct series obtained. Allow = or ≈ etc but must be $y=\ldots$ Allow $\mathrm{f}(x)=\cdots$etc if $y=\mathrm{f}(x)$ has already been defined.

Question Number	Scheme	Marks
4	$4\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-4\dfrac{\mathrm{d}y}{\mathrm{d}x}+y=2\mathrm{e}^{\frac{1}{2}x}$	
(a)	$(\text{AE:})\ 4m^2-4m+1=0$ $\left[\Rightarrow(2m-1)^2=0\right]$ $\Rightarrow m=\dfrac{1}{2}$	M1
	$(\text{CF:}y=)\ (A+Bx)\mathrm{e}^{\frac{1}{2}x}$	A1
	$(\text{PI:}y=)\ \lambda x^2\mathrm{e}^{\frac{1}{2}x}$	B1
	$y'=2\lambda x\mathrm{e}^{\frac{1}{2}x}+\dfrac{\lambda}{2}x^2\mathrm{e}^{\frac{1}{2}x}=\mathrm{e}^{\frac{1}{2}x}\left(\dfrac{\lambda}{2}x^2+2\lambda x\right)$	M1
	$y''=\dfrac{1}{2}\mathrm{e}^{\frac{1}{2}x}\left(\dfrac{\lambda}{2}x^2+2\lambda x\right)+\mathrm{e}^{\frac{1}{2}x}(\lambda x+2\lambda)=\mathrm{e}^{\frac{1}{2}x}\left(\dfrac{\lambda}{4}x^2+2\lambda x+2\lambda\right)$	dM1
	Substituting into given equation to get $\mathrm{e}^{\frac{1}{2}x}\left[\left(\lambda x^2+8\lambda x+8\lambda\right)-\left(2\lambda x^2+8\lambda x\right)+\lambda x^2\right]=2\mathrm{e}^{\frac{1}{2}x}$ $\Rightarrow 8\lambda=2\Rightarrow\lambda=\ldots$	ddM1
	$y=(A+Bx)\mathrm{e}^{\frac{1}{2}x}+\dfrac{1}{4}x^2\mathrm{e}^{\frac{1}{2}x}$	A1ft
		(7)
(b)	Sub $(0,1)\Rightarrow\cdots(1=A)$	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=B\mathrm{e}^{\frac{1}{2}x}+\dfrac{1}{2}(A+Bx)\mathrm{e}^{\frac{1}{2}x}+\dfrac{1}{2}x\mathrm{e}^{\frac{1}{2}x}+\dfrac{1}{8}x^2\mathrm{e}^{\frac{1}{2}x}$	M1
	$\mathrm{e}^{\frac{1}{2}}=B+\dfrac{A}{2}\Rightarrow B=\mathrm{e}^{\frac{1}{2}}-\dfrac{1}{2}$ $[y]=\left(1+\left(\mathrm{e}^{\frac{1}{2}}-\dfrac{1}{2}\right)x\right)\mathrm{e}^{\frac{1}{2}x}+\dfrac{1}{4}x^2\mathrm{e}^{\frac{1}{2}x}$ oe	M1A1
		(4)
		Total 11

Notes
Allow recovery from mixed/incorrect variables etc but if not recovered then A and B marks may be lost (a) **M1:** Forms the auxiliary equation (condone one slip/copying error) and solves 3TQ (usual rules). One consistent solution if no working (could be complex). Implied by a correct CF if no incorrect working shown. **A1:** Correct complementary function $y = ...$ not required. May only be seen in final answer. **B1:** Correct form for particular integral $y = ...$ not required. **M1:** Differentiates their PI using the product rule- allowing for sign/coefficient errors only **dM1:** Differentiates their PI twice using the product rule- allowing for sign/coefficient errors only. **Dependent on the previous M mark.** **ddM1: Dependent on both previous M marks** Substitutes into the given equation, equates coefficients and obtains a value for λ . Must be comparing like terms. **A1ft:** A correct general solution following through on their CF only - **the PI must be correct**. Must have "$y =$" e.g., not "GS =" **Note:** If a candidate uses a PI of the form $\lambda x e^{\frac{1}{2}x}$ then they can score a max of M1A1B0M1M1M0A0 If a candidate uses a PI of the form $\lambda e^{\frac{1}{2}x}$ then they can score a max of M1A1B0M0M0M0A0 (b) **B1:** Uses (0, 1) in their answer to (a) and forms an equation in one or both of their constants. This mark could be implied by a correct value for 'A' from a correct part (a). **M1:** Differentiates their GS, which must contain a term "Bxe^{kx}" and a non-zero PI, to obtain an expression of the correct form for the derivative- product rule must be used, allowing for sign/coefficient errors only **M1:** Substitutes $x = 0, \frac{dy}{dx} = e^{\frac{1}{2}}$ into their equation where their derivative is a changed function and finds values for their B (and their A if not found already). **A1:** Any correct expression. Condone e.g., " PS =... "

Question Number	Scheme	Marks
5(a)	$w=2-\dfrac{3}{z+\text{i}} \Rightarrow w=\dfrac{2z+2\text{i}-3}{z+\text{i}} \Rightarrow wz+w\text{i}=2z+2\text{i}-3$ $\Rightarrow z(w-2)=2\text{i}-3-w\text{i} \Rightarrow z=\dfrac{2\text{i}-3-w\text{i}}{w-2}$ oe e.g. $z=\dfrac{3}{2-w}-\text{i}$	M1 A1
	$(z=)\dfrac{(v-3)+\text{i}(2-u)}{(u-2)+\text{i}v}\times\dfrac{(u-2)-\text{i}v}{(u-2)-\text{i}v}=\ldots$	dM1
	$(z=)\dfrac{(v-3)(u-2)-\text{i}v(v-3)-\text{i}(u-2)^2+v(2-u)}{(u-2)^2+v^2}$ $\Rightarrow x=\dfrac{6-3u}{(u-2)^2+v^2}$ and $y=\dfrac{(-u^2-v^2+4u+3v-4)}{(u-2)^2+v^2}$	ddM1
	Half-line $\arg z=\dfrac{\pi}{4}\Rightarrow y=x$	B1
	$6-3u=-u^2-v^2+4u+3v-4$ $\Rightarrow u^2+v^2-7u-3v+10=0$ $\Rightarrow\left(u-\dfrac{7}{2}\right)^2+\left(v-\dfrac{3}{2}\right)^2-\dfrac{49}{4}-\dfrac{9}{4}+10=0$	dddM1
	$\Rightarrow\left(u-\dfrac{7}{2}\right)^2+\left(v-\dfrac{3}{2}\right)^2=\dfrac{9}{2}$	A1
		(7)
(b)(i)	radius is $\dfrac{3}{\sqrt{2}}$ or $\left(\dfrac{3\sqrt{2}}{2}\right)$	B1ft
(ii)	Centre is $\left(\dfrac{7}{2},\dfrac{3}{2}\right)$	B1ft
		(2)
		Total 9

Notes
(a) **M1:** Completes an attempt to make z the subject. **A1:** Correct expression. Any equivalent correct expression for z. Award when first seen and ISW if they then simplify incorrectly. **dM1**: **Dependent on previous M mark.** Replaces w with $u + \mathrm{i}v$ and shows an appropriate attempt to rationalise the denominator. Accept if $x + \mathrm{i}y$ is used instead for this mark. **ddM1**: **Dependent on both previous M marks.** Extracts the correct real and imaginary parts for their expression, allowing for sign errors only, but allow the 'i' coefficients in the imaginary parts. **B1:** Any valid statement that $y = x$ on the half-line. This could be implied by setting their real and imaginary parts equal, after rationalisation, or equivalent work. **dddM1**: **Dependent on all previous M marks** Attempts to equate their real and imaginary parts and proceed to the equation of a circle via a valid completing the square attempt. Must be a valid circle equation with equal coefficients for the square terms **A1:** Correct circle equation obtained from correct work (b) In this part mark parts (i) and (ii) together but if the radius is clearly stated as a coordinate etc then B0 **B1 ft:** For correct radius ft their circle equation. Can only be scored if they have a valid circle equation **B1ft:** For the centre ft their circle equation. Can only be scored if they have a valid circle equation **Note:** Alternative methods are possible. If in doubt send to review.

5(a) Alt I	$z = x + \mathrm{i}y, y = x \Rightarrow w = \frac{2z+2\mathrm{i}-3}{z+\mathrm{i}} = \frac{2x-3+\mathrm{i}(2x+2)}{x+(x+1)\mathrm{i}}$	B1
	$\Rightarrow (u+\mathrm{i}v)(x+(x+1)\mathrm{i}) = 2x-3+\mathrm{i}(2x+2) \Rightarrow \ldots + \ldots \mathrm{i} = \ldots$	M1
	$ux - v(x+1) + \mathrm{i}(u(x+1)+vx) = (2x-3)+\mathrm{i}(2x+2)$	A1
	$ux - v(x+1) = (2x-3) \Rightarrow x = \frac{v-3}{u-v-2}$ $u(x+1)+vx = (2x+2) \Rightarrow x = \frac{2-u}{u+v-2}$	dM1
	$\Rightarrow \frac{v-3}{u-v-2} = \frac{2-u}{u+v-2}$ $(v-3)(u+v-2) = (2-u)(u-v-2)$ $\Rightarrow uv + v^2 - 2v - 3u - 3v + 6 = 2u - 2v - 4 - u^2 + uv + 2u (= 0)$	ddM1

	$\Rightarrow u^2+v^2-7u-3v+10=0$	
	$\Rightarrow \left(u-\frac{7}{2}\right)^2+\left(v-\frac{3}{2}\right)^2-\frac{49}{4}-\frac{9}{4}+10=0$	dddM1
	$\Rightarrow \left(u-\frac{7}{2}\right)^2+\left(v-\frac{3}{2}\right)^2=\frac{9}{2}$	A1
		(7)
Notes		
(a) **Alt I** **B1**: Use of $y=x$ in the expression for z **M1**: Uses $y=x$ and $z=x+\mathrm{i}y$ in the expression for w and equates to $u+\mathrm{i}v$, cross multiples and expands. **A1**: Correct expression with the i^2 terms simplified. **dM1**: Equates real and imaginary parts and proceeds to find two expressions for x. **Dependent on previous M mark.** **ddM1**: Eliminate x by solving simultaneously then proceed to simplify by cross multiplying and expanding to at least eliminate uv terms. **Dependent on both previous M marks.** **dddM1: Dependent on all previous M marks** proceeds to the equation of a circle via a valid competing the square attempt. Must be a valid circle equation with equal coefficients for the square terms **A1:** Correct circle equation obtained from correct work		

Question Number	Scheme	Marks
6(a)	$z=\cos\theta+\mathrm{i}\sin\theta$ **and** $\frac{1}{z}=\cos\theta-\mathrm{i}\sin\theta$ $\Rightarrow z+\frac{1}{z}=\ldots$ or $\Rightarrow z-\frac{1}{z}=\ldots$	M1
	$\frac{1}{2}\left(z+\frac{1}{z}\right)=\cos\theta$ and $\frac{1}{2\mathrm{i}}\left(z-\frac{1}{z}\right)=\sin\theta$	A1*
		(2)
(b)	$-\frac{1}{8\mathrm{i}}\left(z^3+3z^2\left(\frac{-1}{z}\right)+3z\left(\frac{-1}{z}\right)^2-\frac{1}{z^3}\right)$ **OR** $\frac{1}{16}\left(z^4+4z^3\left(\frac{1}{z}\right)+6z^2\left(\frac{1}{z}\right)^2+4z\left(\frac{1}{z}\right)^3+\left(\frac{1}{z}\right)^4\right)$	M1
	$\frac{\mathrm{i}}{128}\left(z^3-3z+\frac{3}{z}-\frac{1}{z^3}\right)\left(z^4+4z^2+6+\frac{4}{z^2}+\frac{1}{z^4}\right)=\ldots$	M1
ALT for first two M marks	$\left(z^2-\frac{1}{z^2}\right)^3=z^6+3z^4\left(-\frac{1}{z^2}\right)+3z^2\left(-\frac{1}{z^2}\right)^2+\left(-\frac{1}{z^2}\right)^3$	M1
	$=\frac{\mathrm{i}}{128}\left(z^6+3z^4\left(-\frac{1}{z^2}\right)+3z^2\left(-\frac{1}{z^2}\right)^2+\left(-\frac{1}{z^2}\right)^3\right)\left(z+\frac{1}{z}\right)=\cdots$	M1
	$=\frac{\mathrm{i}}{128}\left[\left(z^7-\frac{1}{z^7}\right)+\left(z^5-\frac{1}{z^5}\right)-3\left(z^3-\frac{1}{z^3}\right)-3\left(z-\frac{1}{z}\right)\right]$	M1
	$=\frac{\mathrm{i}}{128}\left(2\mathrm{i}\sin7\theta+2\mathrm{i}\sin5\theta-6\mathrm{i}\sin3\theta-6\mathrm{i}\sin\theta\right)$	M1
	$=-\frac{1}{64}\sin7\theta-\frac{1}{64}\sin5\theta+\frac{3}{64}\sin3\theta+\frac{3}{64}\sin\theta$ o.e.	A1
		(5)
(c)	$\int_0^{\frac{\pi}{2}}\sin^3\theta\cos^4\theta\,\mathrm{d}\theta=-\frac{1}{64}\int_0^{\frac{\pi}{2}}\left(\sin7\theta+\sin5\theta-3\sin3\theta-3\sin\theta\right)\mathrm{d}\theta$ $=-\frac{1}{64}\left[\frac{-\cos7\theta}{7}\frac{-\cos5\theta}{5}+\cos3\theta+3\cos\theta\right]_0^{\frac{\pi}{2}}$	M1A1ft

	$-\frac{1}{64}\left[(0+0+0+0)-\left(-\frac{1}{7}-\frac{1}{5}+1+3\right)\right]$	M1
	$=\frac{2}{35}$	A1
		(4)
		Total 11

Notes

(a)

M1: Uses de Moivre correctly to obtain correct expression for $\frac{1}{z}$ and either adds or subtracts to z. Must connect LHS and RHS in some way for one of the expressions- a collection of trig terms with no connection to z and $\frac{1}{z}$ is M0

A1*: Both correct expressions obtained from correct work

(b)

Note: this is a 'hence' question so candidates must use the identities in (a)

M1: Attempts to expand either bracket using binomial expansion, coefficients must be correct but condone slips if the intention is clear

M1: Attempts to multiply both expanded brackets, in terms of z, together

ALT for first two M marks

M1: (Recognising the product can be written as a difference of two squares)
Expands $\left(z^2-\frac{1}{z^2}\right)^3$. Coefficients must be correct but condone slips if the intention is clear

M1: Attempts to multiply their expansion of $\left(z^2-\frac{1}{z^2}\right)^3$ with $\left(z+\frac{1}{z}\right)$

M1: Groups terms in z correctly to form only sine terms after multiplication - could be implied by correct sine terms seen

M1: Use of general De Moivre $\left(z^n-\frac{1}{z^n}\right)\equiv 2\mathrm{i}\sin n\theta$ to obtain at least one correct term on the RHS- but the i's must be dealt with correctly- either consistently cancelled or remain in place

A1: Correct final result obtained from correct work.

(c)

M1: Integrates an expression of the form $a\sin 7\theta+b\sin 5\theta+c\sin 3\theta+d\sin\theta$ to obtain an expression of the form $\ldots\cos 7\theta+\ldots\cos 5\theta+\ldots\cos 3\theta+\ldots\cos\theta$ but M0 if they are clearly differentiating. Accept if the candidate integrates with coefficients as *a, b, c, d*

A1ft: Correct integration ft their values from (b) and accept in terms of *a, b, c, d*

M1: Substitutes the correct limits and subtracts the correct way round. This can be implied by a correct final answer, once the first M1A1 is scored as no calculator warning given in this question. M0 if they have clearly differentiated.

A1: Fully correct result, from correct integration

Question Number	Scheme		Marks
	$\sec x\frac{\mathrm{d}y}{\mathrm{d}x}-2y\sin x=\mathrm{e}^{x+\sin^2 x}$		
7(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}-2y\sin x\cos x=\cos x\mathrm{e}^{x+\sin^2 x}\Rightarrow \text{I.F.}=\mathrm{e}^{-\int 2\sin x\cos x\mathrm{d}x}$ oe		M1
	$=\mathrm{e}^{-\sin^2 x}$		A1
			(2)
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(y\mathrm{e}^{-\sin^2 x}\right)=\mathrm{e}^{-\sin^2 x}\cos x\mathrm{e}^{x+\sin^2 x}$ $\Rightarrow y\mathrm{e}^{-\sin^2 x}=\int \mathrm{e}^x\cos x\mathrm{d}x$ oe		M1A1
	$I=\int \mathrm{e}^x\cos x\mathrm{d}x=\mathrm{e}^x\sin x-\int \mathrm{e}^x\sin x\mathrm{d}x$	$\mathrm{I}=\int \mathrm{e}^x\cos x\mathrm{d}x=\mathrm{e}^x\cos x+\int \mathrm{e}^x\sin x\mathrm{d}x$	M1
	$=\mathrm{e}^x\sin x-\left[-\mathrm{e}^x\cos x+\int \mathrm{e}^x\cos x\mathrm{d}x\right]$ $\Rightarrow I=\frac{1}{2}\mathrm{e}^x\left(\sin x+\cos x\right)$	$=\mathrm{e}^x\cos x+\left[\mathrm{e}^x\sin x-\int \mathrm{e}^x\cos x\mathrm{d}x\right]$ $\Rightarrow I=\frac{1}{2}\mathrm{e}^x\left(\sin x+\cos x\right)$	M1A1
	$y\mathrm{e}^{-\sin^2 x}=\frac{1}{2}\mathrm{e}^x\left(\sin x+\cos x\right)+C$ $\Rightarrow y=\frac{\mathrm{e}^x\left(\sin x+\cos x\right)}{2\mathrm{e}^{-\sin^2 x}}+\frac{C}{\mathrm{e}^{-\sin^2 x}}$ oe		A1
			(6)
(c)	$x=0, y=2\Rightarrow 2=\frac{1}{2}+C$ $\Rightarrow C=\ldots\left(\frac{3}{2}\right)$		M1
	$\Rightarrow y=\frac{\frac{1}{2}\mathrm{e}^x\left(\sin x+\cos x\right)}{\mathrm{e}^{-\sin^2 x}}+\frac{3}{2\mathrm{e}^{-\sin^2 x}}$ oe		A1
			(2)
			Total 10

Notes

Mark parts (a) and (b) together

(a)

M1: Dividing by sec x (or multiplying by cos x) and obtaining $\text{I.F.} = e^{-\int 2\sin x\cos x\,dx}$ oe e.g. $\text{I.F.} = e^{-\int \sin 2x\,dx}$

A1: Obtains $\text{I.F.} = e^{-\sin^2 x}$

Note: if the candidate integrates $e^{-\int \sin 2x\,dx} = e^{\frac{1}{2}\cos 2x} = \cdots e^{\frac{1}{2}-\sin^2 x}$ then states that the $\text{I.F.} = e^{-\sin^2 x}$ with justification (e.g. any multiple of the IF is also an IF) then full marks can be scored in (a)

If candidates use a correct integration factor in parts (b) and (c) then all marks are available

(b)

M1: Obtains $y \times$ their IF (or correct IF) $= \int$ their IF (or correct IF) $\times \cos x e^{x+\sin^2 x}$

A1: simplifies RHS to obtain $y \times$ their IF (or correct IF) $= \int e^x \cos x\,dx$ or equivalent correct expression: e.g. if their IF is $e^{\frac{1}{2}-\sin^2 x}$ then they will obtain $y \times e^{\frac{1}{2}-\sin^2 x} = e^{\frac{1}{2}} \int e^x \cos x\,dx$

if their IF is $e^{\frac{1}{2}\cos 2x}$ then they will obtain $y \times e^{\frac{1}{2}\cos 2x} = \int e^{\frac{1}{2}\cos 2x} \cos x e^{x+\sin^2 x}\,dx$

or $y \times e^{\frac{1}{2}\cos 2x} = \int e^{x+\frac{1}{2}} \cos x\,dx$

M1: Uses IBP once on RHS (sign errors only)

M1: Uses IBP twice on RHS (sign errors only)

A1: Correct integral for $\int e^x \cos x\,dx$

A1: c.a.o. Constant introduced and treated correctly- correct final answer in any equivalent form

e.g. if their IF is $e^{\frac{1}{2}-\sin^2 x}$ then they will obtain $y = \dfrac{e^x e^{\frac{1}{2}}(\sin x + \cos x)}{2e^{\frac{1}{2}-\sin^2 x}} + \dfrac{C}{e^{\frac{1}{2}-\sin^2 x}}$

e.g. if their IF is $e^{\frac{1}{2}\cos 2x}$ then they will obtain $y = \dfrac{e^x e^{\frac{1}{2}}(\sin x + \cos x)}{2e^{\frac{1}{2}\cos 2x}} + \dfrac{C}{e^{\frac{1}{2}\cos 2x}}$

(c) **M1:** Substitutes the given boundary conditions and re-arranges to obtain a value for the constant from their GS

A1: Correct value and correct particular solution (in any equivalent form)

if their IF is $e^{\frac{1}{2}-\sin^2 x}$ or $e^{\frac{1}{2}\cos 2x}$ their constant will be $C = \ldots\left(\frac{3}{2}e^{\frac{1}{2}}\right)$

Question Number	Scheme	Marks
8		
(a)	POI: $2\cos\left(\theta+\frac{\pi}{6}\right)=\sec\left(\theta-\frac{\pi}{3}\right)\Rightarrow 2\cos\left(\theta+\frac{\pi}{6}\right)\cos\left(\theta-\frac{\pi}{3}\right)=1$	B1
	$2\cos\left(\theta+\frac{\pi}{6}\right)\sin\left(\theta+\frac{\pi}{6}\right)=1\Rightarrow\sin\left(2\theta+\frac{\pi}{3}\right)=1\Rightarrow 2\theta=\cdots\Rightarrow\theta=\cdots$	M1
	$2\theta=\frac{\pi}{6},\ldots\Rightarrow\theta=\frac{\pi}{12}$ and $r=2\cos\frac{\pi}{4}=\sqrt{2}$	A1
		(3)
(b) **Way 1**	**<u>Way1</u>** $\left(\frac{1}{2}\right)\int_0^{\frac{\pi}{12}}\left(\sec\left(\theta-\frac{\pi}{3}\right)\right)^2 \mathrm{d}\theta-\left(\frac{1}{2}\right)\int_0^{\frac{\pi}{12}}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta$ Overall strategy	M1
	$\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(\sec\left(\theta-\frac{\pi}{3}\right)\right)^2 \mathrm{d}\theta=k\tan\left(\theta-\frac{\pi}{3}\right)$ **OR** $\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta=k\left[\theta+\frac{\sin\left(2\theta+\frac{\pi}{3}\right)}{2}\right]$	M1
	For **both** $\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(\sec\left(\theta-\frac{\pi}{3}\right)\right)^2 \mathrm{d}\theta=k\tan\left(\theta-\frac{\pi}{3}\right)$ **and** $\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta=k\left[\theta+\frac{\sin\left(2\theta+\frac{\pi}{3}\right)}{2}\right]$	M1

	For each correct integral $\left(\frac{1}{2}\right)\left[\tan\left(\theta-\frac{\pi}{3}\right)\right]-\left(\frac{1}{2}\right)\left[2\theta+\sin\left(2\theta+\frac{\pi}{3}\right)\right]$	A1 A1
	$\frac{1}{2}\left[\tan\left(\theta-\frac{\pi}{3}\right)\right]_0^{\frac{\pi}{12}}-\left[\theta+\frac{\sin\left(2\theta+\frac{\pi}{3}\right)}{2}\right]_0^{\frac{\pi}{12}}$ $=\frac{1}{2}\left(\sqrt{3}-1\right)-\left[\left(\frac{\pi}{12}+\frac{1}{2}\right)-\left(0+\frac{\sqrt{3}}{4}\right)\right]=\ldots$	dM1
	$=\frac{3}{4}\sqrt{3}-1-\frac{\pi}{12}$	A1
		(7)
8(b) **Way 2**	$\text{small triangle}-\left(\int_0^{\frac{\pi}{12}}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta-\text{large triangle}\right)$ Overall strategy mark	M1
	Area of small triangle $=\frac{1}{2}\left[2-\sqrt{2}\cos\frac{\pi}{12}\right]\left[\sqrt{2}\sin\frac{\pi}{12}\right]=\ldots$	M1
	$=\left[\frac{2\sqrt{3}-3}{4}\right]$	A1
	$\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta=k\left[\theta+\frac{\sin\left(2\theta+\frac{\pi}{3}\right)}{2}\right]$	M1
	$\left(\frac{1}{2}\right)\left[2\theta+\sin\left(2\theta+\frac{\pi}{3}\right)\right]$	A1
	Area = $=\left[\frac{2\sqrt{3}-3}{4}\right]-\left[\left(\frac{\pi}{12}+\frac{1}{2}\right)-\left(0+\frac{\sqrt{3}}{4}\right)\right]+\frac{1}{2}\sqrt{2}\cos\frac{\pi}{12}\sqrt{2}\sin\frac{\pi}{12}=\cdots$	dM1
	$=\frac{3}{4}\sqrt{3}-1-\frac{\pi}{12}$	A1
		(7)

8(b) **Way 3**	Largest triangle - $\int_0^{\frac{\pi}{12}}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta$ Overall strategy mark	M1
	Largest triangle $\frac{1}{2}\times 2\times\sqrt{2}\sin\frac{\pi}{12} \quad = \ldots$	M1
	$=\frac{\sqrt{3}-1}{2}$	A1
	$\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta = k\left[\theta+\frac{\sin\left(2\theta+\frac{\pi}{3}\right)}{2}\right]$	M1
	$\left(\frac{1}{2}\right)\left[2\theta+\sin\left(2\theta+\frac{\pi}{3}\right)\right]$	A1
	Area = $=\frac{\sqrt{3}-1}{2}-\left[\left(\frac{\pi}{12}+\frac{1}{2}\right)-\left(0+\frac{\sqrt{3}}{4}\right)\right]=\cdots$	dM1
	$=\frac{3}{4}\sqrt{3}-1-\frac{\pi}{12}$	A1
		Total 10

Notes

(a)**B1:** Sets both equations equal and replaces $\sec\left(\theta-\frac{\pi}{3}\right)$ with $\frac{1}{\cos\left(\theta-\frac{\pi}{3}\right)}$

M1: Uses the identity $\cos\theta=\sin\left(\theta+\frac{\pi}{2}\right)$ and simplifies using the identity $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to obtain an expression of the form $\sin 2\theta = k$ then solves an equation of the form $\sin 2\theta = k$ to find a value for θ

A1: r and θ correctly obtained from correct work

(b) **Way 1**

M1: Correct overall strategy mark (may not be scored until the end and condone the omission of the $\frac{1}{2}$'s for this mark)

M1: Correct form for the integration of $\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(\sec\left(\theta-\frac{\pi}{3}\right)\right)^2 \mathrm{d}\theta$ **or** correct form for integration of $\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta$

M1: Correct form for the integration of both $\left(\frac{1}{2}\right)\int_{...}^{...}\left(\sec\left(\theta-\frac{\pi}{3}\right)\right)^2 \mathrm{d}\theta$ **and**

$$\left(\frac{1}{2}\right)\int_{...}^{...}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta$$

NOTE: for $\left(\frac{1}{2}\right)\int_{...}^{...}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta$ they may expand and use identities e.g.

$$\left(\frac{1}{2}\right)\int_{...}^{...}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta=\int 1+\cos\left(2\theta+\frac{\pi}{3}\right)=\int 1+\cos 2\theta\cos\frac{\pi}{3}-\sin 2\theta\sin\frac{\pi}{3}$$

$$=\theta+\frac{1}{4}\sin 2\theta+\frac{\sqrt{3}}{4}\cos 2\theta$$

A1: For one correct integral (condone the omission of the $\frac{1}{2}$ for this mark)

A1: For both correct integrals (condone the omission of the $\frac{1}{2}$ for this mark)

dM1: Dependent on the first M mark. Applies the correct limits the correct way round and makes progress to obtain an expression for the area. The $\frac{1}{2}$'s must be included for this mark.

A1: Correct final answer in exact equivalent form

(b) **Way 2**

M1: Correct overall strategy mark (may not be scored until the end and condone the omission of the $\frac{1}{2}$'s for this mark)

M1: Correct attempt at the area of the small triangle. Condone numerical slips etc, but must see evidence of the half and evidence of using the correct horizontal and vertical distances

A1: Correct exact area obtained

M1: Correct form for the integration of $\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2$

A1: Fully correct expression for the integration of $\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2$ but condone the omission of the $\frac{1}{2}$ for this mark

dM1: Dependent on the first M mark. Applies the correct limits the correct way round and makes progress to obtain an expression for the area. The $\frac{1}{2}$'s must be included for this mark

A1: Correct final answer in exact equivalent form

(b) **Way 3**

M1: Correct overall strategy mark (may not be scored until the end and condone the omission of the $\frac{1}{2}$'s for this mark)

M1: Correct attempt at the area of the largest triangle. Condone numerical slips etc, but must see evidence of the half and evidence of using the correct horizontal and vertical distances

A1: Correct exact area obtained

M1: Correct form for the integration of $\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2$

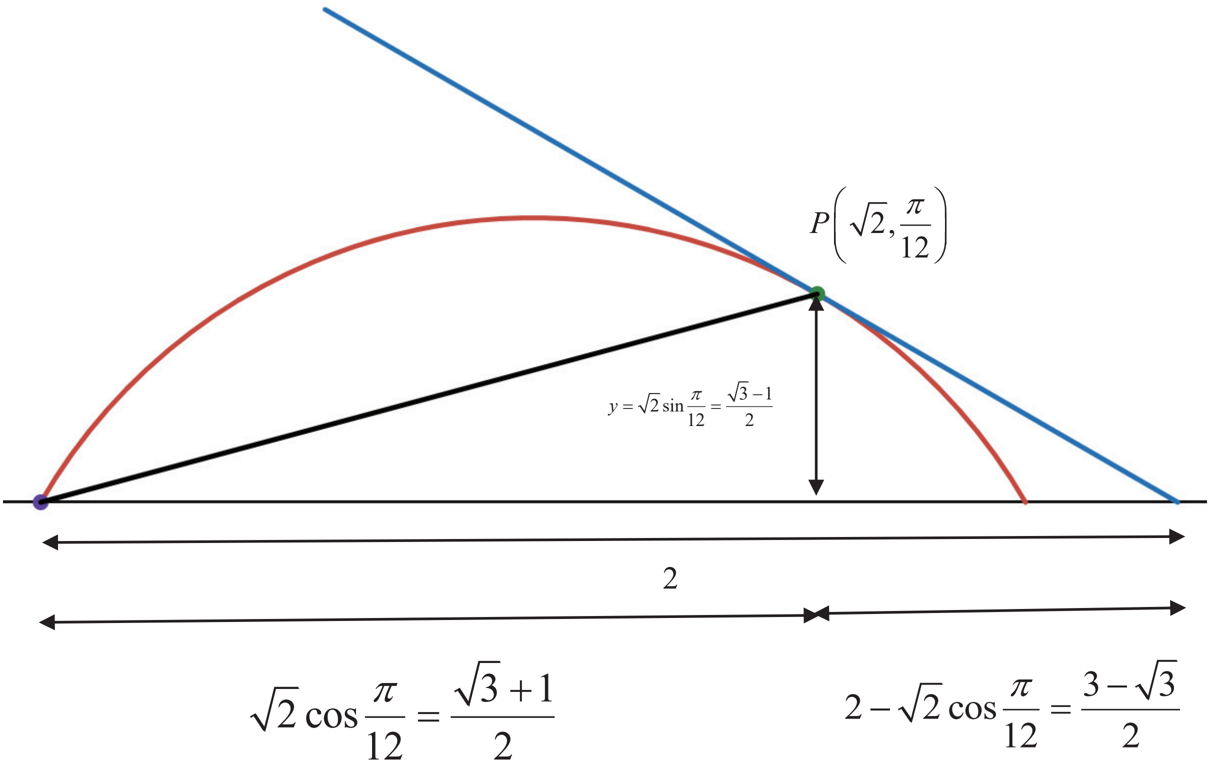
A1: Fully correct expression for the integration of $\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2$ but condone the omission of the $\frac{1}{2}$ for this mark

dM1: Dependent on the first M mark. Applies the correct limits the correct way round and makes progress to obtain an expression for the area. The $\frac{1}{2}$'s must be included for this mark

A1: Correct final answer in exact equivalent form

	TOTAL FOR PAPER: 75 MARKS

Useful diagram for Q8



$$\text{Area of small } \Delta = \frac{1}{2}\left(\frac{3-\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}-1}{2}\right) = \frac{2\sqrt{3}-3}{4}$$

$$\text{Area of large } \Delta = \frac{1}{2}\left(\frac{\sqrt{3}+1}{2}\right)\left(\frac{\sqrt{3}-1}{2}\right) = \frac{1}{4}$$

$$\text{Area of largest } \Delta = \frac{1}{2}(2)\left(\frac{\sqrt{3}-1}{2}\right) = \frac{\sqrt{3}-1}{2}$$