Jun 2026
WFM02 Further Pure Mathematics F2
Mark Scheme

Question Number	Scheme	Notes	Marks
1	$z_1=\frac{1}{2}\left(\cos\frac{4\pi}{5}+\mathrm{i}\sin\frac{4\pi}{5}\right)$	$z_2=\sqrt{3}-3\mathrm{i}$	
(a)(i)	$\frac{1}{(z_1)^3}=(z_1)^{-3}=\left(\frac{1}{2}\right)^{-3}\left(\cos\frac{4\pi}{5}+\mathrm{i}\sin\frac{4\pi}{5}\right)^{-3}$ $=8\left(\cos\frac{-12\pi}{5}+\mathrm{i}\sin\frac{-12\pi}{5}\right)$ or $8\mathrm{e}^{\frac{-12\pi}{5}\mathrm{i}}$	For 8... or ... $\left(\cos\frac{-12\pi}{5}+\mathrm{i}\sin\frac{-12\pi}{5}\right)$ oe etc	M1
	$8\mathrm{e}^{\frac{-2\pi}{5}\mathrm{i}}$	$8\mathrm{e}^{\frac{-2\pi}{5}\mathrm{i}}$ only	A1
(a)(ii)	$\lvert z_2\rvert=\sqrt{\left(\sqrt{3}\right)^2+3^2}\ \left\{=\sqrt{12}=2\sqrt{3}\right\}$	Correct numerical expression for $\lvert z_2\rvert$ or $\lvert z_2^*\rvert$	M1
	$\arg z_2=-\arctan\left(\frac{3}{\sqrt{3}}\right)=-\arctan\left(\sqrt{3}\right)=-\frac{\pi}{3}$	A correct value in radians for a relevant angle. Implied by any of $\pm\frac{\pi}{3}$, $\pm\frac{\pi}{6}$, $\pm\frac{2\pi}{3}$ or $\pm\frac{5\pi}{6}$	M1
	$\frac{24\mathrm{e}^{\frac{4\pi}{5}\mathrm{i}}}{2\sqrt{3}\mathrm{e}^{-\frac{\pi}{3}\mathrm{i}}}=4\sqrt{3}\mathrm{e}^{\frac{17\pi}{15}\mathrm{i}}$	Obtains $p\mathrm{e}^{q\mathrm{i}}$, using their z_1 and $2\sqrt{3}\mathrm{e}^{-\frac{\pi}{3}}$	M1
	$4\sqrt{3}\mathrm{e}^{-\frac{13\pi}{15}\mathrm{i}}$	Correct answer. Allow exact equivalents for $4\sqrt{3}$	A1
			(6)
(b)	$z=(z_1)^{\frac{1}{2}}=\frac{\sqrt{2}}{2}\left(\cos\frac{2\pi}{5}+\mathrm{i}\sin\frac{2\pi}{5}\right)$	For $\frac{\sqrt{2}}{2}$ ... oe or ... $\left(\cos\frac{2\pi}{5}+\mathrm{i}\sin\frac{2\pi}{5}\right)$ oe	M1
	$\frac{\sqrt{2}}{2}\left(\cos\frac{2\pi}{5}+\mathrm{i}\sin\frac{2\pi}{5}\right)$ or $\frac{\sqrt{2}}{2}\left(\cos\frac{7\pi}{5}+\mathrm{i}\sin\frac{7\pi}{5}\right)$	One correct root (any equivalents)	A1
	$\frac{\sqrt{2}}{2}\left(\cos\frac{2\pi}{5}+\mathrm{i}\sin\frac{2\pi}{5}\right)$ and $\frac{\sqrt{2}}{2}\left(\cos\frac{-3\pi}{5}+\mathrm{i}\sin\frac{-3\pi}{5}\right)$	Both correct roots in correct form. Allow exact equivalents for $\frac{\sqrt{2}}{2}$	A1
			(3)
			Total 9

Further Notes

Accept equivalent notations throughout (e.g. cos/sin form instead of exponential form for (a)(ii)) except for final answers.

(a)(i)

M1: A partial attempt at the cube, either the modulus or the argument sorted out correctly, though not necessarily simplified or in the correct range. You can condone if they miss an i for the method mark.

A1cao: Fully correct and simplified with argument in the specified range.

(a)(ii)

M1: Correctly attempts the modulus of z_2 or its conjugate (see alt below).

M1: Attempts the argument of z_2 or its conjugate. If no method shown accept any of the angles given, for method. Look for an attempt at an arctan of a relevant ratio.

M1: Look for an attempt at dividing moduli and subtracting argument for their attempts at z_1 and z_2 of suitable forms (Euler or cis form). Alternative here is to first multiply numerator and denominator by the conjugate of z_2 and then find in Euler form and multiply moduli, add arguments

$$\frac{48z_1}{\sqrt{3}-3\text{i}} \times \frac{\sqrt{3}+3\text{i}}{\sqrt{3}+3\text{i}} = \frac{24\text{e}^{\frac{4\pi\text{i}}{5}}\left(2\sqrt{3}\text{e}^{\frac{\pi}{3}\text{i}}\right)}{12} = 4\sqrt{3}\text{e}^{\frac{12\pi+5\pi}{15}\text{i}}$$

A1: Correct answer with argument in the specified range. Allow simplified exact equivalents of $4\sqrt{3}$ e.g. $\frac{12}{\sqrt{3}}$

(b)

M1: Attempts the square root using De Moivre, with either a correct modulus or correct argument found.

A1: Any one correct answer in any form, need not be simplified.

A1: Both answer correct in the form specified and no other incorrect solutions extra solutions.

Note if they give $\pm\frac{\sqrt{2}}{2}\left(\cos\frac{2\pi}{5}+\text{i}\sin\frac{2\pi}{5}\right)$ and $\pm\frac{\sqrt{2}}{2}\left(\cos\frac{-3\pi}{5}+\text{i}\sin\frac{-3\pi}{5}\right)$ then M1A1A1 can be scored since both correct roots are given in the correct form and no incorrect roots are given (there are repeated roots in an incorrect form). If just $\pm\frac{\sqrt{2}}{2}\left(\cos\frac{2\pi}{5}+\text{i}\sin\frac{2\pi}{5}\right)$ is given it is M1A1A0.

Question Number	Scheme	Notes	Marks
2	$\mathrm{f}(x)=(1+4x)^2\ln(1+4x)$	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+(3x-1)\frac{\mathrm{d}y}{\mathrm{d}x}+2y^2=5$	
(i)(a)	$\mathrm{f}'(x)=2(1+4x)(4)\ln(1+4x)+\frac{4(1+4x)^2}{1+4x}$ $\{=8(1+4x)\ln(1+4x)+4(1+4x)\}$	Differentiates using product and chain rule to obtain an expression of the correct form	**M1**
	$\mathrm{f}''(x)=32\ln(1+4x)+\frac{32(1+4x)}{1+4x}+16\Rightarrow\ldots$	Differentiates using product rule again to obtain $\mathrm{f}''(x)=A\ln(1+4x)+B$	**dM1**
	$32\ln(1+4x)+48$	Correct expression (condone missing brackets)	**A1**
			(3)
(i)(b)	$\mathrm{f}''(0)=48\rightarrow\frac{48}{2}=24$	24 or their $\frac{B}{2}$ Condone $24x^2$	**B1ft**
			(1)
(ii)	$x=1,\ \frac{\mathrm{d}y}{\mathrm{d}x}=-3,\ y=2\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-6+8=5\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=3$	Correct value for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	**B1**
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+3\frac{\mathrm{d}y}{\mathrm{d}x}+(3x-1)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+4y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	M1: Differentiates to obtain an equation with at least 3 terms of the correct form. Condone "=5" for "=0" A1: Correct differentiation	**M1** **A1**
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}-9+6-24=0\Rightarrow\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\ldots\ \{27\}$	Substitutes $x=1,\ \frac{\mathrm{d}y}{\mathrm{d}x}=-3,\ y=2$ and their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ into their equation to find a value for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$	**M1**
	$y=2-3(x-1)+\frac{3}{2}(x-1)^2+\frac{9}{2}(x-1)^3$	M1: Uses $x=1,\ \frac{\mathrm{d}y}{\mathrm{d}x}=-3,\ y=2$ and their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ & $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ to form a correct series expansion (may be unsimplified)	**dM1** **A1**
	A1: Correct expansion with simplified coefficients.		
			(6)
			Total 10

Further Notes

(i)

dM mark, must see evidence of the product rule having been used.

(b) Accept if seen as part of a written out series (ignore other terms). Allow if they make up a B if they don't have a derivative of correct form.

(ii)

2nd M mark accept any value following a suitable attempt at the derivative to imply the substitution if method is not seen (though if using the function from (i) it is M0).

Final dM mark, there must have been attempts at values for the correct derivatives in (ii). Use of the function in (i) will be dM0.

If they do not show a value for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$ before substituing in the Taylor series then you may need to check if their value fits with the third derivative expression. If correct for their expression allow M1dM for the values, if incorrect score M1dM0 (bod).

Final A, the "y =" may be omitted and condone f(x) = …, but x = .. or f(x − 1) = … is A0. Must be from a correct third derivative equation.

Question Number	Scheme/Notes		Marks
3	$\dfrac{x+k}{x-3} \geqslant \dfrac{3x}{x+4}$		
(a)	$\dfrac{x+k}{x-3} - \dfrac{3x}{x+4} \geqslant 0$ $\dfrac{(x+k)(x+4)-3x(x-3)}{(x-3)(x+4)} \geqslant 0$	$(x-3)(x+4)^2(x+k) \geqslant 3x(x+4)(x-3)^2$ $(x-3)(x+4)^2(x+k) - 3x(x+4)(x-3)^2 \geqslant 0$ $\{\Rightarrow (x-3)(x+4)[(x+4)(x+k)-3x(x-3)] \geqslant 0\}$	M1
	Brings terms to one side and attempts common denominator or multiplies by $(x-3)^2(x+4)^2$ and brings terms to one side. See note below.		
	$(x-3)(x+4)(x^2+4x+kx+4k-3x^2+9x) \geqslant 0$ $(x-3)(x+4)(-2x^2+(k+13)x+4k) \geqslant 0$ $(x-3)(x+4)(2x^2-(k+13)x-4k) \leqslant 0$	M1: Reaches $(x-3)(x+4)(2x^2+...x+...) \leqslant 0$ A1: Correct inequality from fully correct work	M1 A1
			(3)
(b)	$k=2 \Rightarrow (x-3)(x+4)(2x^2-15x-8) \leqslant 0$ $2x^2-15x-8=(2x+1)(x-8)=0 \Rightarrow x=-\dfrac{1}{2},\ 8$	Substitutes $k=2$ and solves quadratic.	M1
	If restarting with $\dfrac{x+2}{x-3} \geqslant \dfrac{3x}{x+4}$ then allow for solving the equivalent quadratic however attained from their method (e.g. cross multiplying).		
	$(x-3)(x+4)(2x+1)(x-8) \leqslant 0$ $\Rightarrow$ critical values $x=-4,\ -\dfrac{1}{2},\ 3,\ 8$ $\Rightarrow \alpha <,\leqslant x <,\leqslant \beta, \gamma <,\leqslant x <,\leqslant \delta$	Identifies the correct shape of the regions with cvs $-4,\ 3$ and their $-\dfrac{1}{2}$ and 8. Inequality strictness may be incorrect but the ascending order of roots with correct directions of inequalities must be given	M1
	$\Rightarrow -4 < x \leqslant -\dfrac{1}{2},\quad 3 < x \leqslant 8$	Correct regions condoning errors with inequality strictness	A1
		Fully correct solution set.	A1
			(4)
			Total 7

Further Notes

(a)

For those with with an **initial** incorrect step of inequalities (e.g. multiplying through by $(x-3)(x+4)$ initially and then again later), max M0M1A0 can be scored. If an initial correct step putting over a common denominator is followed by similar partial multiplications which are incorrect but eventually recovered, then M1M1A0 may be scored.

(b)

1st M for solving the quadratic – accept correct roots found for their quadratic or else usual rules if method shown.

2nd M you are looking for the "second" and "fourth" portions of the number line with their ascending roots.

Final mark, allow any suitable notation e.g., $\left(-4,-\dfrac{1}{2}\right],\ (3,\ 8]$. Accet with a comma, "and" or "or" between but if using set notation then they must be using the union (intersection is final A0 but alllow the M and first A).

Question Number	Scheme	Notes	Marks
4	$x\dfrac{dy}{dx}+2y=12x^2\sqrt{x^4+9}$	$x>0$	
(a)	$\dfrac{dy}{dx}+\dfrac{2}{x}y=12x\sqrt{x^4+9}\Rightarrow \text{IF}=e^{\int\frac{2}{x}dx}$	$\text{IF}=e^{\int\pm\frac{2}{x}(dx)}$	M1
	$e^{\int\frac{2}{x}dx}=e^{2\ln x}=e^{\ln x^2}=x^2$	IF $=x^2$ (may be simplified later)	A1
	$x^2y=\int 12x^3\sqrt{x^4+9}\,(dx)$	$\text{IF}\times y=\int(\text{IF}\times f(x))(dx)$ with their IF and f(x)	M1
	$\int 12x^3\sqrt{x^4+9}\,dx=2(x^4+9)^{\frac{3}{2}}\ (+c)$	$\int ...x^3\sqrt{x^4+9}\,(dx)\to ...(x^4+9)^{\frac{3}{2}}$	M1
	$y=\dfrac{2(x^4+9)^{\frac{3}{2}}+c}{x^2}$ or $y=\dfrac{2(x^4+9)^{\frac{3}{2}}}{x^2}+\dfrac{c}{x^2}$	Correct answer in $y=f(x)$ form with constant correctly placed	A1
			(5)
(b)	$y=\dfrac{715}{12},\ x=2\Rightarrow\dfrac{715}{12}=\dfrac{250+c}{4}\Rightarrow c=...\left\{\dfrac{-35}{3}\right\}$	Uses $y=\dfrac{715}{12}$ and $x=2$ to find a value for c	M1
	$y=\dfrac{2(x^4+9)^{\frac{3}{2}}-\frac{35}{3}}{x^2}$ oe e.g., $y=\dfrac{6(x^4+9)^{\frac{3}{2}}-35}{3x^2}$	Correct particular solution	A1
			(2)
			Total 7

Further Notes

(a)

M1A1 may be implied by sight of a correct integrating factor stated or used.

2nd M, condone slips in simplifying $\sqrt{x^4+9}$ to x^2+3 for the method mark, but must have had correct form of expression to score the 3rd M. Condone using $12x^2\sqrt{x^4+9}$ for f(x).

Final A, the constant of integration must have been correctly treated. Accept unsimplified coefficients.

(b)

The M may be scored from their answer to (a) as long as it included a constant of integration.

If no method is shown you will to check their value is correct for their equation.

For the A mark coefficients should be simplified.

Question Number	Scheme	Notes	Marks
5(a)	$\mathrm{f}(r)=\frac{1}{r(r+1)}=\frac{1}{r}-\frac{1}{r+1}$	Correct partial fractions	B1
			(1)
(b)	e.g., $\mathrm{g}(r)=\frac{r(r+1)(12r^2+k)+1}{r(r+1)}=12r^2+k+\frac{1}{r(r+1)}\Rightarrow \mathrm{h}(r)=12r^2+k$ * **or** $\mathbf{h(r)=g(r)-f(r)=\frac{12r^4+12r^3+kr^2+kr+1}{r(r+1)}-\frac{1}{r(r+1)}=\frac{r(r+1)(12r^2+k)}{r(r+1)}=12r^2+k}$ Obtains $\mathrm{h}(r)=12r^2+k$ with factorisation seen or via long division. Can follow B0 in (a) - ft their partial fractions if necessary.		B1*
			(1)
(c)	$\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)=\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\ldots+\frac{1}{n}-\frac{1}{n+1}$	Attempts the differences on the result of (a)	M1
	$\sum_{r=1}^{n}\mathrm{g}(r)=12\times\frac{1}{6}n(n+1)(2n+1)+nk+1-\frac{1}{n+1}$	Applies a standard formula on $\mathrm{h}(r)$ Fully correct expression	M1 A1
	$(n+1)\sum_{r=1}^{n}\mathrm{g}(r)=2n(n+1)^2(2n+1)+kn(n+1)+n+1-1$ $=2n(2n+1)(n^2+2n+1)+kn^2+kn+n$ $=2n(2n^3+5n^2+4n+1)+kn^2+kn+n$ $=4n^4+10n^3+(k+8)n^2+(k+3)n$		ddM1 A1
			(5)
(d)	$n(n+2)(4n^2+2n-1)=\{4n^4+2n^3+8n^3\}-n^2+4n^2-2n$ $\Rightarrow k+8=-1+4\Rightarrow k=\ldots\ \{-5\}$ or $k+3=-2\Rightarrow k=\ldots\ \{-5\}$ Either sets their $k+8=-1+4$ or $k+3=-2$ and solves for k		M1
	$k=-5$	$k=-5$ only, with no incorrect work	A1
			(2)
			Total 9

Further Notes

(b) Other method such as use of long division are possible. Any correct method can score the mark but must be complete – e.g. by long division they must reach remainder 0 to score the mark.

(c)

M1: Uses the difference method with their partial fractions of the correct form (if using their partial fractions they must produce a difference). Implied by a correct expression for their fractions otherwise 3 rows from $r=1,\ 2,\ n-1,\ n$ must be attempted. Note extraction is not required and the rows may be embedded within working for the full summation.

M1: Replaces $\sum_{r=1}^{n}r^2$ with $\frac{1}{6}n(n+1)(2n+1)$ or $\sum_{r=1}^{n}k$ with kn

A1: Correct expression for $\sum_{r=1}^{n}\mathrm{g}(r)$

ddM1: Must have scored both previous M marks. Obtains $(n+1)\sum_{r=1}^{n}\mathrm{g}(r)=$ an expanded quartic expression.

A1: Correct expression.

(d)

Alt: M1 substitutes n = 1 or other appropriate value to generator and solve an equation in k from setting expressions equal.

Question Number	Scheme	Notes	Marks
6(a)	area of $R=\frac{1}{2}\int_0^{\pi} r^2\mathrm{d}\theta=\frac{1}{2}\int_0^{\pi}(k-2\cos\theta)^2\,\mathrm{d}\theta=\frac{1}{2}\int_0^{\pi}\left(k^2-4k\cos\theta+4\cos^2\theta\right)\mathrm{d}\theta$		M1
	$=\frac{1}{2}\int_0^{\pi}\left(k^2-4k\cos\theta+4\left(\frac{1}{2}\cos 2\theta+\frac{1}{2}\right)\right)\mathrm{d}\theta$	Uses $\cos^2\theta=\pm\frac{1}{2}\cos 2\theta\pm\frac{1}{2}$	M1
	$=\frac{1}{2}\int_0^{\pi}\left(k^2+2-4k\cos\theta+2\cos 2\theta\right)\mathrm{d}\theta$ $=\frac{1}{2}\left[\left(k^2+2\right)\theta-4k\sin\theta+\sin 2\theta\right]_0^{\pi}$	Integrates to obtain expression of the correct form	M1
	$\frac{1}{2}\left(k^2+2\right)\pi=\left(4+\sqrt{5}\right)\pi$ $k^2+2=8+2\sqrt{5}\Rightarrow k^2=6+2\sqrt{5}$ *	Reaches $k^2=6+2\sqrt{5}$ with sufficient work and no errors seen	A1*
			(4)
(b)	$x=r\cos\theta\Rightarrow x=\left(1+\sqrt{5}\right)\cos\theta-2\cos^2\theta$	Uses $x=r\cos\theta$	M1
	$\frac{\mathrm{d}x}{\mathrm{d}\theta}=-\left(1+\sqrt{5}\right)\sin\theta+4\cos\theta\sin\theta$	Differentiates to the correct form (oe)	M1
	$\sin\theta\left(4\cos\theta-\left(1+\sqrt{5}\right)\right)=0\Rightarrow\cos\theta=\frac{1+\sqrt{5}}{4}$	Sets = 0 and solves to obtain a valid value for $\cos\theta$ consistent with their derivative (oe).	ddM1
	$\theta=\frac{\pi}{5}$	Correct value for θ. Allow awrt 0.628	A1
	$\left\{r=1+\sqrt{5}-\frac{1}{2}-\frac{1}{2}\sqrt{5}\Rightarrow\right\}$ $r=\frac{1}{2}+\frac{1}{2}\sqrt{5}$ or $\frac{1}{2}\left(1+\sqrt{5}\right)$	Correct exact value for r or awrt 1.62 and no others points.	A1
			(5)
			Total 9

Further notes

(a)

M1: Uses $\frac{1}{2}\int r^2\mathrm{d}\theta$ and expands to expression of the correct form. Condone missing the middle k

2nd M: Applies a suitable attempt at the double angle formula to the integrand (may be in separate working) to achieve an integrable form. Does not have to be within integrand.

Final A: Sufficient working must be shown. There must be a step with the substitution of limits carried out (allow bottom limit zero to be implied) and trig terms evaluated (allow if the sine terms are not seen), set equal to the given area and either expansion or cancellation of π before the final answer. Watch out for cases where the k is missing in the middle term of expansion, which score 1110.

(b)

First M Must be applied to curve, so r must be substituted, but allow with use of k rather than a value or awrt 3.23. Condone a copy error when substituting for r.

Second M Allow attempts to differentiate either $x=r\cos\theta$ or $y=r\sin\theta$ for this mark. Form of derivative must be correct in either case. For $x=r\cos\theta$ may get $A\sin\theta+B\sin 2\theta$. For $y=r\sin\theta$ they should get $\frac{\mathrm{d}y}{\mathrm{d}\theta}=A\cos\theta+B\cos 2\theta$ or $A\cos\theta+B(\cos^2\theta\pm\sin^2\theta)$ (oe). May use k or awrt 3.23 for k.

ddM Must have scored both previous M marks, so must be using $x=r\cos\theta$. Value found should be consistent up to sign errors when rearranging with their derivative. May be in terms of k.

Note they e.g. may use double angle formula to get $\cos 2\theta=\frac{2\sqrt{5}-2}{8}$ – allow up to sign error as before.

A marks, accept answers rounding to 3 s.f. or alternative exact forms. Accept $\frac{1}{2}\sqrt{6+2\sqrt{5}}$ for r

Question Number	Scheme	Notes	Marks
7	$y = z\cos x \qquad \cos^2 x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\sin 2x\dfrac{\mathrm{d}y}{\mathrm{d}x}+2y=12\cos 2x\cos^3 x$		
(a)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\cos x\dfrac{\mathrm{d}z}{\mathrm{d}x}-z\sin x$	Correct differentiation of $y = z\cos x$	B1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\cos x\dfrac{\mathrm{d}^2z}{\mathrm{d}x^2}-\sin x\dfrac{\mathrm{d}z}{\mathrm{d}x}-\sin x\dfrac{\mathrm{d}z}{\mathrm{d}x}-z\cos x$	Attempts to differentiate a second time using the product rule at least once.	M1
	$\cos^2 x\left(\cos x\dfrac{\mathrm{d}^2z}{\mathrm{d}x^2}-2\sin x\dfrac{\mathrm{d}z}{\mathrm{d}x}-z\cos x\right)+\sin 2x\left(\cos x\dfrac{\mathrm{d}z}{\mathrm{d}x}-z\sin x\right)+2z\cos x=12\cos 2x\cos^3 x$ Substitutes their derivatives into differential equation (I)		M1
	$\cos^3 x\dfrac{\mathrm{d}^2z}{\mathrm{d}x^2}-2\sin x\cos^2 x\dfrac{\mathrm{d}z}{\mathrm{d}x}-z\cos^3 x+2\sin x\cos^2 x\dfrac{\mathrm{d}z}{\mathrm{d}x}-2z\sin^2 x\cos x+2z\cos x=12\cos 2x\cos^3 x$ $\cos^3 x\dfrac{\mathrm{d}^2z}{\mathrm{d}x^2}-z\cos^3 x-2z\cos x\left(1-\cos^2 x\right)+2z\cos x=12\cos 2x\cos^3 x$ Uses **correct** identities to obtain an equation in cos only		M1
	$\cos^3 x\dfrac{\mathrm{d}^2z}{\mathrm{d}x^2}+z\cos^3 x=12\cos 2x\cos^3 x$ $\Rightarrow \dfrac{\mathrm{d}^2z}{\mathrm{d}x^2}+z=12\cos 2x$ *	Fully correct proof with no errors seen	A1*
			(5)
(b)	$z=\lambda\cos 2x,\ z'=-2\lambda\sin 2x,\ z''=-4\lambda\cos 2x$ $-4\lambda\cos 2x+\lambda\cos 2x=12\cos 2x$ $\Rightarrow -3\lambda=12\Rightarrow\lambda=-4$	Uses PI of $z=\lambda\cos 2x$, differentiates twice to obtain correct forms and substitutes to obtain a value for λ	M1
	$m^2+1=0\Rightarrow m=\pm\mathrm{i}$ $\Rightarrow \text{CF}: z=A\cos x+B\sin x$	Correct CF. The "$z=$" is not required	B1
	$\Rightarrow (z=)\ A\cos x+B\sin x-4\cos 2x$	Correct GS ft their CF and λ. The "z =" is not required and condone "y=".	M1
	$\Rightarrow y=\cos x\left(A\cos x+B\sin x-4\cos 2x\right)$	Fully correct GS with "$y=$"	A1
			(4)
(c)	$(0,1)\Rightarrow 1=1(A+0-4)\Rightarrow A=5$	Uses coordinates to obtain a value for A	M1
	$\left(\dfrac{3\pi}{4},4\right)\Rightarrow 4=-\dfrac{\sqrt{2}}{2}\left(-\dfrac{5\sqrt{2}}{2}+\dfrac{\sqrt{2}}{2}B-0\right)$ $\Rightarrow 4=\dfrac{5}{2}-\dfrac{1}{2}B\Rightarrow B=-3$	Uses coordinates and their A to obtain a value for B	dM1
	$\left(\dfrac{\pi}{3},p\right)\Rightarrow (p)=\dfrac{1}{2}\left(\dfrac{5}{2}-\dfrac{3\sqrt{3}}{2}+2\right)$	Uses coordinates and their A and B to obtain a numerical expression for p	ddM1
	$(p)=\dfrac{1}{2}\left(\dfrac{9}{2}-\dfrac{3\sqrt{3}}{2}\right)=\dfrac{3}{4}\left(3-\sqrt{3}\right)$	Correct value for p in the correct form	A1
			(4)
			Total 13

Further Notes

Condone poor notation on the derivatives, use of c,s notation for cos$x$ and sin$x$ etc as long as the mathematics is correct.

(a) B0M1 is possible at the start for those do $\frac{\mathrm{d}y}{\mathrm{d}x} = \cos x \frac{\mathrm{d}z}{\mathrm{d}x}$ but must be producing a second derivative from their attempt at the first derivative, not just a general form. For the B1 they may first find $\frac{\mathrm{d}y}{\mathrm{d}z}$ then apply $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}z}\frac{\mathrm{d}z}{\mathrm{d}x}$ but must obtain a correct expression involving $\frac{\mathrm{d}y}{\mathrm{d}x}$ to score the mark.

3rd M The trig identities identities use should be clear, no big jumps.

(b)

First M: The PI may include other terms (e.g. a $\mu \sin 2x$ term). Allow for $\cos 2x \to \ldots \sin 2x$ (where … can be any positive or negative number) as an attempt to differentate. They may must be substituting into the correct equation.

For the B and M marks, if their initial CF is incorrect (so B0) and when the forming the GS it is not is not consitent but still an attempt at a CF, award generously.

Alts:

(a) If they rearrange first they may get (oe) $z = \frac{y}{\cos x} \Rightarrow \frac{\mathrm{d}z}{\mathrm{d}x} = y\sec x \tan x + \frac{\mathrm{d}y}{\mathrm{d}x}\sec x$ which can score the B1, then for M1 the below (oe)

$$\frac{\mathrm{d}^2z}{\mathrm{d}x^2} = y\sec^3 x + y\sec x\tan^2 x + \frac{\mathrm{d}y}{\mathrm{d}x}\sec x\tan x + \frac{\mathrm{d}^2y}{\mathrm{d}x^2}\sec x + \frac{\mathrm{d}y}{\mathrm{d}x}\sec x\tan x$$

Then can substitute or adapt equation for the next M, e.g.

$$\Rightarrow \cos^3 x\frac{\mathrm{d}^2z}{\mathrm{d}x^2} = y + y\sin^2 x + 2\frac{\mathrm{d}y}{\mathrm{d}x}\sin x\cos x + \cos^2 x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$$

$$= \cos^2 x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \sin 2x\frac{\mathrm{d}y}{\mathrm{d}x} + y + y(1-\cos^2 x)$$

$$= \cos^2 x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \sin 2x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y - y\cos^2 x = 12\cos 2x\cos^3 x - z\cos^3 x$$

(both M's scored at this point as equation substituted and in terms of cos only)

$$\Rightarrow \frac{\mathrm{d}^2z}{\mathrm{d}x^2} = 12\cos 2x - z \Rightarrow \frac{\mathrm{d}^2z}{\mathrm{d}x^2} + z = 12\cos 2x$$

Variations on this theme are possible.

(c) If by error they only have one constant of integration in their equation, maximum M1dM0ddM0A0 can be scored for using either coordiante to find their constant.

An incorrect answer to (b) may mean they need to set and solve simultaneous equations to find their constants. M1 for full method to find one constant, dM1 for finding both using both coordinates.

(d)

ddMand A mark, condone p being called y for these marks.

Note if their answer to (c) is $y = A\cos x + B\sin x - 4\cos 2x$ then they can score M1dM1ddM1A0 in (d), they should get $A = 5$, $B = 5 + 4\sqrt{2}$ and $p = \frac{5}{2} + \left(5 + 4\sqrt{2}\right)\frac{\sqrt{3}}{2} + 2$ (though you are looking for the method, not these answers).

Question Number	Scheme	Notes	Marks
8(a)	$y = mx \quad y = \frac{1}{2}\left(x^2 - 4x + 3\right)$ $x^2 - 4x + 3 = 2mx$ $x^2 - (4 + 2m)x + 3 = 0$	Substitutes $y = mx$ into $y = \frac{1}{2}\left(x^2 - 4x + 3\right)$ and rearranges to a 3TQ (the "= 0" is not required)	M1
	$d = (4 + 2m)^2 - 12 = 0$ $4 + 2m = \pm 2\sqrt{3} \Rightarrow m = -2 \pm \sqrt{3}$ **or** $4m^2 + 16m + 4 = m^2 + 4m + 1 = 0$ $\Rightarrow (m + 2)^2 = 3 \Rightarrow m = -2 \pm \sqrt{3}$ or $m = \frac{-4 \pm \sqrt{16 - 4}}{2} = -2 \pm \sqrt{3}$	M1: Applies discriminant = 0 and solves via usual rules to obtain a value for m A1: Both correct exact values.	M1 A1
			(3)
(b)	E.g. $\arctan\left(-2 + \sqrt{3}\right) = -15^\circ, -\frac{\pi}{12}, -0.26..., -0.083\pi$ or $\arctan\left(-2 - \sqrt{3}\right) = -75^\circ, -\frac{5\pi}{12}, -1.3..., -0.42\pi$	Attempts to takes the arctan to find a relevant angle of one of their gradients.	M1
	$-15^\circ \leqslant \arg z \leqslant ...^\circ$ or $...^\circ \leqslant \arg z \leqslant 105^\circ$ $\left\{105^\circ = \frac{7\pi}{12}, 1.8..., 0.58\pi\right\}$	One end of the range correct - value and direction of inequality (not just the value). Accept radians.	A1
	$-15^\circ \leqslant \arg z \leqslant 105^\circ$	Fully correct range for arg z in degrees.	A1
			(3)
(c)	$\Rightarrow w = \mathrm{i}(x - 2\mathrm{i})^2 + 2y - 5\mathrm{i}$ $= \mathrm{i}\left(x^2 - 4\mathrm{i}x - 4\right) + x^2 - 4x + 3 - 5\mathrm{i}$ $= \mathrm{i}x^2 + 4x - 4\mathrm{i} + x^2 - 4x + 3 - 5\mathrm{i}$	Substitutes $y = \frac{1}{2}\left(x^2 - 4x + 3\right)$ and fully expands, condone a slip.	M1
	$u + \mathrm{i}v = x^2 + 3 + \mathrm{i}\left(x^2 - 9\right)$ $\Rightarrow u = x^2 + 3, \quad v = x^2 - 9$ $v = u - 12$	M1: Equates real and imaginary parts to obtain expressions in x for both u and v and eliminates x to get a linear equation. A1: Correct equation	dM1 A1
			(3)
(d)	$\lvert w \rvert_{\min} = \frac{1}{2}\sqrt{12^2 + 12^2}$	Correct numerical expression for $\lvert w \rvert_{\min}$ Allow ft on $v = \pm u \pm k$	M1
	$6\sqrt{2}$	$6\sqrt{2}$ only	A1cso
			(2)
			Total 11
			PAPER TOTAL: 75

Further Notes

Alternative approaches to part (a):

Alt 1

M1: Finds the derivative at x, forms $y =$ "m"x with their derivative in terms of x and equates to the original curve.

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x - 2 \Rightarrow y = (x - 2)x \Rightarrow \frac{1}{2}(x^2 - 4x + 3) = x(x - 2)$$

M1: Expands and solves the resulting equation (usual rules – by calculator must be correct roots for their quadratic) and substitutes back into $m = x - 2$ to find m.

A1: Correct values for m

(Note this is an informal way of doing Alt 2, but works since the equation $y = x^2 - 2x$ formed is representing the points that the line parallel to the tangent will pass through at a given value of x)

Alt 2
M1: Finds the derivative of the curve at a general point, $x = a$ say, and uses the gradient to set up the general equation of the tangent,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x - 2 \Rightarrow m = a - 2 \Rightarrow \text{tangent is } y - \frac{1}{2}(a^2 - 4a + 3) = (a-2)(x-a)$$

M1: Sets the constant term from their tangent equation equal to 0, solves for a (usual rules– by calculator must be correct roots for their quadratic) and substitutes back in to find a value for m. A1 correct values

$$c = 0 \Rightarrow \frac{1}{2}(a^2 - 4a + 3) = (a-2)a \Rightarrow a^2 = 3 \Rightarrow a = \pm\sqrt{3} \Rightarrow m = \pm\sqrt{3} - 2$$

(b)
M1: Correctly takes the arctan of one of their gradients. Could be in radians (awrt 2sf if necessary). Allow for evaluating any of $\arctan(\pm m)$ or $\arctan\left(\pm\frac{1}{m}\right)$. May be done via finding points on the tangents or the point on the curve that led to their m. Allow for any correct attempt to identify a relevant angle.
A1: A range with two endpoints and one correct and in the right place. Condone strict inequalities and radians. Degrees symbol not required.
A1: Degrees symbol not required. Any acceptable notation.

(c)
The substitution of y into the coordinates may be done after extracting real and imaginary terms, but the first M is not awarded until the substitution has been made. The dM is not awarded until x is eliminated and a a linear equation in u and v only is formed.

Alternative: Two points on C are e.g., $\left(0, \frac{3}{2}\right)$ and $(1, 0)$ and these transform to $(3, -9)$ and $(4, -8)$ (M1)

Equation of l is then, e.g., $\frac{v-(-9)}{-8-(-9)} = \frac{u-3}{4-3}$ (dM1) $\Rightarrow v = u - 12$ (A1)

(d)
May first find the closest point using $u = -v$ to solve and obtain $(6, -6)$, then find distance to origin.
Alt: uses $w = u + (u - 12)\mathrm{i}$ to find the magnitude and minimises.

$$\begin{aligned} |w| &= \sqrt{u^2 + (u-12)^2} = \sqrt{2u^2 - 24u + 144} \\ &= \sqrt{2(u-6)^2 - 72 + 144} \\ &\Rightarrow \text{Mininum} = \sqrt{72} = 6\sqrt{2} \end{aligned}$$

Final A is cso so must have come from the correct line.