Question Number	Scheme		Notes	Marks
1(a)	$\frac{1}{(3n-1)(3n+5)} \equiv \frac{A}{3n-1} + \frac{B}{3n+5}$ $\Rightarrow A = \ldots \quad \left\{\frac{1}{6}\right\}, B = \ldots \quad \left\{-\frac{1}{6}\right\}$		Uses correct partial fractions and finds values for both A and B	**M1**
	$\equiv \frac{1}{6(3n-1)} - \frac{1}{6(3n+5)}$		Correct **partial fractions** in any form e.g., $\frac{1}{18n-6} - \frac{1}{18n+30}$, $\frac{\frac{1}{6}}{3n-1} - \frac{\frac{1}{6}}{3n+5}$, $\frac{1}{6}\left(\frac{1}{3n-1} + \frac{-1}{3n+5}\right)$	**A1**
				(2)
(b)	Allow the M marks if r is used for n Ignore errors with terms that are cancelled			
	$\frac{20}{6}\sum_{r=1}^{n}\left(\frac{1}{3n-1} - \frac{1}{3n+5}\right) =$ $\frac{20}{6}\left(\frac{1}{2} - \frac{1}{8} + \frac{1}{5} - \frac{1}{11} + \frac{1}{8} - \frac{1}{14} + \ldots + \frac{1}{3n-7} - \frac{1}{3n-1} + \frac{1}{3n-4} - \frac{1}{3n+2} + \frac{1}{3n-1} - \frac{1}{3n+5}\right)$ Uses the method of differences for at least the first and last rows. Must attempt to put their rows together. Allow with their partial fractions provided there are two which form a difference and have different 2 term linear denominators in n and numeric numerators. Ignore errors/omission with the 20 or the "$\frac{1}{6}$". Allow this mark to be implied if their 4 terms in the correct positions are the only ones that result.			**M1**
	$= \frac{20}{6}\left(\frac{1}{2} + \frac{1}{5} - \frac{1}{3n+2} - \frac{1}{3n+5}\right)$		Any correct expression for the required sum. The "20" may appear later - allow this mark if a correct sum without the 20 was seen but is then misprocessed before the 20 is reapplied.	**A1**
	$= \frac{20}{6}\left(\frac{7(3n+2)(3n+5) - 10(6n+7)}{10(3n+2)(3n+5)}\right)$		Achieves a consistent single fraction expression. Must come from adding 4 fractions - 2 numeric and 2 with different two term linear denominators and numeric numerators. Ignore omission/error with the 20 and/or the "$\frac{1}{6}$".	**dM1**
	$= \frac{1}{3}\left(\frac{63n^2 + 87n}{(3n+2)(3n+5)}\right) = \frac{n(21n+29)}{(3n+2)(3n+5)}$		Correct expression in the correct form. Not just values for the constants unless form is seen	**A1**
				(4)
				Total 6

Question Number	Scheme	Notes	Marks
2(a)	$y=(1-3x)^A+e^{Bx}\Rightarrow\dfrac{dy}{dx}=-3A(1-3x)^{A-1}+Be^{Bx}$	Correct first derivative. Accept unsimplified	**B1**
	$\dfrac{d^2y}{dx^2}=9A(A-1)(1-3x)^{A-2}+B^2e^{Bx}$ $\dfrac{d^3y}{dx^3}=-27A(A-1)(A-2)(1-3x)^{A-3}+B^3e^{Bx}$ or e.g., $-27(A^3-3A^2+2A)(1-3x)^{A-3}+B^3e^{Bx}$ M1: Obtains $\dfrac{d^3y}{dx^3}=g(A)(1-3x)^{A-3}+h(B)e^{Bx}$ A1: Fully correct third derivative. Could be unsimplified. Condone poor bracketing unless further work confirms error. If g(*A*) is only seen expanded then it must be correct		**M1** **A1**
			(3)
(b)	$f'(0)=-3A+B=0$	Substitutes $x = 0$ into their first derivative and sets = 0 and achieves a linear equation in *A* and *B*	**M1**
	$f''(0)=9A(A-1)+B^2=0$ $\Rightarrow 9A^2-9A+(3A)^2=0$ or $\Rightarrow 9\left(\dfrac{B}{3}\right)\left(\dfrac{B}{3}-1\right)+B^2=0$	Substitutes $x = 0$ into their second derivative, sets = 0 and obtains a quadratic equation in *A* (or *B*) only	**dM1**
	$18A^2-9A=0\Rightarrow 9A(2A-1)=0\Rightarrow A=...$ or $2B^2-3B=0\Rightarrow B(2B-3)=0\Rightarrow B=...$	Solves for *A* and *B*. No working is required but must get real values and neither can be 0 unless the zero solution is additional	**ddM1**
	$A=\dfrac{1}{2},\ B=\dfrac{3}{2}$	Both correct values. Accept equivalent fractions/decimals. Ignore extra zero solution	**A1**
	Allow all marks in (b) even if full marks were not obtained in (a)		**(4)**
(c)	$\dfrac{d^3y}{dx^3}=-27A(A-1)(A-2)(1-3x)^{A-3}+B^3e^{Bx}$ $\left(\dfrac{d^3y}{dx^3}\right)_{x=0}=-27\left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}-1\right)\left(\dfrac{1}{2}-2\right)+\left(\dfrac{3}{2}\right)^3=...\ \left(-\dfrac{27}{4}\right)$ Substitutes their *A* and their *B* and $x = 0$ into their 3rd derivative and obtains a value. *A* and *B* must be non-zero and must have come from simultaneous equations (one linear, one quadratic). If *k* = ... they must have divided by 6		**M1**
	$k=-\dfrac{27}{4}\div 6=...$ Divides their value by 6 or 3! to find a value for *k*. These two M marks are likely to scored at the same time. If just a value is given without a substitution attempt it must be consistent.		**dM1**
	$k=-\dfrac{9}{8}$	Correct value for *k*. This fraction or $-1\frac{1}{8}$ This mark requires a fully correct solution.	**A1**
			(3)
			Total 10
	Alternative for (b) & (c) by series expansions: $y=(1-3x)^A+e^{Bx}=1-3Ax+\dfrac{A(A-1)}{2}(-3x)^2+\dfrac{A(A-1)(A-2)}{3!}(-3x)^3+...+1+Bx+\dfrac{(Bx)^2}{2}+\dfrac{(Bx)^3}{3!}+...$ $=2+kx^3+...\Rightarrow -3A+B=0\quad$ (M1) $\dfrac{9A(A-1)}{2}+\dfrac{B^2}{2}=0\Rightarrow 9A^2-9A+9A^2=0\quad$ (dM1) $2A^2-A=A(2A-1)=0\Rightarrow A=\dfrac{1}{2},\quad B=\dfrac{3}{2}\quad$ (ddM1A1) $k=-\dfrac{9}{2}A(A-1)(A-2)+\dfrac{B^3}{6}\quad$ (M1) $\Rightarrow k=-\dfrac{9}{2}\left(\dfrac{1}{2}\right)\left(-\dfrac{1}{2}\right)\left(-\dfrac{3}{2}\right)+\dfrac{1}{6}\left(\dfrac{3}{2}\right)^3=...,\ -\dfrac{9}{8}\quad$ (dM1, A1)		

Question Number	Scheme	Notes	Marks
3(a)	$4\frac{dy}{dx} = y - 2xy^5 \quad u = y^{-4} \quad \left\{y = u^{-\frac{1}{4}}\right\}$		
	e.g., $\frac{du}{dx} = -\frac{4}{y^5}\frac{dy}{dx}, \quad \frac{dy}{dx} = -\frac{1}{4}u^{-\frac{5}{4}}\frac{du}{dx}$	Correct differentiation of $u = y^{-4}$ to obtain an equation in $\frac{du}{dx}$ and $\frac{dy}{dx}$ May be implied by later work	**B1**
	$4\left(-\frac{y^5}{4}\right)\frac{du}{dx} = y - 2xy^5, \quad -u^{-\frac{5}{4}}\frac{du}{dx} = y - 2xy^5$	Use the given differential equation and substitutes. Requires a correct form from differentiation of $u = y^{-4}$	**M1**
	$4\left(-\frac{y^5}{4}\right)\frac{du}{dx} = y - 2xy^5 \Rightarrow \frac{du}{dx} + \frac{1}{y^4} = 2x \Rightarrow \frac{du}{dx} + u = 2x$* or, e.g., $-u^{-\frac{5}{4}}\frac{du}{dx} = y - 2xy^5 \Rightarrow \frac{du}{dx} = 2u^{\frac{5}{4}}xy^5 - u^{\frac{5}{4}}y$ or $2x - u \Rightarrow \frac{du}{dx} + u = 2x$* Achieves the given answer with no errors and an intermediate step after substitution shown		**A1***
			(3)
(b)	$I = e^{\int dx} = e^x$	Correct integrating factor	**B1**
	$ue^x = \int 2xe^x \, dx$	For $Iu = \int 2xI \, dx$ Allow with y for u. I must be a function of x	**M1**
	Note that a tabular "DI" approach may be used for parts. dM1 for a correct form		
	$ue^x = 2xe^x - \int 2e^x \, dx$	Applies parts to RHS and achieves correct form - allow with y for u and condone + used for - in the parts formula	**dM1**
	$ue^x = 2xe^x - 2e^x \, (+c)$	Correct equation with or without a constant of integration. Must have u	**A1**
	$u = 2x - 2 + ce^{-x}$	Any correct equation with "u=..." and the constant correctly placed	**A1**
	If a 2nd order method is used, CF: $\{A\}e^{-x}$(B1), PF: $\lambda x + \mu$(M1), valid method for λ and μ(dM1), A marks as above		**(5)**
(c)	$1 = c - 2 \Rightarrow c = 3$	Uses the given conditions (y=1 or u=1 and x=0) to find the constant of integration. Condone poor algebra e.g, $\frac{1}{a} + \frac{1}{b}$ used for $\frac{1}{a+b}$	**M1**
	$y^4 = \frac{1}{2x - 2 + 3e^{-x}}$	Any correct equation in the required $y^4 = \ldots$ form e.g., $y^4 = \frac{1}{2(x-1) + \frac{3}{e^x}}$ Allow $y^4 = \frac{1}{2x - 2 + ce^{-x}}, \ c = 3$	**A1**
			(2)
			Total 10

Question Number	Scheme/Notes	Marks
4(a)	$\frac{d^2y}{dx^2} = x^2y + \frac{dy}{dx} \Rightarrow \frac{d^3y}{dx^3} = 2xy + x^2\frac{dy}{dx} + \frac{d^2y}{dx^2}$ M1: Differentiates x^2y to $Axy + Bx^2\frac{dy}{dx}$ A1: Any correct expression for the 3rd derivative	**M1** **A1**
	$\frac{d^4y}{dx^4} = 2y + 2x\frac{dy}{dx} + x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + \frac{d^3y}{dx^3}$ Differentiates again to obtain an expression of the form $\frac{d^4y}{dx^4} = Cy + Dx\frac{dy}{dx} + x^2\frac{d^2y}{dx^2} + \frac{d^3y}{dx^3}$ Terms may be uncollected	**dM1**
	$\frac{d^4y}{dx^4} = \frac{d^3y}{dx^3} + x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y$ Fully correct 4th derivative in correct form	**A1**
		(4)
(b)	$\frac{dy}{dx} = 3$ and $y = 1$ at $x = 1$ $\Rightarrow \frac{d^2y}{dx^2} = 1 + 3 = 4$ Correct value for the 2nd derivative	**B1**
	$\left(\frac{d^3y}{dx^3}\right)_{x=1} = 2(1)(1) + 1\times3 + 4 = 9,\ \left(\frac{d^4y}{dx^4}\right)_{x=1} = 9 + 4 + 4\times3 + 2 = 27$ Finds values for their 3rd and 4th derivatives at $x = 1$. Allow slips	**M1**
	$\{y=\}1 + 3(x-1) + \frac{"4"(x-1)^2}{2!} + \frac{"9"(x-1)^3}{3!} + \frac{"27"(x-1)^4}{4!} + ...$ Applies Taylor's expansion correctly with their 3 values. There is no credit for quoting a correct formula - it must be applied appropriately	**dM1**
	$\{y=\}1 + 3(x-1) + 2(x-1)^2 + \frac{3(x-1)^3}{2} + \frac{9(x-1)^4}{8} + ...$ Correct series with simplified coefficients	**A1**
		(4)
		Total 8

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td rowspan="7">5</td><td colspan="2">"Solutions relying entirely on calculator technology are not acceptable"</td><td></td></tr>
<tr><td>$\left|\frac{x^2+5x-2}{x^2+1}\right| < 2 \Rightarrow \left|x^2+5x-2\right| < 2\left(x^2+1\right)$
$x^2+5x-2=2\left(x^2+1\right) \Rightarrow x=...$
or
$x^2+5x-2=-2\left(x^2+1\right) \Rightarrow x=...$</td><td>Multiplies through by x^2+1 and attempts to find at least one pair of cvs from an initially correct equation (no fraction). Usual rules (2 solutions) if 3TQ with solving method seen. If 2TQ allow $ax^2+bx=0 \Rightarrow \{x=0,\}\quad x=\pm\frac{b}{a}$ and condone this without factorisation seen</td><td>M1</td></tr>
<tr><td colspan="2">$x^2-5x+4=(x-1)(x-4) \Rightarrow x=1,\ 4$ or $3x^2+5x\{=x(3x+5)\}=0 \Rightarrow x=0,\ -\frac{5}{3}$
Identifies 1 correct pair of cvs. The "0" may appear later</td><td>A1</td></tr>
<tr><td>$x^2+5x-2=2\left(x^2+1\right) \Rightarrow x=...$
and
$x^2+5x-2=-2\left(x^2+1\right) \Rightarrow x=...$</td><td>Complete method to find all 4 cvs (rules as previous M1)</td><td>dM1</td></tr>
<tr><td colspan="2">$x^2-5x+4=(x-1)(x-4) \Rightarrow x=1,\ 4$ or $3x^2+5x\{=x(3x+5)\}=0 \Rightarrow x=0,\ -\frac{5}{3}$
All 4 cvs correct. The "0" may appear later. Ignore extra cvs but no further marks unless they are disregarded.</td><td>A1</td></tr>
<tr><td>$x<-\frac{5}{3},\quad 0<x<1,\quad x>4$</td><td>M1: With 4 cvs a, b, c, d where $a<b<c<d$ identifies
$x<a,\quad b<x<c,\quad x>d$
Condone non-strict inequalities.
A1: Fully correct. Allow any equivalent correct notation e.g.
$\left(-\infty,-\frac{5}{3}\right)\cup(0,1)\cup(4,\infty)$
Condone "and" but do not allow $\cap$ for this last mark</td><td>M1
A1</td></tr>
<tr><td colspan="2">Note that if no 3TQ solving method is seen 110011 is possible but the last two marks cannot be scored if there is no algebra at all. Condone use of a calculator for the last two marks provided there has been some algebra at some point.
If squaring is used this leads to:
$3x^4-10x^3-13x^2+20x=x\left(3x^3-10x^2-13x+20\right)=x(3x+5)(x-1)(x-4)$
but the first 2 M marks are only scored if there is clear evidence of a non-calculator method for factorising (e.g., factor theorem, grid, etc.)
You may also see the difference of two squares e.g.,
$2^2\left(x^2+1\right)^2-\left(x^2+5x-2\right)^2=\left(3x^2+5x\right)\left(x^2-5x+4\right)=x(3x+5)(x-1)(x-4)=0\Rightarrow...$
We must see the factorisation of the 3TQ as with the main scheme</td><td>(6)</td></tr>
<tr><td colspan="2"></td><td>Total 6</td></tr>
</table>

Question Number	Scheme	Notes	Marks
6	$(x+1)^2+(y-1)^2=1 \Rightarrow \lvert z-(-1+\mathrm{i})\rvert=1$	Correct locus e.g., $\lvert z+1-\mathrm{i}\rvert=1$	**B1**
	$w=\dfrac{z+2}{3z+4} \Rightarrow z=\dfrac{2-4w}{3w-1}$	M1: Attempts to make *z* the subject A1: Any correct rearrangement	**M1** **A1**
	$\lvert z-(-1+\mathrm{i})\rvert=1 \Rightarrow \left\lvert \dfrac{2-4w}{3w-1}+1-\mathrm{i}\right\rvert=1$	Uses a locus of the form $\lvert z\pm1\pm\mathrm{i}\rvert=1$ and their expression for *z* in terms of *w* and sets = 1	**dM1**
	$\Rightarrow \left\lvert \dfrac{2-4w+3w-3w\mathrm{i}-1+\mathrm{i}}{3w-1}\right\rvert=1 \Rightarrow \lvert 1-w-3w\mathrm{i}+\mathrm{i}\rvert=\lvert 3w-1\rvert$ $\Rightarrow \lvert 1-u-\mathrm{i}v-3u\mathrm{i}+\mathrm{i}+3v\rvert=\lvert 3u+3\mathrm{i}v-1\rvert$ M1: Finds common denominator, multiplies up and introduces $w=u+\mathrm{i}v$		**ddM1**
	$(1-u+3v)^2+(1-v-3u)^2=(3u-1)^2+9v^2$	Applies Pythagoras correctly to obtain an equation in *u* and *v*	**dddM1**
	$u^2+v^2-2u+4v+1=0$	Correct equation in this form	**A1**
			(7)
	There may be various alternatives. If using the main scheme above benefits the learners for any different approaches then please use that. Approaches which attempt to map points can access the first mark as above (or by choosing three correct points (or complex numbers) for the circle). The remaining marks would score as follows: M1 for transforming 3 points using the given transformation A1: 3 correct points dM1ddM1dddM1: For a complete valid method to establish the equation of circle *D* Could find the intersection point of two perpendicular bisectors and then find the radius and produce a circle equation, or use the transformed points in a general circle equation and solve the 3 simultaneous equations involving 3 unknowns and produce a circle equation A1: Correct equation Another possibility is: $u+\mathrm{i}v=\dfrac{x+\mathrm{i}y+2}{3(x+\mathrm{i}y)+4}$ or $x+\mathrm{i}y=\dfrac{2-4(u+\mathrm{i}v)}{3(u+\mathrm{i}v)-1}$ (the latter could get M1A1 already) $\Rightarrow x=\dfrac{-12u^2-12v^2+10u-2}{(3u-1)^2+9v^2}\quad y=\dfrac{-2v}{(3u-1)^2+9v^2}\quad$ (M1A1) Could write this as $x+\mathrm{i}y$ and note that $(3u-1)^2+9v^2=9u^2-6u+1+9v^2$ $\left(\dfrac{-12u^2-12v^2+10u-2}{(3u-1)^2+9v^2}+1\right)^2+\left(\dfrac{-2v}{(3u-1)^2+9v^2}-1\right)^2=1$ (dM1ddM1dddM1, B1 if all correct) $u^2+v^2-2u+4v+1=0\quad$ (A1)		

Question Number	Scheme	Notes	Marks
7(a)	$\sin\theta = \frac{1}{2\mathrm{i}}\left(\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta}\right)$ $\Rightarrow \sin^4\theta = \left\{\frac{1}{(2\mathrm{i})^4}\right\}\left(\left(\mathrm{e}^{\mathrm{i}\theta}\right)^4 + 4\left(\mathrm{e}^{\mathrm{i}\theta}\right)^3\left(-\mathrm{e}^{-\mathrm{i}\theta}\right) + 6\left(\mathrm{e}^{\mathrm{i}\theta}\right)^2\left(-\mathrm{e}^{-\mathrm{i}\theta}\right)^2 + 4\mathrm{e}^{\mathrm{i}\theta}\left(-\mathrm{e}^{-\mathrm{i}\theta}\right)^3 + \left(\mathrm{e}^{-\mathrm{i}\theta}\right)^4\right)$ Expands $\left(\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta}\right)^4$ or squares twice. Allow slips but must have at least 4 terms and include an attempt at binomial coefficients which may be uncalculated e.g., $\binom{4}{3}$... Allow via $\left\{\frac{1}{(2\mathrm{i})^4}\right\}\left(z - \frac{1}{z}\right)^4 = \left\{\frac{1}{16}\right\}\left(z^4 - 4z^2 + 6 - 4z^{-2} + z^{-4}\right)$		**M1**
	$= \frac{1}{16}\left(\mathrm{e}^{4\mathrm{i}\theta} - 4\mathrm{e}^{2\mathrm{i}\theta} + 6 - 4\mathrm{e}^{-2\mathrm{i}\theta} + \mathrm{e}^{-4\mathrm{i}\theta}\right)$ or $\frac{1}{16}\left(z^4 - 4z^2 + 6 - 4z^{-2} + z^{-4}\right)$ Correct simplified expansion. The $\frac{1}{16}$ may be applied later		**A1**
	$= \frac{1}{16}\left(\cos 4\theta + \mathrm{i}\sin 4\theta + \cos 4\theta - \mathrm{i}\sin 4\theta - 4\left(\cos 2\theta + \mathrm{i}\sin 2\theta + \cos 2\theta - \mathrm{i}\sin 2\theta\right) + 6\right)$ or $= \frac{1}{16}\left(\mathrm{e}^{4\mathrm{i}\theta} + \mathrm{e}^{-4\mathrm{i}\theta} - 4\left(\mathrm{e}^{2\mathrm{i}\theta} + \mathrm{e}^{-2\mathrm{i}\theta}\right) + 6\right) = \frac{1}{16}\left(2\cos 4\theta - 4\left(2\cos 2\theta\right) + 6\right)$ Completes an attempt to use $\mathrm{e}^{n\theta\mathrm{i}} = \cos n\theta + \mathrm{i}\sin n\theta$ or $\cos n\theta = \frac{1}{2}\left(\mathrm{e}^{\mathrm{i}n\theta} + \mathrm{e}^{-\mathrm{i}n\theta}\right)$ to express the expansion in terms of trig functions. The $\frac{1}{16}$ may be incorrect or missing		**dM1**
	$\frac{1}{16}\left(2\cos 4\theta - 8\cos 2\theta + 6\right) \Rightarrow \frac{1}{8}\left(\cos 4\theta - 4\cos 2\theta + 3\right)$ Correct expression in the required form		**A1**
			(4)
(b)	$\Rightarrow \sin^4\left(\frac{1}{2}\pi - \theta\right) = \frac{1}{8}\left(\cos 4\left(\frac{1}{2}\pi - \theta\right) - 4\cos 2\left(\frac{1}{2}\pi - \theta\right) + 3\right)$ $\cos^4\theta = \frac{1}{8}\left(\cos 4\left(\frac{1}{2}\pi - \theta\right) - 4\cos 2\left(\frac{1}{2}\pi - \theta\right) + 3\right)$ $= \frac{1}{8}\left(\cos\left(2\pi - 4\theta\right) - 4\cos\left(\pi - 2\theta\right) + 3\right)$ $= \frac{1}{8}\left(\cos 4\theta + 4\cos 2\theta + 3\right)$ oe e.g., $\frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$ M1: $\sin^4\theta = A\cos 4\theta + B\cos 2\theta + C \Rightarrow \cos^4\theta = D\cos 4\theta + E\cos 2\theta + F$ Do not allow if something other than $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$ is clearly being used A1: Any correct expression in terms of $\cos 4\theta$ and $\cos 2\theta$ (not fortuitous)		**M1** **A1**
			(2)
(c)	$\int\left(\sin^4\theta + \cos^4\theta\right)\mathrm{d}\theta = \int\left(\frac{1}{8}\left(\cos 4\theta - 4\cos 2\theta + 3\right) + \frac{1}{8}\left(\cos 4\theta + 4\cos 2\theta + 3\right)\right)\mathrm{d}\theta$ $\left\{= \int\left(\frac{1}{4}\cos 4\theta + \frac{3}{4}\right)\mathrm{d}\theta \text{ or e.g., } \frac{1}{8}\int\left(2\cos 4\theta + 6\right)\mathrm{d}\theta\right\}$ Uses their expressions from (a) and (b) both of correct form and different and substitutes correctly within an integral. M0 if expression would reduce to just k. Must be consistent if only seen with terms collected		**M1**
	$= \frac{1}{16}\sin 4\theta + \frac{3}{4}\theta\,(+c)$	Correct 2 term result with or without $+ c$	**A1**
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
8(a)	e.g., $u = xy \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = x\frac{\mathrm{d}y}{\mathrm{d}x} + y$ or $y = ux^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -ux^{-2} + x^{-1}\frac{\mathrm{d}u}{\mathrm{d}x}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x\frac{\mathrm{d}u}{\mathrm{d}x} - u}{x^2}$ Differentiates $u = xy$ oe to obtain any correct equation in $\frac{\mathrm{d}u}{\mathrm{d}x}$ and $\frac{\mathrm{d}y}{\mathrm{d}x}$		**B1**
	$\frac{\mathrm{d}u}{\mathrm{d}x} = x\frac{\mathrm{d}y}{\mathrm{d}x} + y \Rightarrow \frac{\mathrm{d}^2u}{\mathrm{d}x^2} = x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} + \frac{\mathrm{d}y}{\mathrm{d}x}$ or e.g., $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -x^{-2}\frac{\mathrm{d}u}{\mathrm{d}x} + 2x^{-3}u - x^{-2}\frac{\mathrm{d}u}{\mathrm{d}x} + x^{-1}\frac{\mathrm{d}^2u}{\mathrm{d}x^2} \Rightarrow -2x^{-2}\left(x\frac{\mathrm{d}y}{\mathrm{d}x} + y\right) + 2x^{-2}y + x^{-1}\frac{\mathrm{d}^2u}{\mathrm{d}x^2}$ Differentiates again and obtains an equation of the correct form before given answer i.e., $A\frac{\mathrm{d}^2u}{\mathrm{d}x^2} = Bx\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + C\frac{\mathrm{d}y}{\mathrm{d}x}$ oe Allow if there are clearly cancelling terms involving other variables/derivatives or the work makes completely clear how the given answer is reached		**M1**
	$\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{1}{x}\left(\frac{\mathrm{d}^2u}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ *	Obtains the given answer with no errors. No intermediate step is required	**A1***
			(3)
(b)	$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2(2x+1)\frac{\mathrm{d}y}{\mathrm{d}x} + 13xy = 17\mathrm{e}^{3x} - 4y$ $\Rightarrow x\frac{1}{x}\left(\frac{\mathrm{d}^2u}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x}\right) + 4x\frac{1}{x}\left(\frac{\mathrm{d}u}{\mathrm{d}x} - y\right) + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 13xy = 17\mathrm{e}^{3x} - 4y$ Substitutes the given second derivative and their first derivative into DE (I)		**M1**
	$\Rightarrow \frac{\mathrm{d}^2u}{\mathrm{d}x^2} + 4\frac{\mathrm{d}u}{\mathrm{d}x} - 4y + 13u = 17\mathrm{e}^{3x} - 4y$ $\Rightarrow \frac{\mathrm{d}^2u}{\mathrm{d}x^2} + 4\frac{\mathrm{d}u}{\mathrm{d}x} + 13u = 17\mathrm{e}^{3x}$ *	Obtains the given answer with an intermediate step after substitution and no errors	**A1***
			(2)
(c)	$m^2 + 4m + 13 = 0 \Rightarrow m = -2 \pm 3\mathrm{i}$	Solves $m^2 + 4m + 13 = 0$ (allow miscopy and apply usual rules if necessary but may use calculator but if so must get a correct root for their 3TQ)	**M1**
	$\{u =\}\ \mathrm{e}^{-2x}(A\cos 3x + B\sin 3x)$	Correct CF	**A1**
	Note other CFs are possible: $\{u =\}\ A\mathrm{e}^{(-2+3\mathrm{i})x} + B\mathrm{e}^{(-2-3\mathrm{i})x}$, $A\mathrm{e}^{-2x}\cos(3x + B)$		
	$u = \lambda\mathrm{e}^{3x} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 3\lambda\mathrm{e}^{3x} \Rightarrow \frac{\mathrm{d}^2u}{\mathrm{d}x^2} = 9\lambda\mathrm{e}^{3x}$ $\Rightarrow \frac{\mathrm{d}^2u}{\mathrm{d}x^2} + 4\frac{\mathrm{d}u}{\mathrm{d}x} + 13u = (9\lambda + 12\lambda + 13\lambda)\mathrm{e}^{3x}$	Starts with the correct PI form, differentiates twice (obtaining correct (changed) forms) and substitutes into LHS	**M1**
	$\Rightarrow 9\lambda + 12\lambda + 13\lambda = 17 \Rightarrow \lambda = ...$	Solves to find the unknown in the PI	**dM1**
	$u = \mathrm{e}^{-2x}(A\cos 3x + B\sin 3x) + \frac{1}{2}\mathrm{e}^{3x}$	Correct GS of equation (II). Must be $u = ...$	**A1**
			(5)
(d)	$y = \frac{1}{x}\left(\mathrm{e}^{-2x}(A\cos 3x + B\sin 3x) + \frac{1}{2}\mathrm{e}^{3x}\right)$	Correct follow through equation. Allow with their CF + PI provided both are functions of x (allow e.g., CF with no constant, PI with constant) Must be $y = ...$	**B1ft**
			(1)
			Total 11

Question Number	Scheme	Notes	Marks
9	"Solutions relying entirely on calculator technology are not acceptable"		
(a)	$1+\sin\theta = 1+\cos 2\theta \Rightarrow \sin\theta = 1-2\sin^2\theta \Rightarrow 2\sin^2\theta+\sin\theta-1=0$ Sets $1+\sin\theta = 1+\cos 2\theta$ and obtains a 3TQ in $\sin\theta$ (terms may not all be on one side) by using $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ or equivalent work condoning sign errors only in identities		**M1**
	$2\sin^2\theta+\sin\theta-1=0 \Rightarrow (2\sin\theta-1)(\sin\theta+1)=0$ $\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = ...$	Solves 3TQ in $\sin\theta$ for θ Must see factorisation or equivalent work	**dM1**
	$\left(\frac{3}{2}, \frac{\pi}{6}\right)$ Allow $r=\frac{3}{2},\ \theta=\frac{\pi}{6}$ even if followed by $\left(\frac{\pi}{6}, \frac{3}{2}\right)$ Correct polar coordinates. Ignore extra coordinate pairs unless correct *P* is rejected		**A1**
			(3)

(b)	Note that the $\frac{1}{2}$s may have already been applied	
	$\int (1+\sin\theta)^2\, d\theta = \int \left(1+2\sin\theta+\sin^2\theta\right) d\theta = \int \left(1+2\sin\theta+\frac{1}{2}-\frac{1}{2}\cos 2\theta\right) d\theta$ $\int (1+\cos 2\theta)^2\, d\theta = \int \left(1+2\cos 2\theta+\cos^2 2\theta\right) d\theta = \int \left(1+2\cos 2\theta+\frac{1}{2}+\frac{1}{2}\cos 4\theta\right) d\theta$	
	$\int (1+\sin\theta)^2\, d\theta = \int \left(1+2\sin\theta+\sin^2\theta\right) d\theta$ and uses $\sin^2\theta = \pm\frac{1}{2}\pm\frac{1}{2}\cos 2\theta$ $\int (1+\cos 2\theta)^2\, d\theta = \int \left(1+2\cos 2\theta+\cos^2 2\theta\right) d\theta$ and uses $\cos^2 2\theta = \pm\frac{1}{2}\pm\frac{1}{2}\cos 4\theta$	**M1(one)** **dM1(both)**
	Allows the Ms for $r_1^2 \pm r_2^2$ but $(r_1 \pm r_2)^2$ is M0dM0	
	$\int \left(1+2\sin\theta+\frac{1}{2}-\frac{1}{2}\cos 2\theta\right) d\theta = \frac{3}{2}\theta - 2\cos\theta - \frac{1}{4}\sin 2\theta (+c)$ $\int \left(1+2\cos 2\theta+\frac{1}{2}+\frac{1}{2}\cos 4\theta\right) d\theta = \frac{3}{2}\theta + \sin 2\theta + \frac{1}{8}\sin 4\theta (+c)$	**A1(one)** **Requires 1 previous M mark** **A1(both)**
	Area of *R*: $\frac{1}{2}\left[\frac{3}{2}\theta - 2\cos\theta - \frac{1}{4}\sin 2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{6}} + \frac{1}{2}\left[\frac{3}{2}\theta + \sin 2\theta + \frac{1}{8}\sin 4\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $= \frac{1}{2}\left(\frac{\pi}{4} - \sqrt{3} - \frac{\sqrt{3}}{8} - \left(-\frac{3\pi}{4}\right)\right) + \frac{1}{2}\left(\frac{3\pi}{4} - \left(\frac{\pi}{4} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{16}\right)\right)$ or $\frac{1}{2}\left(\pi - \frac{9\sqrt{3}}{8}\right) + \frac{1}{2}\left(\frac{\pi}{2} - \frac{9\sqrt{3}}{16}\right)$ A completely valid method using the $\frac{1}{2}$ twice and the correct limits, obtaining a numerical expression with trig terms evaluated. May not be consistent - if the limits are seen correctly on the square brackets accept the **sum** of two substitutions. If just $a\pi + b\sqrt{3}$ results it must be $\frac{3\pi}{4} - \frac{27}{32}\sqrt{3}$	**ddM1**
	$= \frac{3\pi}{4} - \frac{27}{32}\sqrt{3}$ Correct expression in this form. Requires all previous marks.	**A1**
		(6)
		Total 9