Question	Scheme	Marks
1(a)	$\frac{dy}{dx} - 3y\tan x = \sec^2 x$ with $y = 4$ when $x = \frac{\pi}{4}$	
	Mark parts (a) and (b) together	
	$\text{IF} = \exp\left(\int -3\tan x \, dx\right) = \ldots$	M1
	$= \exp(3\ln\cos x) = \cos^3 x$ *	A1*
		(2)
Alt (a)	$\frac{d}{dx}\left(y\cos^3 x\right) = \frac{dy}{dx}\cos^3 x + y \times 3\cos^2 x \times -\sin x$	M1
	$= \cos^3 x\left(\frac{dy}{dx} - 3y\tan x\right)$ hence integrating factor	A1*
		(2)
(b)	$\Rightarrow y\cos^3 x = \int \sec^2 x \times \cos^3 x \, dx$	M1
	$\Rightarrow y\cos^3 x = \sin x \ \ (+c)$ o.e. **OR** $y = \sin x\sec^3 x + (c\sec^3 x)$ **OR** $y = \tan x\sec^2 x + (c\sec^3 x)$ o.e.	A1
	$\Rightarrow 4\cos^3\frac{\pi}{4} = \sin\frac{\pi}{4} + c \Rightarrow c = \frac{\sqrt{2}}{2} \Rightarrow y = \tan x\sec^2 x + \frac{\sqrt{2}}{2}\sec^3 x$ o.e.	M1 A1
		(4)
		(6 marks)

Notes:

(a)

M1: Attempts the integrating factor directly. Allow for an attempt at $\exp\left(\pm\int k\tan x\right)$ but no need to evaluate the integral for this mark.
Must be shown, but could be implied by an integrated expression of the correct form e.g. $\exp(-3\ln\sec x)$ or $\exp(3\ln\cos x)$.
If $(\sec x)^{-3}$ is all that is seen then accept for M1 only

A1*: Correct simplified integrating factor. Correct processing seen, so exp and ln inverse applied correctly, and correct integration of $-3\tan x \longrightarrow -3\ln\sec x$ *or* $3\ln\cos x$ seen. **No sight of exp and ln being correctly processed, or no evidence of correct integration is max M1A0**

Alt by verification

M1: Attempts to differentiate $y\cos^3 x \rightarrow \cos^3 x\frac{\mathrm{d}y}{\mathrm{d}x} \pm ky\sin x\cos^2 x$

A1*: Shows correctly that the derivative is $\cos^3 x$ times equation (I) **and makes minimal conclusion.**

(b)

M1: Applies the integrating factor. Look for $y \times \text{their IF} = \int \sec^2 x \times \text{their IF}\,\mathrm{d}x$ or equivalent

A1: Correct general form. The constant may be missing for this mark.
M1: Applies the initial condition to find a value or c and treats it correctly (i.e. having the correct position, and not just appearing in isolation). Incorrect treatment of c is M0 (e.g. $y = \tan x\sec^2 x + c \rightarrow c = \ldots$ is M0)
A1: Correct answer. Accept alternatives in terms of $\cos x$ etc that are in the form $y = \ldots$ so e.g.

$$y = \frac{\sin x}{\cos^3 x} + \frac{\sqrt{2}}{2\cos^3 x}$$

etc

Note: c could be given as $\frac{1}{\sqrt{2}}$. ISW once a correct answer is seen.

Question	Scheme	Marks
2(a)	$\frac{2x}{x^2+1}=\frac{1}{x+4}\Rightarrow 2x(x+4)=x^2+1$ **OR** $\frac{2x}{x^2+1}-\frac{1}{x+4}=0\Rightarrow\frac{2x(x+4)-(x^2+1)}{(x^2+1)(x+4)}=0$	M1
	$(\Rightarrow x^2+8x-1=0)\Rightarrow x=\frac{-8\pm\sqrt{64-4\times1\times-1}}{2}=\ldots$	dM1
	$x=-4\pm\sqrt{17}$	A1
		(3)
(b)	$x<-4-\sqrt{17}$ **or** $-4<x<-4+\sqrt{17}$	M1
	Both $x<-4-\sqrt{17},\ -4<x<-4+\sqrt{17}$	A1
		(2)
(c)	$x<-4+\sqrt{17}, x\neq-4$ **OR** $x<-4\ \ ,-4<x<-4+\sqrt{17}$ **OR** $x\in(-\infty,-4)\cup\left(-4,-4+\sqrt{17}\right)$	M1 A1
		(2)
		(7 marks)

Notes:

(a)

M1: Sets equal and cross multiplies to reach an unsimplified quadratic equation, OR takes terms to one side, combines together with common denominator to reach an unsimplified quadratic on their numerator.

dM1: Solves the quadratic. Usual rules. May use a calculator. **Dependent on the previous M Mark.**

Note: If the answer given is incorrect, and no QF is quoted then M0. But if the correct formula is quoted and there are errors in the substitution of values or incorrect answer given after formula quoted then the M mark can still be scored.

A1: Correct roots. Must be exact.

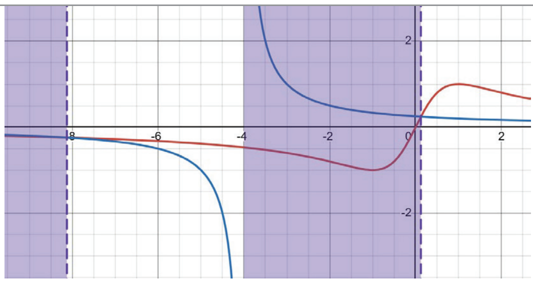
(b)

NOTE: decimal answers can score M marks only in parts (b) and (c)

M1: For $x<\alpha$ **OR** $-4<x<\beta$ for their roots α,β, where $\alpha<\beta$ and their $\alpha<-4$, their $\beta>-4$ (their roots need to be on opposite sides of the asymptote)

Selects a solution set from one of those above with their critical values (provided their CVs are within the parameters allowed). Allow if any non-strict inequalities are used for this mark.

A1: Correct answer. **Must be exact.**



Useful graph of solution

(c)
M1: Deduces partially correct range by including all the values less than −4 in their answer (with possible exception of their negative root α). May also include the −4 and boundary value(s) for this mark. **i.e. allow boundary values to be included/excluded as long as the reflected part of the reciprocal graph is included in the solution.**
e.g. $x \leq -4$ **M1A0** (values less than -4 included, boundary allowed, but incomplete solution)

$x < -4$ **M1A0** (values less than -4 included, but incomplete solution)

$-4-\sqrt{17} < x < -4$ and $x < -4-\sqrt{17}$ **M1A0** (values less than -4 included, excluding the negative root)

$-4-\sqrt{17} < x < -4$ **M0A0** (values less than -4 not included in the solution set)

$x \leq -4+\sqrt{17}$, $x \neq -4$ **M1A0** (values less than -4 included, but includes end point)

$x \leq -4+\sqrt{17}$ **M1A0** (values less than -4 included, but incomplete solution)

A1: Deduces fully correct range. Allow equivalent notations such as $x \in (-\infty, -4) \cup (-4, -4+\sqrt{17})$ with e.g. 'or/and' between the brackets, but not $\cap$. **Values must be exact.**

Useful graph of solution

Question	Scheme	Marks
3(a)	$2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+y\frac{\mathrm{d}y}{\mathrm{d}x}-6xy=0$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=3$, $y=\frac{1}{2}$ when $x=2$	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{2x}\left(6xy-y\frac{\mathrm{d}y}{\mathrm{d}x}\right)\Rightarrow\left.\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right\|_{x=2}=\frac{1}{4}\left(6-\frac{3}{2}\right)=\frac{9}{8}$	M1A1
	E.g. $2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2x\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+\underline{y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}-6y-6x\frac{\mathrm{d}y}{\mathrm{d}x}=0$ Or $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-\frac{1}{2x^2}\left(6xy-y\frac{\mathrm{d}y}{\mathrm{d}x}\right)+\frac{1}{2x}\left(6x\frac{\mathrm{d}y}{\mathrm{d}x}+6y\underline{-\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}}\right)$ Or $2x\frac{d^2y}{dx^2}=-y\frac{dy}{dx}+6xy\Rightarrow 2\frac{d^2y}{dx^2}+2x\frac{d^3y}{dx^3}=\underline{-\left(\frac{dy}{dx}\right)^2-y\frac{d^2y}{dx^2}}+6y+6x\frac{dy}{dx}$	M1<u>A1</u> A1
	$\left.\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\right\|_{x=2}=\frac{435}{64}$	A1
		(6)
(b)	$(y)=\frac{1}{2}+3(x-2)+\frac{9/8}{2}(x-2)^2+\frac{435/64}{3!}(x-2)^3$	M1
	$y=\frac{1}{2}+3(x-2)+\frac{9}{16}(x-2)^2+\frac{145}{128}(x-2)^3$	A1
		(2)
		(8 marks)

Notes:

(a)

M1: Attempts to find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at $x=2$ (May see after the 3rd derivative has been calculated)

A1: Correct value. Note if a method is used in (a) that does not find this value, score for the attempt at it in (b). A correct value for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ implies M1A1

M1: Attempts to differentiate the equation (may have been rearranged first) with **at least one** use of **product rule** in evidence (sign/coefficient errors only)

A1: For $y\frac{\mathrm{d}y}{\mathrm{d}x}\to y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$

A1: Fully correct expression in the third derivative (can be implicit)

A1: Correct value for the third derivative following a correct expression

(b)

M1: Applies the series formula correctly with their values. Must have the correct factorials/implied factorials if not shown. If series is incorrect for their values and no formula shown then M0

A1: Correct answer with <u>simplified</u> coefficients. **Must be $\boldsymbol{y}=\cdots$ so $\boldsymbol{f(x)}=\cdots$ is A0**

(a) **Alt**
M1: Attempts to differentiate the equation (may have been rearranged first) with at least one use of product rule.

A1: For $y\dfrac{\mathrm{d}y}{\mathrm{d}x} \to y\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$

M1: Attempts to replace $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ in their expression to find $\dfrac{\mathrm{d}^3 y}{\mathrm{d}x^3}$ in terms of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, x and y only.

A1: Correct replacement at least once

A1: Fully correct expression in the third derivative in terms of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, x and y only.

A1: Correct value for the third derivative at $x = 2$
Note: If in doubt about this method being used, please send to review

Question	Scheme	Marks
4(a)	$\dfrac{r+4}{r(r+1)(r+2)} \equiv \dfrac{A}{r}+\dfrac{B}{r+1}+\dfrac{C}{r+2}$	B1
	$\Rightarrow r+4 \equiv A(r+1)(r+2)+Br(r+2)+Cr(r+1)$; $r=0 \Rightarrow A=\ldots$	M1
	Two of $A=2,\ B=-3,\ C=1$	A1
	$\dfrac{r+4}{r(r+1)(r+2)} \equiv \dfrac{2}{r}-\dfrac{3}{r+1}+\dfrac{1}{r+2}$	A1
		(4)
(b)	$\displaystyle\sum_{r=1}^{n}\frac{r+4}{r(r+1)(r+2)}=\sum_{r=1}^{n}\frac{2}{r}-\frac{3}{r+1}+\frac{1}{r+2}$ $=\left(\frac{2}{1}-\frac{3}{2}+\frac{1}{3}\right)$ $+\left(\frac{2}{2}-\frac{3}{3}+\frac{1}{4}\right)$ $+\left(\frac{2}{3}-\frac{3}{4}+\frac{1}{5}\right)$ $+\cdots$ $+\left(\frac{2}{n-1}-\frac{3}{n}+\frac{1}{n+1}\right)$ $+\left(\frac{2}{n}-\frac{3}{n+1}+\frac{1}{n+2}\right)$	M1A1
	$=\left(\dfrac{2}{1}-\dfrac{1}{2}\right)+\left(-\dfrac{2}{n+1}+\dfrac{1}{n+2}\right)$ o.e.	A1
	$=\dfrac{3}{2}-\dfrac{2}{n+1}+\dfrac{1}{n+2}=\dfrac{3(n+1)(n+2)-4(n+2)+2(n+1)}{2(n+1)(n+2)}=\ldots$	M1
	$=\dfrac{n(3n+7)}{2(n+1)(n+2)}$	A1
	All marks can be awarded if working in terms of r and then replaced at the end. If no replacement made, then deduct the final A mark	
		(5)
		(9 marks)

Notes:

(a)

B1: Correct form for the partial fractions attempted. **Note:** some candidates may start with $\frac{r+4}{r(r+1)(r+2)} \equiv \frac{A}{r(r+1)}+\frac{B}{(r+1)(r+2)}$ and proceed to split further into four partial fractions, equivalent to the form stated. If in doubt about this method, please send to review.

M1: Method to find at least one of the coefficients, may be the cover-up rule

A1: Two correct coefficients

A1: Fully correct partial fractions found (not just stating the values), but accept if seen in (b) if only the coefficients are found in (a).

(b)
M1: Applies the method of differences, with at least three of the given brackets evaluated, and showing some evidence of appropriate cancellation of terms on the 'diagonal' e.g. $\frac{1}{3}-\frac{3}{3}+\frac{2}{3}$ etc
A1: Shows sufficient terms in the method of differences to establish complete cancellation of terms on the main diagonal (so e.g at least 3 terms at start and 2 terms at the end or for 2 terms at the start and 3 terms at the end)
Alt: establishes the correct non-cancelling numeric (accept e.g 3/2 for the numeric terms) **OR** algebraic terms (which would imply M1A1)
A1: Correct non-cancelling terms extracted.
M1: Puts over the correct common denominator and attempts to simplify the numerator to achieve a quadratic of the form an^2+bn but accept an^2+bn+c with $a\neq 0$
A1: Correct answer or values for P, Q, R and S.
Allow recovery if the terms are given in terms of r instead of n and recovered at the end. If no recovery then score final A0.
Alt for the first 3 marks in (b)
Re-writing the PFs $\frac{2}{r}-\frac{3}{r+1}+\frac{1}{r+2}$ as $\frac{2}{r}-\frac{2}{r+1}+\frac{-1}{r+1}+\frac{1}{r+2}$ and then using method of differences on two separate sums

(b) Alt	$\sum_{r=1}^{n}\left[\frac{2}{r}-\frac{2}{r+1}\right]+\sum_{r=1}^{n}\left[-\frac{1}{r+1}+\frac{1}{r+2}\right]$ $=\left(\frac{2}{1}-\frac{2}{2}\right)+\left(-\frac{1}{2}+\frac{1}{3}\right)$ $+\left(\frac{2}{2}-\frac{2}{3}\right)+\left(-\frac{1}{3}+\frac{1}{4}\right)+\cdots$ $+\left(\frac{2}{n-1}-\frac{2}{n}\right)+\left(-\frac{1}{n}+\frac{1}{n+1}\right)$ $+\left(\frac{2}{n}-\frac{2}{n+1}\right)+\left(-\frac{1}{n+1}+\frac{1}{n+2}\right)$	**M1A1**
	$=\frac{2}{1}-\frac{2}{n+1}-\frac{1}{2}+\frac{1}{n+2}$	**A1**
	$=\frac{3}{2}-\frac{2}{n+1}+\frac{1}{n+2}=\frac{3(n+1)(n+2)-4(n+2)+2(n+1)}{2(n+1)(n+2)}=\ldots$	**M1**
	$=\frac{n(3n+7)}{2(n+1)(n+2)}$	**A1**
		(5)

Notes:
(b) **M1:** Applies the method of differences, for both summations, with at least two of the brackets evaluated for both, and some evidence of cancellation of terms on the 'diagonal' **A1:** Shows sufficient terms in the method of differences for the cancelling of terms on the main diagonal (e.g. first two and last, or first and last two etc) **Alt:** correct non-cancelling numeric OR algebraic terms extracted (which would imply M1A1) **A1:** Correct non-cancelling terms extracted. **M1:** Puts over the correct common denominator and attempts to simplify, to achieve a quadratic of the form $an^2 + bn$ but accept $an^2 + bn + c$ with $a \neq 0$ **A1:** Correct answer or values for P, Q, R and S. Allow recovery if the terms are given in terms of r instead of n and recovered at the end. If no recovery then score final A0.

Question	Scheme	Marks
5(a)	$y = r\sin\theta = \left(\sqrt{3} + \dfrac{1-\cos\theta}{\sin\theta}\right)\sin\theta = \sqrt{3}\sin\theta + 1 - \cos\theta$ $\Rightarrow \dfrac{dy}{d\theta} = \sqrt{3}\cos\theta + \sin\theta$ or $\dfrac{dy}{d\theta} = \sqrt{3}\cos\theta + \cos\theta\tan\dfrac{\theta}{2} + \sin\theta\dfrac{1}{2}\sec^2\dfrac{\theta}{2}$ Or equivalent **NOTE** that $\sin\theta \times \frac{1}{2}\sec^2\frac{\theta}{2} \equiv \tan\frac{\theta}{2} \equiv \frac{1-\cos\theta}{\sin\theta}$ so final term may be written this way	M1 A1
	$\dfrac{dy}{d\theta} = 0 \Rightarrow \sin\theta = -\sqrt{3}\cos\theta \Rightarrow \tan\theta = ... \Rightarrow \theta = ..$	M1
	$\theta\left(= \arctan\left(-\sqrt{3}\right)\right) = \dfrac{2\pi}{3}$	A1
		(4)
Alt (i) **For the 2nd M1A1**	$\sqrt{3}\cos\theta + \cos\theta\dfrac{1-\cos\theta}{\sin\theta} + \dfrac{1}{2}\sin\theta\left(1 + \dfrac{(1-\cos\theta)^2}{\sin^2\theta}\right) = 0$ $\Rightarrow 2\sqrt{3}\cos\theta\sin\theta + 2\cos\theta - 2\cos^2\theta + \left(\sin^2\theta + 1 - 2\cos\theta + \cos^2\theta\right) = 0$ $\Rightarrow \sqrt{3}\sin 2\theta - \cos 2\theta + 1 = 0$ $\Rightarrow 2\sin\left(2\theta - \dfrac{\pi}{6}\right) = -1 \Rightarrow \theta = ...$	
Alt (ii) **For the 2nd M1A1**	$\sqrt{3}\cos\theta + \cos\theta\tan\dfrac{\theta}{2} + \sin\theta\dfrac{1}{2}\sec^2\dfrac{\theta}{2} = 0 \Rightarrow 2\sqrt{3} + 2\tan\dfrac{\theta}{2} + \tan\theta\left(1 + \tan^2\dfrac{\theta}{2}\right) = 0$ $\Rightarrow 2\sqrt{3} + 2\tan\dfrac{\theta}{2} + \dfrac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\left(1 + \tan^2\dfrac{\theta}{2}\right) = 0 \Rightarrow \sqrt{3} + 2\tan\dfrac{\theta}{2} - \sqrt{3}\tan^2\dfrac{\theta}{2} = 0$ $\Rightarrow \theta = 2\arctan\sqrt{3} = ..$	
(b)	$\displaystyle\int r^2 d\theta = \int 3 + 2\sqrt{3}\tan\frac{\theta}{2} + \tan^2\frac{\theta}{2}\,d\theta$	M1
	$\displaystyle = \int 3 + 2\sqrt{3}\tan\frac{\theta}{2} + \sec^2\frac{\theta}{2} - 1\,d\theta$	M1
	$= 3\theta + 2\sqrt{3} \times -2\ln\cos\dfrac{\theta}{2} + 2\tan\dfrac{\theta}{2} - \theta$ (+c) o.e.	M1 A1
	Substitutes their limits to reach Area $= \left(\dfrac{1}{2}\right)\left[2\dfrac{2\pi}{3} - 4\sqrt{3}\ln\cos\dfrac{\pi}{3} + 2\tan\dfrac{\pi}{3} - 0\right]$ o.e.	M1

	$= 2\sqrt{3}\ln 2 + \frac{2\pi}{3} + \sqrt{3}$	**A1**
		(6)
		(10 marks)

Notes:

(a)

M1: Applies $y = r\sin\theta$ and attempts the differentiation. The given identity may be applied first, or later, or not at all. Condone sign/coefficient errors only. Use of $y = r\cos\theta$ scores 0 marks in (a)

A1: Correct derivative.

M1: Sets the derivative equal to zero and solves for θ. If identity was used initially expect as scheme but alternative routes are possible, e.g. see examples above. Allow for any plausible attempt that reaches a value of θ if there are minor errors but attempts at trig identities should be correct up to sign error.

A1: Correct value- must be in correct quadrant. If not explicitly written in (a) but used in (b) as their limit then allow this mark to score in (b)

(b)

M1: Shows intention to find $\int r^2$ by expanding r^2 to obtain three terms in the integrand. There may be errors in the coefficients when squaring, but three terms should be achieved.

M1: Applies $\tan^2 A = \pm 1 \pm \sec^2 A$ to their integral. Not dependent so may be scored as long as they have a $\tan^2$ term. Note: some candidates may use the identity given in the question, but they must progress to terms which can be integrated.

M1: Attempts the integration including $\sec^2 ... \to k\tan ...$ and $\tan ... \to p\ln\cos ...$ or equivalent (e.g. $p\ln\sec ...$). Some candidates may attempt to use the given identity to integrate the $\tan^2\frac{\theta}{2}$ term, but there must be a complete method shown to get the correct form for the integral. Not dependent but must have achieved appropriate tan and $\sec^2$ terms.

A1: Fully correct integration (though the $\frac{1}{2}$ and limits may be missing for this mark).

M1: Applies their limits to their integral. Must see evidence of substitution into their integrated expression. Must have had appropriate tan and $\sec^2$ terms in their integral

A1: Correct answer in the form specified.

NOTE: If a mix of variables is seen, and later corrected prior to substitution of limits, then full marks can still be achieved, but if not recovered then deduct the penultimate A mark

Note: Omission of the ½ could result in M1M1M1A1M1A0

Question	Scheme	Marks
6(a)	$4\dfrac{d^2y}{dx^2}-4\dfrac{dy}{dx}+37y=6e^{5x}$	
	Aux. Eqn. $4m^2-4m+37=0\Rightarrow m=\dfrac{4\pm\sqrt{16-16\times37}}{8}=\dfrac{1}{2}\pm3i$	M1
	C.F. is $(y=)e^{\frac{1}{2}x}(A\cos3x+B\sin3x)$	A1
	PI try $(y=)\ ke^{5x}$	B1
	$\dfrac{dy}{dx}=5ke^{5x},\dfrac{d^2y}{dx^2}=25ke^{5x}\Rightarrow4(25ke^{5x})-4(5ke^{5x})+37(ke^{5x})=6e^{5x}$	M1
	$\Rightarrow 100k-20k+37k=6\Rightarrow k=\cdots\ \ (k=\dfrac{2}{39}\,or\,\dfrac{6}{117}\ o.e.)$	M1
	So gen sol. is $y=\dfrac{2}{39}e^{5x}+e^{\frac{1}{2}x}(A\cos3x+B\sin3x)$	A1ft
		(6)
(b)	$x=0,y=0\Rightarrow0=\dfrac{2}{39}+A$	M1
	$\dfrac{dy}{dx}=\dfrac{10}{39}e^{5x}+\dfrac{1}{2}e^{\frac{1}{2}x}(A\cos3x+B\sin3x)+e^{\frac{1}{2}x}(-3A\sin3x+3B\cos3x)$	M1A1ft
	$x=0,\dfrac{dy}{dx}=0\Rightarrow\dfrac{10}{39}+\dfrac{1}{2}A+3B=0\Rightarrow B=...,A=...$	M1
	$(y=)\ \frac{2}{39}e^{5x}+e^{\frac{1}{2}x}\left(-\frac{2}{39}\cos3x-\frac{1}{13}\sin3x\right)$	A1
		(5)
		(11 marks)

Notes:

(a)

M1: Forms correct auxiliary equation and attempts to solve by any valid means. Implied by correct roots. Usual rules for solving a quadratic apply.

A1: Correct complementary function, allow without the "$y=$". Accept $y=Ae^{\left(\frac{1}{2}+3i\right)x}+Be^{\left(\frac{1}{2}-3i\right)x}$ or from De Moivre can use e.g. $y=Ce^{\frac{1}{2}x}\cos(3x+\alpha)$ etc. Condone mixed variables for this mark. May not be seen until the end.

B1: Correct form for the particular integral selected. Must include ke^{5x} but accept with any extra terms that will disappear when constants found.

M1: Differentiates their PI twice and substitutes into the equation. Allow sign/coefficient errors only.

M1: Proceeds to find the value of the constant(s) from a valid PI. Note that the constant value need not be simplified, so accept exact equivalents such as $\frac{6}{117}$ etc

A1ft: Correct general solution. **Must have $y=\dots$ and be in terms of x *only*. Follow through their CF in terms of x only but the PI must be correct.** May appear at the start of (b) instead of the end of (a) which is fine.

(b)

M1: Uses the initial condition for *x* to find the value for "*A*" (their cosine coefficient). If by error of CF they obtain an equation in *A* and *B* then score for the attempt of producing a suitable equation.

M1: Differentiates the GS. Allow sign/coefficient errors only, so a full method for product rule.

A1ft: Correct differentiation following through their PI- but must have been of the correct form. Also allow ft for an incorrect value of A if found before differentiation

M1: Uses the initial conditions to find values for all constants in their equation.

A1: Correct particular solution. Allow exact equivalents for the coefficients. Accept without 'y = …' so e.g. PS = is ok here. **Must be in terms of *x only* with no mixed variables etc**

NOTE: Alt method – it is possible for candidates to form two simultaneous equations in A and B from the first and second derivatives and solve to find A and B. If you see this then please send to review.

Question	Scheme	Marks
7(a)(i)+(ii)	$\cos 5\theta + i\sin 5\theta = (\cos\theta + i\sin\theta)^5$ o.e.	M1
	$= c^5 + 5c^4si - 10c^3s^2 - 10c^2s^3i + 5cs^4 + s^5i$ $\Rightarrow \sin 5\theta = \ldots, \cos 5\theta = \ldots$	M1
	$\sin 5\theta \equiv 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$ *	A1*
	$\cos 5\theta \equiv \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	A1
		(4)
(b)	$\tan 5\theta = \dfrac{\sin 5\theta}{\cos 5\theta} = \dfrac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta} \times \dfrac{\cos^{-5}\theta}{\cos^{-5}\theta}$	M1
	$\dfrac{\dfrac{5\sin\theta}{\cos\theta} - \dfrac{10\sin^3\theta}{\cos^3\theta} + \dfrac{\sin^5\theta}{\cos^5\theta}}{1 - \dfrac{10\sin^2\theta}{\cos^2\theta} + \dfrac{5\sin^4\theta}{\cos^4\theta}} = \dfrac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$	A1*
		(2)
(c)	$x = \tan\theta,\ 2x^5 - 15x^4 - 20x^3 + 30x^2 + 10x - 3 = 0$ $\Rightarrow \dfrac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta} = \dfrac{3}{2} \Rightarrow \tan 5\theta = \dfrac{3}{2}$	M1
	$\theta = \dfrac{1}{5}\tan^{-1}\dfrac{3}{2}$ $(= 0.19655 \text{ radians}, 11.2619\ldots°)$	A1
	$\Rightarrow x = \tan 0.196555\ldots, \ldots$	M1
	Two of: $x = 0.1991\ldots$, $1.0822\ldots$, $8.464\ldots$, $-1.7847\ldots$, $-0.4607\ldots$	A1
	All of: $x = -1.785$, -0.461 , 0.199 , 1.082 , 8.464	A1
		(5)
		(11 marks)

Notes:

(a)(i)+(ii)

M1: Demonstrates use of De Moivre such as $\cos 5\theta + \mathrm{i}\sin 5\theta = (\cos\theta + i\sin\theta)^5$, $\sin 5\theta = Im\,(\cos\theta + i\sin\theta)^5$ or $\cos 5\theta$ = their real terms from the expansion of $(\cos\theta + i\sin\theta)^5$

M1: Attempts to expand, need not be simplified. Accept sign errors, but should have the correct evaluated coefficients and the correct powers on the sin and cos terms and equate **either** the real or imaginary parts

A1*: Correct expression for $\sin 5\theta$ derived from **fully correct working**. Allow notational slips with nCr etc as long as the intention is clear. To score this mark we need to see either a full expansion if working with $(\cos\theta + i\sin\theta)^5$ or the imaginary terms if working with $Im\,(\cos\theta + i\sin\theta)^5$

Note: Consistent omission of i's is 2$^{\text{nd}}$ M0, but condone the occasional slip for the M marks

A1: Correct expression for $\cos 5\theta$. Allow recovery from slips, e.g. missing arguments for this mark, and allow if seen in part (b).

(b)

M1: Applies $\tan 5\theta = \dfrac{\sin 5\theta}{\cos 5\theta}$ and shows or clearly states the intention to divide numerator and denominator through by $\cos^5\theta$ or equivalent work (e.g. factorising $\sin\theta$ from the numerator and $\cos\theta$ from the denominator and then dividing all terms by $\cos^4\theta$ to obtain $\tan\theta$ terms)

A1*: Correct given expression reached from **fully correct work with no errors**

Note: Do not penalise slips in notation or missing arguments already penalised in (a)

(c)

Note: Must use (b) so calculator methods to solve by e.g. tracing the points or eq. solver scores 0

M1: Substitutes $x = \tan\theta$ into the equation and rearranges to identify the $\tan 5\theta$ within the equation. May be slips when rearranging, but look for the attempt to reach $\tan 5\theta$

A1: Obtains a correct value for θ either in degrees or radians, or as an expression. May be implied.

M1: Uses at least one value of θ and undoes the substitution to find at least one value for x

A1: At least two correct values of x found correct to at least 2 s.f. (So accept 0.20,1.1,8.5 etc)

A1: All five solutions correct to 3 decimal places, and no extra solutions.

Question	Scheme	Marks
8(a)	Real line is $y=0$, so have $u+\mathrm{i}v=\dfrac{(\sqrt{3}-\mathrm{i})(x-2)}{x+2}=\dfrac{\sqrt{3}(x-2)}{x+2}-\dfrac{(x-2)}{x+2}\mathrm{i}$	M1
	$\Rightarrow \dfrac{u}{v}=\dfrac{\sqrt{3}}{-1}\Rightarrow v=-\dfrac{1}{\sqrt{3}}u$ *	M1A1*
		(3)
	For any attempts at solutions which use image points under the transformation T for parts (a) or (b) please send to review	
(a) **Alt (i)**	$w=\dfrac{(\sqrt{3}-i)(z-2)}{z+2}\Rightarrow wz+2w=\sqrt{3}z-2\sqrt{3}-iz+2i$ $\Rightarrow z=\dfrac{2(i-w-\sqrt{3})}{i+w-\sqrt{3}}=\dfrac{-2(u+\sqrt{3})-2(v-1)i}{(u-\sqrt{3})+(v+1)i}\times\dfrac{(u-\sqrt{3})-(v+1)i}{(u-\sqrt{3})-(v+1)i}$	M1
	$=\dfrac{-2(u^2-3)+2(v+1)(u+\sqrt{3})i-2i(uv-\sqrt{3}v-u+\sqrt{3})-2(v^2-1)}{(u-\sqrt{3})^2+(v+1)^2}$ $y=0\ \Rightarrow 2(v+1)(u+\sqrt{3})-2(uv-\sqrt{3}v-u+\sqrt{3})=0$	M1
	$2(uv+\sqrt{3}v+u+\sqrt{3})-2uv+2\sqrt{3}v+2u-2\sqrt{3}=0$ $\Rightarrow 4\sqrt{3}v+4u=0\Rightarrow v=-\dfrac{1}{\sqrt{3}}u$ *	A1*
Alt (ii)	$w=\dfrac{(\sqrt{3}-i)(z-2)}{z+2}\Rightarrow ux+2u+ivx+2iv=\sqrt{3}x-2\sqrt{3}-ix+2i$ $\Rightarrow ux+2u=\sqrt{3}x-2\sqrt{3}$ and $vx+2v=-x+2$	M1
	$\Rightarrow x=\frac{-2u-2\sqrt{3}}{u-\sqrt{3}}$ and $x=\frac{-2v+2}{v+1}$ $\Rightarrow \dfrac{-2u-2\sqrt{3}}{u-\sqrt{3}}=\dfrac{-2v+2}{v+1}$	M1
	$\Rightarrow -2uv-2u-2\sqrt{3}v-2\sqrt{3}=-2uv+2\sqrt{3}v+2u-2\sqrt{3}$ $4\sqrt{3}v=-4u\Rightarrow v=-\frac{1}{\sqrt{3}}u$*	A1*

	Note: If a candidate finds an expression for z in (a) then this may be used/seen in (b) and can score M1A1 in part (b) if used in (b)	
(b)	$w=\frac{(\sqrt{3}-\mathrm{i})(z-2)}{z+2}\Rightarrow w(z+2)=(\sqrt{3}-\mathrm{i})z-2(\sqrt{3}-\mathrm{i})\Rightarrow z=\ldots$ or $kz=\ldots$	**M1**
	$z=\frac{-2w-2(\sqrt{3}-\mathrm{i})}{w-\sqrt{3}+\mathrm{i}}$ or $z(w-\sqrt{3}+\mathrm{i})=-2w-2(\sqrt{3}-\mathrm{i})$	**A1**
	$\lvert z\rvert=2\Rightarrow 2\lvert u+\mathrm{i}v-\sqrt{3}+\mathrm{i}\rvert=\lvert -2u-2\mathrm{i}v-2\sqrt{3}+2\mathrm{i}\rvert$	**M1**
	$\Rightarrow 4(u-\sqrt{3})^2+4(v+1)^2=(2u+2\sqrt{3})^2+(2v-2)^2$ $\Rightarrow \cancel{4u^2}-8\sqrt{3}u+12+\cancel{4v^2}+8v+4=\cancel{4u^2}+8\sqrt{3}u+12+\cancel{4v^2}-8v+4$	**M1**
	$\Rightarrow 16v=16\sqrt{3}u\Rightarrow v=\sqrt{3}u$ o.e.	**A1**
		(5)
(b) Alt (i)	$w=\frac{(\sqrt{3}-\mathrm{i})(z-2)}{z+2}\Rightarrow w(z+2)=(\sqrt{3}-\mathrm{i})z-2(\sqrt{3}-\mathrm{i})\Rightarrow z=\ldots$ or $kz=\ldots$	**M1**
	$z=\frac{-2w-2(\sqrt{3}-\mathrm{i})}{w-\sqrt{3}+\mathrm{i}}$ or $z(w-\sqrt{3}+\mathrm{i})=-2w-2(\sqrt{3}-\mathrm{i})$	**A1**
	$\lvert z\rvert=2\Rightarrow\lvert w-\sqrt{3}+i\rvert=\lvert -w-\sqrt{3}+i\rvert$ $\lvert w-(\ldots)\rvert=\lvert w-(\ldots)\rvert$	**M1**
	$\lvert w-(\sqrt{3}-i)\rvert=\lvert w-(-\sqrt{3}+i)\rvert$ So perp bisector of $(\sqrt{3},-1)$ and $(-\sqrt{3},1)\Rightarrow v=\sqrt{3}u$ o.e.	**M1A1**
(c)	$R=\{z\in\mathbb{C}:\lvert z\rvert<2\}\cap\{z\in\mathbb{C}:Im\,z>0\}$	
	$v=\sqrt{3}u\Rightarrow arg\,w=arctan\sqrt{3}=\ldots\left(\frac{\pi}{3}\right)$	**M1**
	$v=-\frac{1}{\sqrt{3}}u\Rightarrow arg\,w=arctan-\frac{1}{\sqrt{3}};=-\frac{\pi}{6}$ or $\frac{5\pi}{6}$	**M1A1**
	$z=0\Rightarrow w=\mathrm{i}-\sqrt{3}$ so need $\beta=\frac{5\pi}{6}$	**M1**
	Image region is $\left\{w\in\mathbb{C}:\frac{\pi}{3}<arg\,w<\frac{5\pi}{6}\right\}$	**A1***
		(5)
		(13 marks)

Notes:

(a)
M1: Realises real line is $y = 0$ and substitutes into the equation for w
M1: Equates real and imaginary parts and solves simultaneously to eliminate x or equivalent work
A1*: Correct work leading to the given equation, no errors seen. At least one intermediate step required after identifying- which may be implied- the real and imaginary parts (u and v in terms of x)

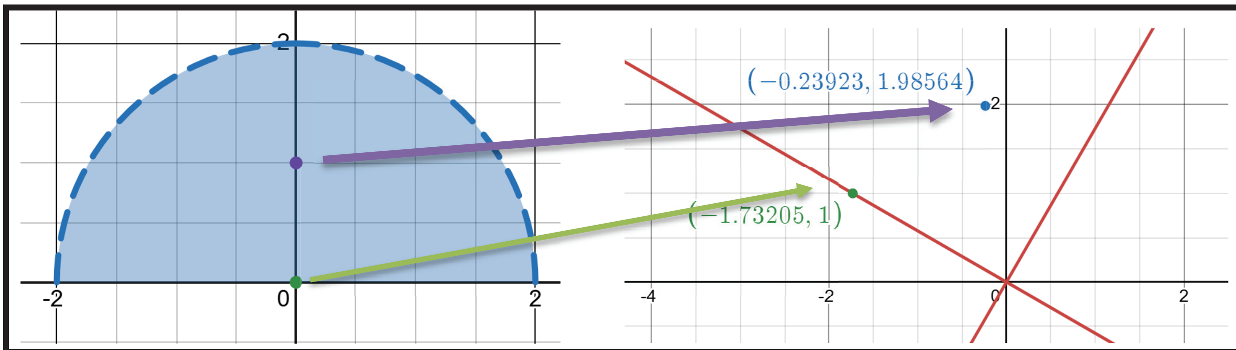
Alt (i):
M1: Makes z the subject and multiplies by a valid conjugate. Accept any suitable attempt to make z the subject
M1: Extracts the imaginary part and sets equal to 0 to achieve an unsimplified line equation in u and v
A1*: Achieves the correct line equation with no errors seen

Alt (ii)
M1: Replaces $w = u + iv$ and $z = x$, then rearranges the given transformation and equates the real and imaginary parts
M1: Finds two expressions for x and equates
A1*: Achieves the correct equation with no errors seen

(b)
M1: Rearranges the transformation to make z the subject, or at least as far as isolating all the z terms together.
A1: Correct expression for z or with z isolated within the expression.
M1: Applies the circle equation and substitutes $w = u + \mathrm{i}v$
M1: Expands the moduli and cancels the square terms to arrive at an equation for a line.
A1: Correct equation (any equivalent form acceptable)

(b) Alt (i)
M1: Rearranges the transformation to make z the subject, or at least as far as isolating all the z terms together.
A1: Correct expression for z or with z isolated within the expression.
M1: Processes to an expression of the form $|z - a| = |z - b|$
M1: Recognises that the locus is a perpendicular bisector of $(\sqrt{3}, -1)$ and $(-\sqrt{3}, 1)$
A1: Achieves the correct equation

(c)
M1: Attempts the argument for the line from the image of the circle from their line equation from part (b).
M1: Attempts the argument for the image of the real axis as given in (a)
A1: Correct possible arguments for the image of real axis identified. Need not select at this stage, so accept if just $-\frac{\pi}{6}$ is given.
M1: Uses a point on the real line segment of the boundary, or in the region R or elsewhere to determine which arguments are needed. May see image of i used (which is $\frac{4-3\sqrt{3}}{5} + \frac{3+4\sqrt{3}}{5}\mathrm{i}$) to identify the correct quadrant, which is fine.
Alt: Uses the condition that Im $z > 0$ in their imaginary part for z to **show that** $v > -\frac{1}{\sqrt{3}}u$ and $|z| < 2$ in their expression for z to **show that** $v > \sqrt{3}u$
A1: Correct answer obtained with full justification. Condone the omission of $w \in \mathbb{C}$ etc



In (c) candidates are effectively finding which side of the lines found in (a), (b) the points inside the semicircle with radius 2 and positive imaginary part, are mapped to under T.

The origin is mapped to point $\left(-\sqrt{3}, 1\right)$ which is $(-1.73\ \ldots, 1)$ shown above, which is on the half line with argument $\frac{5\pi}{6}$

The point i which is $(0, 1)$ is mapped to $\frac{4-3\sqrt{3}}{5}+\frac{3+4\sqrt{3}}{5}\mathrm{i}$ which is $(-0.239.., 1.985..)$ shown above