

WFM02 Further Pure Mathematics F2 Mark Scheme

Question Number	Scheme	Notes	Marks
1	$z^5 - 32i = 0 \Rightarrow r^5 = 32 \Rightarrow r = 2$	Correct value for r . May be shown explicitly or used correctly.	B1
	$5\theta = \frac{\pi}{2} + 2n\pi \Rightarrow \theta = \frac{\pi}{10} + \frac{2n\pi}{5}$	Applies a correct strategy for establishing at least 2 values of θ . This can be awarded if the initial angle $\left(\frac{\pi}{2} \text{ or } \frac{\pi}{10}\right)$ is incorrect but otherwise their strategy is correct.	M1
	$z = 2e^{i\frac{\pi}{10}}, 2e^{i\frac{\pi}{2}}, 2e^{i\frac{9\pi}{10}}, 2e^{i\frac{13\pi}{10}}, 2e^{i\frac{17\pi}{10}}$ or	At least 2 correct, follow through their r	A1ft
	$z = 2e^{i\left(\frac{\pi}{10} + \frac{2n\pi}{5}\right)}, n = 0, 1, 2, 3, 4$	All correct. Must have $r = 2$	A1
			(4)
			Total 4

Question Number	Scheme	Notes	Marks
2	$\frac{x}{2-x} > \frac{x+3}{x}$		
Way 1	$\frac{x}{2-x} > \frac{x+3}{x} \Rightarrow \frac{x}{2-x} - \frac{x+3}{x} > 0$	Collects to one side	M1
	$\frac{x}{2-x} - \frac{x+3}{x} > 0 \Rightarrow \frac{x^2 - (2-x)(x+3)}{x(2-x)} > 0$ M1: Attempt common denominator A1: Correct fraction		M1 A1
	$x = 0, 2$	These critical values	B1
	$x^2 - (2-x)(x+3) = 0$ $\Rightarrow 2x^2 + x - 6 = 0 \Rightarrow x = \dots$	Solves the 3TQ in the numerator	M1
	$x = \frac{3}{2}, -2$	These critical values	A1
	$x > -2, 0 < x < \frac{3}{2}, x > 2$ A1: Any 2 of these with strict inequalities allowed A1: All correct with inequalities as shown. Ignore what they have between their inequalities e.g. allow "or", "and", ":", etc. but not \cap		A1A1
			(8)
			Total 8
Alternative 1: $\times x^2(2-x)^2$			
$x^3(2-x), x(x+3)(2-x)^2$	Multiplies by a positive expression	M1	
$x^3(2-x) - x(x+3)(2-x)^2 > 0$	Collects to one side Correct inequality	M1 A1	
$x = 0, 2$	These critical values	B1	
$x(2-x)[x^2 - (x+3)(2-x)] = 0$ $x^2 - (x+3)(2-x) = 0$ $\Rightarrow 2x^2 + x - 6 = 0 \Rightarrow x = \dots$	Attempts to factorise by taking out a factor of $x(2-x)$ and solves resulting 3TQ. May have quartic and apply the factor theorem.	M1	
$x = \frac{3}{2}, -2$	These critical values	A1	
$x > -2, 0 < x < \frac{3}{2}, x > 2$ A1: Any 2 of these with strict inequalities allowed A1: All correct with inequalities as shown. Ignore what they have between their inequalities e.g. allow "or", "and", ":", etc. but not \cap		A1A1	

Question Number	Scheme	Notes	Marks
3	$w = \frac{(2+i)z+4}{z-i} \Rightarrow wz - wi = (2+i)z + 4$ $\Rightarrow z = \dots$	Attempts to make z the subject	M1
	$z = \frac{wi+4}{w-2-i}$	Correct equation in any form	A1
	$z = \frac{(u+iv)i+4}{u+iv-2-i}$ $z = \frac{((u+iv)i+4)(u-2-(v-1)i)}{(u-2+(v-1)i)(u-2-(v-1)i)}$	Introduces $w = u + iv$ and multiplies numerator and denominator by the conjugate of the denominator	M1
	$u(v-1) + (4-v)(u-2) = 0$	Sets real part = 0 (with or without denominator) Depends on both M marks above	dM1
		Any correct equation	A1
	$3u + 2v - 8 = 0$	Correct equation in the required form (allow any integer multiple)	A1
Way 2	$w = \frac{(2+i)z+4}{z-i}, z = yi \Rightarrow w = \frac{(2+i)yi+4}{yi-i}$ $w = \frac{(2+i)yi+4}{yi-i} \times \frac{i}{i}$	Solves simultaneously and multiplies numerator and denominator by i	M1
	$u = \frac{2y}{y-1}, v = \frac{y-4}{y-1}$	Correct real and imaginary parts	A1
	$u = \frac{2y}{y-1} \Rightarrow y = \frac{u}{u-2}$	Attempts y in terms of u or v	M1
	$y = \frac{u}{u-2} \Rightarrow v = \frac{\frac{u}{u-2} - 4}{\frac{u}{u-2} - 1}$	Obtains an equation connecting u and v	M1
		Any correct equation	A1
	$3u + 2v - 8 = 0$	Correct equation in the required form (allow any integer multiple)	A1
Way 3	Apply the transformation to any point on the imaginary axis	Eg $(0,0) \rightarrow (0,4)$ $(0,1) \rightarrow (4,-2)$	M1
	Apply the transformation to a second point on the imaginary axis	This is the second M mark on e-PEN	M1
	Both transformations correct	This is the first A mark on e-PEN	A1
	Complete method to obtain an equation for the line thro' their 2 points in the w -plane		M1
	Correct equation in any form		A1
	$3u + 2v - 8 = 0$	Correct equation in the required form (allow any integer multiple)	A1
			Total 6

Question Number	Scheme	Notes	Marks
4(a)	$(x+1)\frac{dy}{dx} - xy = e^{3x} \quad x > -1$		
	$\frac{dy}{dx} - \frac{xy}{(x+1)} = \frac{e^{3x}}{(x+1)}$	Correctly rearranged equation	B1
	$I = e^{\int \frac{-x}{x+1} dx} = e^{\int \left(-1 + \frac{1}{x+1}\right) dx}$	Correct strategy for the integrating factor including an attempt at the integration	M1
	$= e^{-x + \ln(x+1)}$	For $-x + \ln(x+1)$	A1
	$= (x+1)e^{-x}$	Correct integrating factor	A1
	$y(x+1)e^{-x} = \int \frac{e^{3x}}{x+1} \times (x+1)e^{-x} dx$	Uses their integrating factor to reach the form $yI = \int QI dx$	M1
	$y(x+1)e^{-x} = \frac{1}{2}e^{2x} + c$	Correct equation (with or without $+ c$)	A1
	$y = \frac{e^{3x}}{2(x+1)} + \frac{ce^x}{(x+1)}$	Correct answer (allow equivalent forms). Must have $y = \dots$	A1
			(7)
(b)	$x = 0, y = 5 \Rightarrow 5 = \frac{1}{2} + c \Rightarrow c = \frac{9}{2}$	Substitutes $x = 0$ and $y = 5$ and attempts to find a value for c .	M1
	$y = \frac{e^{3x}}{2(x+1)} + \frac{9e^x}{2(x+1)}$	Ca0 (oe) Must have $y = \dots$	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
5(a)	$y = \tan^2 x \Rightarrow \frac{dy}{dx} = 2 \tan x \sec^2 x$	Correct first derivative any correct form	B1
	$\frac{dy}{dx} = 2 \tan x \sec^2 x \Rightarrow \frac{d^2y}{dx^2} = 2 \sec^4 x + 4 \sec^2 x \tan^2 x$ M1: Correct application of the product rule and chain rule A1: Correct expression		M1A1
	$\frac{d^2y}{dx^2} = 2 \sec^4 x + 4 \sec^2 x \tan^2 x \Rightarrow \frac{d^3y}{dx^3} = 8 \sec^4 x \tan x + 8 \sec^2 x \tan^3 x + 8 \sec^4 x \tan x$ Or $\frac{d^2y}{dx^2} = 6 \sec^4 x - 4 \sec^2 x \Rightarrow \frac{d^3y}{dx^3} = 24 \sec^4 x \tan x - 8 \sec^2 x \tan x$ M1: Attempt to differentiate using product and chain rule. At least one term to be correct		M1
	$= 8 \sec^4 x \tan x + 8 \sec^2 x \tan x (\sec^2 x - 1) + 8 \sec^4 x \tan x$ $= 24 \sec^4 x \tan x - 8 \sec^2 x \tan x = 8 \sec^2 x \tan x (3 \sec^2 x - 1)$ Fully correct expression		A1
			(5)
(b)	$(y)_{\frac{\pi}{3}} = 3, (y')_{\frac{\pi}{3}} = 8\sqrt{3}, (y'')_{\frac{\pi}{3}} = 80, (y''')_{\frac{\pi}{3}} = 352\sqrt{3}$	Attempts the values up to the third derivative when $x = \frac{\pi}{3}$	M1
	$y = 3 + 8\sqrt{3} \left(x - \frac{\pi}{3}\right) + \frac{80}{2!} \left(x - \frac{\pi}{3}\right)^2 + \frac{352\sqrt{3}}{3!} \left(x - \frac{\pi}{3}\right)^3 + \dots$ Correct application of the Taylor series 2! or 2, 3! or 6		M1
	$y = 3 + 8\sqrt{3} \left(x - \frac{\pi}{3}\right) + 40 \left(x - \frac{\pi}{3}\right)^2 + \frac{176\sqrt{3}}{3} \left(x - \frac{\pi}{3}\right)^3 + \dots$ Correct expansion Must start $y = \dots$ or $\tan^2 x = \dots$ $f(x)$ only accepted if $f(x)$ has been defined to be $\tan^2 x$		A1
			(3)
			Total 8

Question Number	Scheme	Notes	Marks
6(a)	$ z+1-13i =3 z-7-5i \Rightarrow (x+1)^2+(y-13)^2=9\{(x-7)^2+(y-5)^2\}$ Correct application of Pythagoras Accept 3 or 9 on RHS		M1
	$\Rightarrow x^2+y^2-16x-8y+62=0$	Correct equation in any form with terms collected	A1
	Centre (8, 4)	Correct centre. i included scores A0	A1
	$r^2=64+16-62=...$	Correct method for r or r^2	M1
	$r=\sqrt{18}$ or $3\sqrt{2}$	Correct radius. Must be exact.	A1
(b)	$\arg(z-8-6i)=-\frac{3\pi}{4} \Rightarrow y-6=x-8$	Converts the given locus to the correct Cartesian form	B1
	$\Rightarrow x^2+y^2-16x-8y+62=0$ $\Rightarrow x^2+(x-2)^2-16x-8(x-2)+62=0 \Rightarrow x=...$ or $\Rightarrow (y+2)^2+y^2-16x-8(y+2)+62=0 \Rightarrow y=...$	Uses both Cartesian equations to obtain an equation in one variable and attempts to solve	M1
	$x=7-2\sqrt{2}$ or $y=5-2\sqrt{2}$	One correct "coordinate"	A1
	R is $7-2\sqrt{2}+(5-2\sqrt{2})i$ or $x=7-2\sqrt{2}$ and $y=5-2\sqrt{2}$	Correct complex number or coordinates and no others. Must be exact	A1
			Total 9

Question Number	Scheme	Notes	Marks
7(a)	$x = t^2 \Rightarrow \frac{dx}{dy} = 2t \frac{dt}{dy}$ oe	Correct application of the chain rule	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2t} \frac{dy}{dt}$ (or e.g. $\frac{1}{2\sqrt{x}} \frac{dy}{dt}$)	Any correct expression for $\frac{dy}{dx}$ or equivalent equation	A1
	$2\sqrt{x} \frac{dy}{dx} = \frac{dy}{dt} \Rightarrow x^{-\frac{1}{2}} \frac{dy}{dx} + 2\sqrt{x} \frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} \frac{dt}{dx}$ (NB $\frac{d^2y}{dt^2} = 2 \frac{dy}{dx} + 4x \frac{d^2y}{dx^2}$)	Fully correct strategy to obtain an equation involving $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{dt^2}$ Chain rule used on at least one term. Depends on the first M mark	dM1
	$4x \frac{d^2y}{dx^2} + 2(1 + 2\sqrt{x}) \frac{dy}{dx} - 15y = 15x \Rightarrow 4x \frac{d^2y}{dx^2} + 4\sqrt{x} \frac{dy}{dx} + 2 \frac{dy}{dx} - 15y = 15x$ $\Rightarrow \frac{d^2y}{dt^2} + 2 \frac{dy}{dt} - 15y = 15t^2 *$ ddM1: Substitutes into the given differential equation. The full substitution must be seen. Depends on both M marks. A1*: Cso		ddM1 A1*
			(5)
(b)	$m^2 + 2m - 15 = 0 \Rightarrow m = 3, -5$	Attempts to solve $m^2 + 2m - 15 = 0$	M1
	$y = Ae^{-5t} + Be^{3t}$	Correct CF	A1
	$y = at^2 + bt + c \Rightarrow \frac{dy}{dt} = 2at + b \Rightarrow \frac{d^2y}{dt^2} = 2a$ $\Rightarrow 2a + 4at + 2b - 15at^2 - 15bt - 15c = 15t^2$ Starts with the correct PI form and differentiates twice and substitutes		M1
	$-15a = 15 \Rightarrow a = \dots$ $4a - 15b = 0 \Rightarrow b = \dots$ $2a + 2b - 15c = 0 \Rightarrow c = \dots$	Complete method to find a, b and c by comparing coefficients. Values for all 3 needed. Depends on the second M mark.	dM1
	$y = Ae^{-5t} + Be^{3t} - t^2 - \frac{4}{15}t - \frac{38}{225}$	Correct GS. Must start $y = \dots$	A1
			(5)
(c)	$y = Ae^{-5\sqrt{x}} + Be^{3\sqrt{x}} - x - \frac{4}{15}\sqrt{x} - \frac{38}{225}$	Correct equation (follow through their answer to (b)) Must start $y = \dots$	B1ft
			(1)
			Total 11

Question Number	Scheme	Notes	Marks
8(a)	$x = r \cos \theta = (1 + \sin \theta) \cos \theta$ $\Rightarrow \frac{dx}{d\theta} = \cos^2 \theta - (1 + \sin \theta) \sin \theta$ <p style="text-align: center;">or</p> $\Rightarrow \frac{dx}{d\theta} = -\sin \theta + \cos 2\theta$	Differentiates $r \cos \theta$ using product rule or double angle formula	M1
	$\cos^2 \theta - (1 + \sin \theta) \sin \theta = 0 \Rightarrow 1 - \sin^2 \theta - \sin \theta - \sin^2 \theta = 0 \Rightarrow 2 \sin^2 \theta + \sin \theta - 1 = 0$ <p style="text-align: center;">or</p> $-\sin \theta + \cos 2\theta = 0 \Rightarrow -\sin \theta + 1 - 2 \sin^2 \theta = 0 \Rightarrow 2 \sin^2 \theta + \sin \theta - 1 = 0$ <p style="text-align: center;">Sets $\frac{dx}{d\theta} = 0$ and proceeds to a 3TQ in $\sin \theta$</p> <p style="text-align: center;">Depends on the first M mark</p>	Correct derivative in any form	A1
	$\Rightarrow 2 \sin^2 \theta + \sin \theta - 1 = 0$ $\Rightarrow \sin \theta = \frac{1}{2}, (-1) \Rightarrow \theta = \dots$	Solves for θ . Depends on both M marks above.	ddM1
	$\left(\frac{3}{2}, \frac{\pi}{6} \right)$	Correct coordinates and no others. Need not be in coordinate brackets.	A1
			(5)
(b)	$\int (1 + \sin \theta)^2 d\theta = \int (1 + 2 \sin \theta + \sin^2 \theta) d\theta$ $= \int \left(1 + 2 \sin \theta + \frac{1}{2} - \frac{1}{2} \cos 2\theta \right) d\theta$	Attempts $\left(\frac{1}{2} \right) \int r^2 d\theta$ and applies $\sin^2 \theta = \pm \frac{1}{2} \pm \frac{1}{2} \cos 2\theta$ Ignore any limits shown	M1
	$\int (1 + \sin \theta)^2 d\theta = \frac{3}{2} \theta - 2 \cos \theta - \frac{1}{4} \sin 2\theta (+c)$	Correct integration (Ignore limits)	A1
	$\frac{1}{2} \left[\frac{3}{2} \theta - 2 \cos \theta - \frac{1}{4} \sin 2\theta \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $= \frac{1}{2} \left[\frac{3\pi}{4} - \left(\frac{\pi}{4} - \sqrt{3} - \frac{\sqrt{3}}{8} \right) \right] \left(= \frac{\pi}{4} + \frac{9\sqrt{3}}{16} \right)$	Applies the limits of $\frac{\pi}{2}$ and their $\frac{\pi}{6}$ Substitution must be shown but no simplification needed	M1
	<p style="text-align: center;">Trapezium:</p> $\frac{1}{2} \left(2 + \left(2 - \frac{3}{2} \sin \frac{\pi}{6} \right) \right) \times \frac{3}{2} \cos \frac{\pi}{6}$ $\left(= \frac{39\sqrt{3}}{32} \right)$	Uses a correct strategy for the area of trapezium $OQSP$	M1
	$\text{Area of } R = \frac{39\sqrt{3}}{32} - \frac{\pi}{4} - \frac{9\sqrt{3}}{16}$	Fully correct method for the required area. Depends on all previous method marks.	dm1
	$\frac{1}{32} (21\sqrt{3} - 8\pi)$	Cao	A1
			(6)

Question Number	Scheme	Notes	Marks
9(a)	$n^5 - (n-1)^5 = n^5 - (n^5 - 5n^4 + 10n^3 - 10n^2 + 5n - 1) = \dots$ <p>Starts the proof by expanding the bracket</p>		M1
	$5n^4 - 10n^3 + 10n^2 - 5n + 1^*$	Correct proof with no errors. Full expansion of $(n-1)^5$ must be shown.	A1*
			(2)
(b)	$1^5 - 0^5 = 5(1)^4 - 10(1)^3 + 10(1)^2 - 5(1) + 1$ $2^5 - 1^5 = 5(2)^4 - 10(2)^3 + 10(2)^2 - 5(2) + 1$ <p>.....</p> $(n-1)^5 - (n-2)^5 = 5(n-1)^4 - 10(n-1)^3 + 10(n-1)^2 - 5(n-1) + 1$ $(n)^5 - (n-1)^5 = 5(n)^4 - 10(n)^3 + 10(n)^2 - 5(n) + 1$ $n^5 = 5 \sum_{r=1}^n r^4 - 10 \sum_{r=1}^n r^3 + 10 \sum_{r=1}^n r^2 - 5 \sum_{r=1}^n r + n$ <p>M1: Applies the result from part (a) between 1 and n and sums both sides Min 3 lines shown A1: Correct equation If only the last line is seen, award M1A1 These marks can be implied by a correct following stage.</p>		M1A1
	$n^5 = 5 \sum_{r=1}^n r^4 - 10 \times \frac{1}{4} n^2 (n+1)^2 + 10 \times \frac{1}{6} n(n+1)(2n+1) - 5 \times \frac{1}{2} n(n+1) + n$ <p>M1: Introduces at least 2 correct summation formulae A1: Correct equation</p>		M1A1
	$5 \sum_{r=1}^n r^4 = \frac{5}{2} n^2 (n+1)^2 - \frac{5}{3} n(n+1)(2n+1) + \frac{5}{2} n(n+1) + n^5 - n = \dots$ $5 \sum_{r=1}^n r^4 = n(n+1) \left[\frac{5}{2} n(n+1) - \frac{5}{3} (2n+1) + \frac{5}{2} + n^3 - n^2 + n - 1 \right]$ <p>Makes $5 \sum_{r=1}^n r^4$ or $\sum_{r=1}^n r^4$ the subject and takes out a factor of $n(n+1)$</p>		M1
	$\sum_{r=1}^n r^4 = \frac{1}{30} n(n+1) [15n(n+1) - 10(2n+1) + 15 + 6(n^3 - n^2 + n - 1)]$ $= \frac{1}{30} n(n+1) [6n^3 + 9n^2 + n - 1] = \frac{1}{30} n(n+1)(2n+1)(\dots)$ <p>Takes out a factor of $n(n+1)(2n+1)$ Depends on all previous method marks</p>		dM1
	$= \frac{1}{30} n(n+1)(2n+1)(3n^2 + 3n - 1)$		cao
		(7)	
			Total 9