| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 1 1(a) | $\frac{2}{r(r+1)(r-1)}=\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}$ | M1A1A1 (3) |
| 1(b) | $\begin{array}{ll} r=2 & 1-\frac{2}{2}+\frac{1}{3} \\ r=3 & \frac{1}{2}-\frac{2}{3}+\frac{1}{4} \end{array}$ |  |
|  | $r=4 \quad \frac{1}{3}-\frac{2}{4}+\frac{1}{5}$ | M1 |
|  | $r=n-1 \quad \frac{1}{n-2}-\frac{2}{n-1}+\frac{1}{n}$ |  |
|  | $r=n \quad \frac{1}{n-1}-\frac{2}{n}+\frac{1}{n+1}$ | M1 |
|  | $\sum_{r=2}^{n}\left(\frac{1}{r-1}-\frac{2}{r}+\frac{1}{r+1}\right)=\left(1-\frac{2}{2}+\frac{1}{2}+\frac{1}{n}-\frac{2}{n}+\frac{1}{n+1}\right)$ | A1 |
|  | $\frac{1}{2} \sum_{r=1}^{n} \frac{2}{r(r+1)(r-1)}=\frac{1}{2} \times\left(\frac{1}{2}-\frac{1}{n}+\frac{1}{n+1}\right)=\frac{n^{2}+n-2}{4 n(n+1)}$ | dM1A1 (5) |
|  |  | [8] |
| (a) |  |  |
| M1 | Attempt PFs by any valid method (by implication if 3 correct fractions seen) |  |
| A1A1 <br> (b) | A1 any 2 fractions correct; A1 third fraction correct |  |
|  |  |  |
|  | Method of differences with at least 3 terms at start an | and 3 at end. |
| M1 | Must start at 2 and end at $n$ One M mark for the initial terms and a second for the final. Last lines may be missing $k /(n-1)$ and $c /(n-2)$ These 2 M marks may be implied by a |  |
|  | Last lines may be missing $k /(n-1)$ and $\mathrm{c} /(n-2)$ T correct extraction of terms. If starting from 1, M0M1 | implied by a |
| A1 | Extract the remaining terms. $1-2 / 2$ may be missing and $1 / n-2 / n$ may be combined |  |
| dM1 | Include the $1 / 2$ and attempt a common denominator of the required form. Depends on both previous M marks |  |
|  | $n^{2}+n-2$ |  |
|  | $\overline{4 n(n+1)}$ |  |


| Question <br> Number | Scheme | Marks |
| :---: | :--- | :---: |
|  | Special Case: <br> $\mathbf{1 ( a )}$ | $\frac{2}{r\left(r^{2}-1\right)}=\frac{2 r}{r^{2}-1}-\frac{2}{r}$ seen, award M1A1A0 <br> Award M1A0A0 provided of the form $\frac{2}{r\left(r^{2}-1\right)}=\frac{A r}{r^{2}-1}-\frac{B}{r}$ <br> $\mathbf{1 ( b )}$Terms listed as described above - award M1M1. Further progress unlikely as too many <br> terms needed to establish the cancellation. |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 2 | $\begin{aligned} & w=\frac{z+2}{z-\mathrm{i}} \quad z \neq \mathrm{i} \\ & z=\frac{2+\mathrm{i} w}{w-1} \\ & \|z\|=2 \Rightarrow\left\|\frac{2+\mathrm{i} w}{w-1}\right\|=2 \Rightarrow\|2+\mathrm{i} w\|=2\|w-1\| \\ & \|2+\mathrm{i} u-v\|=2\|u+\mathrm{i} v-1\| \\ & (2-v)^{2}+u^{2}=4\left((u-1)^{2}+v^{2}\right) \\ & 3 u^{2}+3 v^{2}-8 u+4 v=0 \quad \text { oe } \\ & \left(u-\frac{4}{3}\right)^{2}+\left(v+\frac{2}{3}\right)^{2}=\frac{20}{9} \text { or } u^{2}+v^{2}-\frac{8}{3} u+\frac{4}{3} v=0 \end{aligned}$ <br> (i) centre is $\left(\frac{4}{3},-\frac{2}{3}\right)$ <br> (ii) radius is $\frac{2 \sqrt{5}}{3}$ oe | M1 <br> M1 A1 <br> M1 A1 <br> dM1 <br> A1 <br> A1 <br> [8] |
| M1 M1 A1 M1 A1 dM1 (i)A1 (ii)A1 ALT 1 M1 M1 A1 | Rearrange equation to $z=\ldots$ <br> Change $w$ to $u+\mathrm{i} v$ and use $\|z\|=2$ Allow if a different pair of letters used. <br> Correct equation <br> Correct use of Pythagoras on either side. Allow with 2 or 4 (RHS) <br> Correct unsimplified equation <br> Attempt the circle form. Coefficients for $u^{2}$ and $v^{2}$ must be 1 . Depends on all marks <br> Correct centre given (no decimals) (Use of rounded decimals changes the va Correct radius given, any equivalent form (but no decimals) <br> NB: These 2 A marks can only be awarded if the results have been deduced circle equation. <br> Change $w$ to $u+\mathrm{i} v$ Allow a different pair of letters. <br> Rearrange equation to $z=\ldots$ and use $\|z\|=2$ <br> Correct equation <br> Then as above. | 3 previous M <br> lues) <br> from a correct |
| ALT 2 | Very rare but may be seen: i maps to $\infty \Rightarrow \pm 2 \mathrm{i}$ map to a diameter of $C$ So $\frac{2 i+2}{i}$ and $\frac{-2 i+2}{-3 i}$ are ends of a diameter Calculate centre and radius | M1A1 <br> M2A1 <br> M1A1A1 |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 3(a) | $y=r \sin \theta=\sin \theta+\sin \theta \cos \theta \quad \text { OR } \quad r \sin \theta=\sin \theta+\frac{1}{2} \sin 2 \theta$ | B1 |
|  | $\begin{aligned} & \frac{\mathrm{d} y}{\mathrm{~d} \theta}=\cos \theta-\sin ^{2} \theta+\cos ^{2} \theta \quad \text { OR } \quad \frac{\mathrm{d} y}{\mathrm{~d} \theta}=\cos \theta+\cos 2 \theta \\ & 0=\cos \theta+2 \cos ^{2} \theta-1 \quad=(2 \cos \theta-1)(\cos \theta+1) \end{aligned}$ | M1 |
|  | $\cos \theta=\frac{1}{2} \quad(\cos \theta=-1 \quad$ outside range for $\theta) \quad \theta=\frac{\pi}{3}$ $A$ is $\left(1 \frac{1}{2}, \frac{\pi}{3}\right)$ | M1 A1 (4) |
| 3(b) | $\text { Area }=\frac{1}{2} \int_{0}^{\frac{\pi}{3}}(1+\cos \theta)^{2} \mathrm{~d} \theta$ | B1 |
|  | $=\frac{1}{2} \int\left(1+2 \cos \theta+\frac{1}{2}(\cos 2 \theta+1)\right) \mathrm{d} \theta$ | M1A1 |
|  | $=\frac{1}{2}\left[\frac{3}{2} \theta+2 \sin \theta+\frac{1}{4} \sin 2 \theta\right]_{0}^{3}$ | dM1A1 |
|  | $\begin{equation*} =\frac{\pi}{4}+\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{16}=\frac{\pi}{4}+\frac{9 \sqrt{3}}{16} \tag{6} \end{equation*}$ | A1 |
|  |  | [10] |
| (a) |  |  |
| B1 | Use of $r \sin \theta$ Award if not seen explicitly but a correct result following use of double angle formula is seen. |  |
| M1 | Differentiate $r \sin \theta$ or $r \cos \theta$ |  |
| M1 | Set $\frac{\mathrm{d}(r \sin \theta)}{\mathrm{d} \theta}=0$ and solve the resulting equation. Only the solution used need be shown. |  |
| A1 | Correct coordinates of $A$ |  |
| (b)B1 | Use of Area $=\frac{1}{2} \int r^{2} \mathrm{~d} \theta$ with $r=1+\cos \theta$, limits not needed. |  |
| M1 | Attempt $(1+\cos \theta)^{2}$ (minimum accepted is $\left(1+k \cos \theta+\cos ^{2} \theta\right)$ ) and change $\cos ^{2} \theta$ to an expression in $\cos 2 \theta$ using $\cos ^{2} \theta=\frac{1}{2}( \pm \cos 2 \theta \pm 1)$ |  |
| A1 | Correct integrand; limits not needed. $\frac{1}{2}$ may be missing. |  |
| dM1 | Attempt to integrate all terms. $\cos 2 \theta \rightarrow \pm \frac{1}{k} \sin 2 \theta k= \pm 1$ or $\pm 2$ Limits not needed. |  |
|  | Depends on the previous M mark |  |
| A1 | Correct integration and correct limits seen |  |
| A1 | Substitute correct limits and obtain the correct answer in the required form. |  |


| Question Number | Scheme Marks |
| :---: | :---: |
|  | Alternative for (b) using integration by parts (Very rare but may be seen) $\begin{aligned} & \text { Area }=\frac{1}{2} \int_{0}^{\frac{\pi}{3}}(1+\cos \theta)^{2} \mathrm{~d} \theta \\ & =\frac{1}{2}\left[\int(1+2 \cos \theta) \mathrm{d} \theta+\int \cos ^{2} \theta \mathrm{~d} \theta\right] \\ & =\frac{1}{2}\left[\int(1+2 \cos \theta) \mathrm{d} \theta+\cos \theta \sin \theta+\int \sin ^{2} \theta \mathrm{~d} \theta\right] \\ & =\frac{1}{2}\left[\theta+2 \sin \theta+\sin \theta \cos \theta+\int\left(1-\cos ^{2} \theta\right) \mathrm{d} \theta\right]_{0}^{\frac{\pi}{3}} \\ & =\frac{1}{2}\left[\theta+2 \sin \theta+\frac{1}{2}(\sin \theta \cos \theta+\theta)\right]_{0}^{\frac{\pi}{3}} \\ & =\frac{\pi}{4}+\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{16}=\frac{\pi}{4}+\frac{9 \sqrt{3}}{16} \end{aligned}$ |
| B1 M1 A1 dM1 A1A1 | Use of Area $=\frac{1}{2} \int r^{2} \mathrm{~d} \theta$ with $r=1+\cos \theta$, limits not needed. <br> Attempt $(1+\cos \theta)^{2}$ (minimum accepted is $\left(1+k \cos \theta+\cos ^{2} \theta\right)$ ) and attempt first stage of $\int \cos ^{2} \theta \mathrm{~d} \theta$ by parts. Reach $\int \cos ^{2} \theta \mathrm{~d} \theta=\cos \theta \sin \theta \pm \int \sin ^{2} \theta \mathrm{~d} \theta$ Limits not needed Correct so far. Limits not needed. <br> Attempt to integrate all terms. $\int(1+2 \cos \theta) \mathrm{d} \theta$ and attempt to complete $\int \cos ^{2} \theta \mathrm{~d} \theta$ using Pythagoras identity. Limits not needed. Depends on the previous M mark As main scheme |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 4 (a) | $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}=\frac{4}{y}\left(\frac{\mathrm{~d} y}{\mathrm{~d} x}\right)^{2}-3$ | M1 |
|  | $\frac{\mathrm{d}^{3} y}{\mathrm{~d} x^{3}}=-\frac{4}{y^{2}}\left(\frac{\mathrm{~d} y}{\mathrm{~d} x}\right)^{3}+\frac{8}{y} \times \frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}} \times \frac{\mathrm{d} y}{\mathrm{~d} x}$ | M1A1A1 |
|  | $\frac{\mathrm{d}^{3} y}{\mathrm{~d} x^{3}}=-\frac{4}{y^{2}}\left(\frac{\mathrm{~d} y}{\mathrm{~d} x}\right)^{3}+\frac{8}{y}\left(\frac{4}{y}\left(\frac{\mathrm{~d} y}{\mathrm{~d} x}\right)^{2}-3\right)\left(\frac{\mathrm{d} y}{\mathrm{~d} x}\right)$ |  |
|  | $\frac{\mathrm{d}^{3} y}{\mathrm{~d} x^{3}}=\frac{28}{y^{2}}\left(\frac{\mathrm{~d} y}{\mathrm{~d} x}\right)^{3}-\frac{24}{y}\left(\frac{\mathrm{~d} y}{\mathrm{~d} x}\right) *$ | $\mathrm{A1} * *(5)$ |
| ALT | $\frac{\mathrm{d} y}{\mathrm{~d} x} \frac{\mathrm{~d}^{2} y}{\mathrm{~d} x^{2}}+y \frac{\mathrm{~d}^{3} y}{\mathrm{~d} x^{3}}-8 \frac{\mathrm{~d} y}{\mathrm{~d} x} \times \frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}+3 \frac{\mathrm{~d} y}{\mathrm{~d} x}=0$ | M1A1A1 |
|  | $\frac{\mathrm{d}^{3} y}{\mathrm{~d} x^{3}}=\frac{1}{y}\left(7 \frac{\mathrm{~d} y}{\mathrm{~d} x}\right)\left(\frac{4}{y}\left(\frac{\mathrm{~d} y}{\mathrm{~d} x}\right)^{2}-3\right)-\frac{3}{y} \frac{\mathrm{~d} y}{\mathrm{~d} x}$ | M1 |
|  | $\frac{\mathrm{d}^{3} y}{\mathrm{~d} x^{3}}=\frac{28}{y^{2}}\left(\frac{\mathrm{~d} y}{\mathrm{~d} x}\right)^{3}-\frac{24}{y}\left(\frac{\mathrm{~d} y}{\mathrm{~d} x}\right) *$ | A1 * (5) |
| 4(b) | At $x=0 \quad \frac{\mathrm{~d}^{2} y}{\mathrm{dx}^{2}}=\frac{4}{8}(1)^{2}-3=-\frac{5}{2} \quad$ oe | B1 |
|  | $\frac{\mathrm{d}^{3} y}{\mathrm{~d} x^{3}}=\frac{28}{64} \times 1^{3}-\frac{24}{8} \times 1=-\frac{41}{16}$ | M1 |
|  | $y=8+x-\frac{5}{2} \times \frac{x^{2}}{2!}-\frac{41}{16} \times \frac{x^{3}}{3!}+\ldots$ | M1 |
|  | $y=8+x-\frac{5}{4} x^{2}-\frac{41}{96} x^{3}+\ldots$ | $\begin{array}{cc} \mathrm{A} 1 \quad(4) \\ & {[9]} \\ \hline \end{array}$ |


| Question <br> Number | Scheme | Marks |
| :---: | :--- | :---: |
| $\mathbf{5 ( a )}$ | Divide through by $y$ No need to re-arrange the equation until later |  |
| M1 | $\mathrm{d}^{3} y$ |  |
| M1 | Attempt the differentiation using product rule and chain rule and obtain $\frac{1}{\mathrm{~d} x^{3}}=\ldots$ |  |
| A1A1 | A1 Either RHS term correct A1 Second RHS term correct and no extras |  |
| A1* | Eliminate $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}$ and obtain the given result |  |
| ALT | Re-arrange the equation (Will probably be seen later in work) |  |
| M1 | Attempt the differentiation using product rule and chain rule |  |
| M1 | A1 correct and no extras |  |
| A1A1 | A1 Two terms correct A1 All |  |
| A1* | Eliminate $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}$ and obtain the correct result |  |
| 5(b)B1 | Correct value for $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}} \quad$ |  |
| M1 | Use the $g i v e n$ expression from (a) to obtain a value for $\frac{\mathrm{d}^{3} y}{\mathrm{~d} x^{3}} \quad$ Award if correct value seen. |  |
| M1 | Taylor's series formed using their values for the derivatives $(2!$ or $2,3!$ or 6$)$ <br> A1 | Correct series, must start (or end) $y=\ldots$ Correct terms must be seen, order may be different. <br> Can have $\mathrm{f}(x)=\ldots$ provided $\mathrm{f}(x)=y$ is defined somewhere. |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 5 NB | Question states "Use algebra..." so purely graphical solutions (using calculator?) score $0 / 7$. A sketch and some algebra to find intersection points can score. $\begin{aligned} & 2 x^{2}+x-3 \geq 0 \\ & 2 x^{2}+x-3=3(1-x) \Rightarrow 2 x^{2}+4 x-6=0 \\ & 2 x^{2}+4 x-6 \Rightarrow x^{2}+2 x-3=(x+3)(x-1)=0 \\ & x=-3,1 \\ & 2 x^{2}+x-3 \leq 0 \\ & -2 x^{2}-x+3=3(1-x) \Rightarrow 2 x^{2}-2 x=0 \\ & 2 x(x-1)=0, x=0,1 \\ & x<-3 \quad 0<x<1 \quad x>1 \end{aligned}$ | M1 <br> A1 <br> M1 <br> A1 <br> dM1A1A1 |
| M1 <br> A1 <br> M1 <br> A1 <br> dM1 <br> A1 <br> A1 | The first 4 marks can be awarded with any inequality sign or $=$ <br> Assume $2 x^{2}+x-3 \geq 0$ and obtain a 3TQ <br> Correct CVs obtained from a correct equation. <br> Assume $2 x^{2}+x-3 \leq 0$ and obtain a 2 or 3 TQ <br> Correct CVs obtained from a correct equation. <br> Form 3 distinct inequalities with their 3 CVs. Can have $<$ or $\leq,>$ or $\geq$. Must have scored both previous M marks. Accept $x<-3 \quad 0<x \quad x \neq 1$ <br> All 3 correct CVs used correctly <br> Inequalities fully correct. "and" between the inequalities is acceptable. If $\cap$ is used, award A0 here. Fully correct set language accepted. |  |
| ALT | Squaring both sides $\begin{aligned} & \quad\left(2 x^{2}+x-3\right)^{2}>9(1-x)^{2} \\ & 4 x^{4}+4 x^{3}-20 x^{2}+12 x>0 \\ & x(x+3)(x-1)(x-1)>0 \\ & \text { CVs : } x=0,-3,1 \end{aligned}$ <br> Then as main scheme | $\begin{array}{\|l} \text { M1A1 } \\ \text { M1 } \\ \text { A1 } \end{array}$ |
| $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { M1 } \\ & \text { A1 } \\ & \hline \end{aligned}$ | These 4 marks can be awarded with any inequality sign or $=$ Square both sides and collect terms to obtain a quartic with 4 or 5 terms Correct quartic Factorise their quartic 3 correct CVs |  |


| Question Number | Scheme Marks |
| :---: | :---: |
| 6(a) | $\begin{align*} & m^{2}-6 m+8=0 \\ & (m-2)(m-4)=0, m=2,4 \\ & (\mathrm{CF}=) A \mathrm{e}^{2 x}+B \mathrm{e}^{4 x} \\ & \mathrm{PI}: y=\lambda x^{2}+\mu x+v \\ & y^{\prime}=2 \lambda x+\mu \quad y^{\prime \prime}=2 \lambda \\ & 2 \lambda-6(2 \lambda x+\mu)+8\left(\lambda x^{2}+\mu x+v\right)=2 x^{2}+x \\ & \lambda=\frac{1}{4},-12 \lambda+8 \mu=1,2 \lambda-6 \mu+8 v=0 \\ & \lambda=\frac{1}{4}, \mu=\frac{1}{2}, v=\frac{5}{16} \\ & y=A \mathrm{e}^{2 x}+B \mathrm{e}^{4 x}+\frac{1}{4} x^{2}+\frac{1}{2} x+\frac{5}{16} \tag{8} \end{align*}$ |
| $\begin{gathered} \hline \text { (a)M1 } \\ \\ \text { A1 } \\ \text { B1 } \\ \text { M1 } \\ \text { M1 } \\ \text { A1 } \\ \text { A1 } \\ \text { A1ft } \end{gathered}$ | Form aux equation and attempt to solve (any valid method). Equation need not be shown if CF is correct or complete solution $(m=2,4)$ is shown <br> Correct CF $y=$. . not needed. <br> Correct form for PI <br> Their PI (minimum 2 terms) differentiated twice and substituted in the equation <br> Coefficients equated <br> Any 2 values correct <br> All 3 values correct <br> A complete solution, follow through their CF and PI. All 3 M marks must have been earned. <br> Must start $y=\ldots$ |
| 6(b) | $\begin{align*} & y=A \mathrm{e}^{2 x}+B \mathrm{e}^{4 x}+\frac{1}{4} x^{2}+\frac{1}{2} x+\frac{5}{16} \\ & 1=A+B+\frac{5}{16} \\ & \frac{\mathrm{~d} y}{\mathrm{~d} x}=2 A \mathrm{e}^{2 x}+4 B \mathrm{e}^{4 x}+\frac{1}{2} x+\frac{1}{2} \quad 0=2 A+4 B+\frac{1}{2} \\ & A=\frac{13}{8} \quad B=-\frac{15}{16} \quad \text { oe } \\ & y=\frac{13}{8} \mathrm{e}^{2 x}-\frac{15}{16} \mathrm{e}^{4 x}+\frac{1}{4} x^{2}+\frac{1}{2} x+\frac{5}{16} \quad \text { oe } \tag{5} \end{align*}$ |
| (b) <br> M1 <br> M1 <br> dM1 <br> A1 <br> A1ft | Substitute $y=1$ and $x=0$ in their complete solution from (a) <br> Differentiate and substitute $\frac{\mathrm{d} y}{\mathrm{~d} x}=0, x=0$ <br> Solve the 2 equations to $A=\ldots$ or $B=\ldots$. Depends on the two previous M marks <br> Both values correct <br> Particular solution, follow through their general solution and $A$ and $B$. Must start $y=\ldots$ |



| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
|  | Alternative for first 4 marks of 7(a): $\begin{aligned} & \sin 4 \theta=\frac{1}{2 \mathrm{i}}\left(z^{4}-z^{-4}\right)=\frac{1}{2 \mathrm{i}}\left((\cos \theta-\mathrm{i} \sin \theta)^{4}-(\cos \theta+\mathrm{i} \sin \theta)^{-4}\right) \\ & =\frac{1}{2 \mathrm{i}}\left(\cos ^{4} \theta+4 \mathrm{i} \cos ^{3} \theta \sin \theta-6 \cos ^{2} \theta \sin ^{2} \theta-4 \mathrm{i} \cos \theta \sin ^{3} \theta+\sin ^{4} \theta\right) \\ & -\frac{1}{2 \mathrm{i}}\left(-\cos ^{4} \theta+4 \mathrm{i} \cos ^{3} \theta \sin \theta+6 \cos ^{2} \theta \sin ^{2} \theta-4 \mathrm{i} \cos \theta \sin ^{3} \theta-\sin ^{4} \theta\right) \\ & =4 \cos ^{3} \theta \sin \theta-4 \cos \theta \sin ^{3} \theta \end{aligned}$ <br> Similar work leads to $\cos 4 \theta=\cos ^{4} \theta-6 \cos ^{2} \theta \sin ^{2} \theta+\sin ^{4} \theta$ Remaining 2 marks as main scheme | M1 <br> M1 <br> A1 <br> A1 |
| $\begin{gathered} \text { M1 } \\ \text { A1 } \\ \text { M1 } \\ \text { A1 } \end{gathered}$ | For the expression derived from de Moivre for either $\sin 4 \theta$ or $\cos 4 \theta$ Both shown and correct Attempt the binomial expansion for either, reaching a simplified expression Both simplified expressions correct |  |


| Question Number | Scheme Marks |
| :---: | :---: |
| 8(a) | $\begin{aligned} & v=y^{-2} \quad \frac{\mathrm{~d} v}{\mathrm{~d} y}=-2 y^{-3} \\ & \frac{\mathrm{~d} y}{\mathrm{~d} x}=\frac{\mathrm{d} y}{\mathrm{~d} v} \times \frac{\mathrm{d} v}{\mathrm{~d} x}=-\frac{y^{3}}{2} \frac{\mathrm{~d} v}{\mathrm{~d} x} \\ & -\frac{y^{3}}{2} \frac{\mathrm{~d} v}{\mathrm{~d} x}+6 x y=3 x \mathrm{e}^{x^{2}} y^{3} \\ & \frac{1}{2} \frac{\mathrm{~d} v}{\mathrm{~d} x}-\frac{6 x y}{y^{3}}=-3 x \mathrm{e}^{x^{2}} \\ & \frac{\mathrm{~d} v}{\mathrm{~d} x}-12 v x=-6 x \mathrm{e}^{x^{2}} \end{aligned}$ |
| ALT 1 | $\begin{aligned} & y=v^{-\frac{1}{2}} \quad \frac{\mathrm{~d} y}{\mathrm{~d} v}=-\frac{1}{2} v^{-\frac{3}{2}} \\ & \frac{\mathrm{~d} y}{\mathrm{~d} x}=\frac{\mathrm{d} y}{\mathrm{~d} v} \times \frac{\mathrm{d} v}{\mathrm{~d} x}=-\frac{1}{2} v^{-\frac{3}{2}} \frac{\mathrm{~d} v}{\mathrm{~d} x} \\ & -\frac{1}{2} v^{-\frac{3}{2}} \frac{\mathrm{~d} v}{\mathrm{~d} x}+6 x v^{-\frac{1}{2}}=3 x \mathrm{e}^{x^{2}} v^{-\frac{3}{2}} \\ & -\frac{1}{2} \frac{\mathrm{~d} v}{\mathrm{~d} x}+6 x v=3 x \mathrm{e}^{x^{2}} \\ & \frac{\mathrm{~d} v}{\mathrm{~d} x}-12 v x=-6 x \mathrm{e}^{x^{2}} \quad * \end{aligned}$ |
| ALT 2 | $\begin{aligned} & v=y^{-2} \quad \frac{\mathrm{~d} v}{\mathrm{~d} y}=-2 y^{-3} \\ & \frac{\mathrm{~d} v}{\mathrm{~d} x}=\frac{\mathrm{d} v}{\mathrm{~d} y} \times \frac{\mathrm{d} y}{\mathrm{~d} x}=-2 y^{-3} \frac{\mathrm{~d} y}{\mathrm{~d} x} \\ & -2 y^{-3} \frac{\mathrm{~d} y}{\mathrm{~d} x}-12 y^{-2} x=-6 x \mathrm{e}^{x^{2}} \\ & \frac{\mathrm{~d} y}{\mathrm{~d} x}+6 x y=3 x \mathrm{e}^{x^{2}} y^{3} \quad x>0 \end{aligned}$ |
| 8(a) <br> B1 <br> M1 <br> A1 <br> dM1 <br> A1* | All Methods: <br> Correct derivative <br> Attempt $\frac{\mathrm{d} y}{\mathrm{~d} x}$ or $\frac{\mathrm{d} v}{\mathrm{~d} x}$ using the chain rule <br> Correct derivative <br> Substitute in equation (I) to obtain an equation in $v$ and $x$ only OR in equation (II) to obtain an equation in $x$ and $y$ only (ALT 2) <br> Correct completion with no errors seen |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 8(b) | IF: $\mathrm{e}^{\int-12 x d x}=\mathrm{e}^{-6 x^{2}}$ | M1A1 |
|  | $v \mathrm{e}^{-6 x^{2}}=\int-6 x \mathrm{e}^{x^{2}} \times\left(\mathrm{e}^{-6 x^{2}}\right) \mathrm{d} x=\int-6 x \mathrm{e}^{-5 x^{2}} \mathrm{~d} x$ | dM1 |
|  | $v \mathrm{e}^{-6 x^{2}}=\frac{6}{10} \mathrm{e}^{-5 x^{2}}(+c)$ | A1 |
|  | $v\left(=y^{-2}\right)=\frac{6}{10} \mathrm{e}^{x^{2}}+c \mathrm{e}^{6 x^{2}}$ | ddM1 |
|  | $y^{2}=\frac{1}{\frac{6}{10} \mathrm{e}^{x^{2}}+c \mathrm{e}^{6 x^{2}}} \text { oe eg } y^{2}=\frac{10}{6 \mathrm{e}^{x^{2}}+k \mathrm{e}^{6 x^{2}}}$ | A1 (6) |
|  |  | [11] |
| (b) |  |  |
| M1 | IF of form $\mathrm{e}^{\int \pm 12 \mathrm{ddx}}$ and attempt the integration. |  |
| A1 | Correct IF |  |
| dM1 | Multiply through by their IF and integrate the LHS. Depends on first M mark of (b) |  |
| A1 | Correct integration of the complete equation with or without constant |  |
| ddM1 |  |  |
| A1 | Any equivalent to that shown. (No need to change letter used for constant when rearranging) |  |

