Pearson Edexcel IAL Further Mathematics

Further Mathematics 1

Past Paper Collection (from 2020)

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Comments and suggestions to DrYuFromShanghai@QQ.com

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number

Candidate Number

Tuesday 14 January 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
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- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
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Advice

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Leave blank

1.

$$\mathbf{A} = \begin{pmatrix} p & -5 \\ -2 & p+3 \end{pmatrix}$$

(a) Determine the values of the constant p for which $\mathbf{A}$ is singular.

(3)

Given that $p = 3$

(b) determine $\mathbf{A}^{-1}$

(3)

Leave blank

Question 1 continued

Q1

(Total 6 marks)

Leave blank

2. Given that $x = -\frac{1}{3}$ is a root of the equation

$$3x^3 + kx^2 + 33x + 13 = 0 \qquad k \in \mathbb{R}$$

determine

(a) the value of k,

(2)

(b) the other 2 roots of the equation in the form $a + \mathrm{i}b$, where a and b are real numbers.

(4)

Leave blank

Question 2 continued

Q2

(Total 6 marks)

Leave blank

3. (a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that for all positive integers n

$$\sum_{r=1}^{n} r^2(2r+3) = \frac{n}{2}(n+1)(n^2+3n+1)$$

(4)

(b) Hence calculate the value of $\sum_{r=10}^{25} r^2(2r+3)$

(2)

Leave blank

Question 3 continued

Q3

(Total 6 marks)

Leave blank

4. $z_1 = p + 5\text{i}, \quad z_2 = 9 + 8\text{i} \quad \text{and} \quad z_3 = \frac{z_1}{z_2}$

where p is a real constant.

(a) Determine z_3 in the form $x + \text{i}y$, where x and y are in terms of p

(3)

(b) Determine the exact value of the modulus of z_2

(1)

Given that the argument of z_1 is $\frac{\pi}{3}$

(c) (i) determine the exact value of p

(ii) determine the exact value of the modulus of z_3

(3)

Leave blank

Question 4 continued

Q4

(Total 7 marks)

Leave blank

5. $$f(x) = x^4 - 12x^{\frac{3}{2}} + 7 \qquad x \geqslant 0$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval [2, 3].
(2)

(b) Taking 2.5 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 2 decimal places.
(4)

(c) Show that your answer to (b) gives α correct to 2 decimal places.
(2)

Leave blank

Question 5 continued

(Total 8 marks)

Q5

Leave blank

6. $$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$$

The transformation represented by **A** maps the point $R(3p - 13, p - 4)$, where p is a constant, onto the point $R'(7, -2)$

(a) Determine the value of p

(3)

The point S has coordinates $(0, 7)$

Given that O is the origin,

(b) determine the area of triangle ORS

(2)

The transformation represented by **A** maps the triangle ORS onto the triangle $OR'S'$

(c) Hence, using your answer to part (b), determine the area of triangle $OR'S'$

(2)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 7 marks)

Leave blank

7. The equation $3x^2 + px - 5 = 0$, where p is a constant, has roots α and β.

(a) Determine the value of

(i) $\alpha\beta$

(ii) $\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$

(3)

(b) Obtain an expression, in terms of p, for

(i) $\alpha + \beta$

(ii) $\left(\alpha + \frac{1}{\beta}\right) + \left(\beta + \frac{1}{\alpha}\right)$

(3)

Given that

$$\left(\alpha + \frac{1}{\beta}\right) + \left(\beta + \frac{1}{\alpha}\right) = 2\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$$

(c) determine the value of p.

(1)

(d) Using the value of p found in part (c), obtain a quadratic equation, with integer coefficients, that has roots $\left(\alpha + \frac{1}{\beta}\right)$ and $\left(\beta + \frac{1}{\alpha}\right)$

(2)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 9 marks)

Leave blank

8. A rectangular hyperbola, H, has Cartesian equation $xy = 16$

The point $P\left(4t, \dfrac{4}{t}\right)$, $t \neq 0$, lies on H.

(a) Use calculus to show that an equation of the normal to H at P is

$$ty - t^3x = 4 - 4t^4$$

(5)

The point A on H has parameter $t = 2$

The normal to H at A meets H again at the point B.

(b) Determine the exact value of the length of AB.

(6)

The tangent to H at A meets the y-axis at the point C.

(c) Determine the exact area of triangle ABC.

(3)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 14 marks)

Leave blank

9. (i)

$$f(n) = 7^n(3n + 1) - 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$, $f(n)$ is a multiple of 9

(6)

(ii) A sequence of numbers is defined by

$$u_1 = 2 \qquad u_2 = 6$$

$$u_{n+2} = 3u_{n+1} - 2u_n \qquad n \in \mathbb{Z}^+$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 2(2^n - 1)$$

(6)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Wednesday 21 October 2020

Afternoon (Time: 1 hour 30 minutes)	Paper Reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

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Leave blank

1. $$f(x) = x^3 - \frac{10\sqrt{x} - 4x}{x^2} \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [1.4, 1.5]

(2)

(b) Determine $f'(x)$.

(3)

(c) Using $x_0 = 1.4$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to calculate a second approximation to α, giving your answer to 3 decimal places.

(2)

Leave blank

Question 1 continued

Q1

(Total 7 marks)

Leave blank

2. The quadratic equation

$$5x^2 - 2x + 3 = 0$$

has roots α and β.

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(1)

(b) determine, giving each answer as a simplified fraction, the value of

(i) $\alpha^2 + \beta^2$

(ii) $\alpha^3 + \beta^3$

(4)

(c) determine a quadratic equation that has roots

$$(\alpha + \beta^2) \text{ and } (\beta + \alpha^2)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers.

(4)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 9 marks)

Leave blank

3. $$f(z) = z^4 + az^3 + bz^2 + cz + d$$

where a, b, c and d are integers.

The complex numbers $3 + i$ and $-1 - 2i$ are roots of the equation $f(z) = 0$

(a) Write down the other roots of this equation.

(2)

(b) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram.

(2)

(c) Determine the values of a, b, c and d.

(5)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 9 marks)

Leave blank

4. (a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n}(2r-1)^2 = \frac{1}{3}n(4n^2-1)$$

for all positive integers n.

(5)

(b) Hence find the exact value of the sum of the squares of the odd numbers between 200 and 500

(4)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 9 marks)

Leave blank

5. The rectangular hyperbola H has equation $xy = 64$

The point $P\left(8p, \frac{8}{p}\right)$, where $p \neq 0$, lies on H.

(a) Use calculus to show that the normal to H at P has equation

$$p^3x - py = 8(p^4 - 1)$$

(5)

The normal to H at P meets H again at the point Q.

(b) Determine, in terms of p, the coordinates of Q, giving your answers in simplest form.

(4)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 9 marks)

Leave blank

6. (i) $$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

(a) Describe fully the single transformation represented by the matrix **A**.

(2)

The matrix **B** represents a rotation of 45° clockwise about the origin.

(b) Write down the matrix **B**, giving each element of the matrix in exact form.

(1)

The transformation represented by matrix **A** followed by the transformation represented by matrix **B** is represented by the matrix **C**.

(c) Determine **C**.

(2)

(ii) The trapezium T has vertices at the points (–2, 0), (–2, k), (5, 8) and (5, 0), where k is a positive constant. Trapezium T is transformed onto the trapezium T' by the matrix

$$\begin{pmatrix} 5 & 1 \\ -2 & 3 \end{pmatrix}$$

Given that the area of trapezium T' is 510 square units, calculate the exact value of k.

(5)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 10 marks)

Leave blank

7. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The line l with equation $3x - 4y + 48 = 0$ is a tangent to C at the point P.

(a) Show that $a = 9$

(4)

(b) Hence determine the coordinates of P.

(2)

Given that the point S is the focus of C and that the line l crosses the directrix of C at the point A,

(c) determine the exact area of triangle PSA.

(4)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 10 marks)

Leave blank

8. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} \frac{2r^2 - 1}{r^2(r+1)^2} = \frac{n^2}{(n+1)^2}$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 12^n + 2 \times 5^{n-1}$$

is divisible by 7

(6)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Friday 8 January 2021

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

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Leave blank

1. (a) Show that the equation $4x - 2\sin x - 1 = 0$, where x is in radians, has a root α in the interval [0.2, 0.6]

(2)

(b) Starting with the interval [0.2, 0.6], use interval bisection twice to find an interval of width 0.1 in which α lies.

(3)

Leave blank

Question 1 continued

Q1

(Total 5 marks)

Leave blank

2. Given that $x = \frac{3}{8} + \frac{\sqrt{71}}{8}\text{i}$ is a root of the equation

$$4x^3 - 19x^2 + px + q = 0$$

(a) write down the other complex root of the equation.

(1)

Given that $x = 4$ is also a root of the equation,

(b) find the value of p and the value of q.

(4)

Leave blank

Question 2 continued

Q2

(Total 5 marks)

Leave blank

3. The matrix **M** is defined by

$$\mathbf{M} = \begin{pmatrix} k+5 & -2 \\ -3 & k \end{pmatrix}$$

(a) Determine the values of k for which **M** is singular.

(2)

Given that **M** is non-singular,

(b) find $\mathbf{M}^{-1}$ in terms of k.

(2)

Leave blank

Question 3 continued

Q3

(Total 4 marks)

Leave blank

4. The equation $2x^2 + 5x + 7 = 0$ has roots α and β

Without solving the equation

(a) determine the exact value of $\alpha^3 + \beta^3$

(3)

(b) form a quadratic equation, with integer coefficients, which has roots

$$\frac{\alpha^2}{\beta} \text{ and } \frac{\beta^2}{\alpha}$$

(5)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 8 marks)

Leave blank

5. (a) Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that

$$\sum_{r=1}^{n}(r+1)(r+5) = \frac{n}{6}(n+7)(2n+7)$$

for all positive integers n.

(5)

(b) Hence show that

$$\sum_{r=n+1}^{2n}(r+1)(r+5) = \frac{7n}{6}(n+1)(an+b)$$

where a and b are integers to be determined.

(2)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 7 marks)

Leave blank

6. The complex number z is defined by

$$z = -\lambda + 3\text{i} \quad \text{where } \lambda \text{ is a positive real constant}$$

Given that the modulus of z is 5

(a) write down the value of λ

(1)

(b) determine the argument of z, giving your answer in radians to one decimal place.

(2)

In part (c) you must show detailed reasoning.

Solutions relying on calculator technology are not acceptable.

(c) Express in the form $a + \text{i}b$ where a and b are real,

(i) $\dfrac{z + 3\text{i}}{2 - 4\text{i}}$

(ii) z^2

(5)

(d) Show on a single Argand diagram the points A, B, C and D that represent the complex numbers

$$z,\ z^*,\ \frac{z + 3\text{i}}{2 - 4\text{i}} \text{ and } z^2$$

(3)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 11 marks)

Leave blank

7. The matrix **A** is defined by

$$\mathbf{A} = \begin{pmatrix} 4 & -5 \\ -3 & 2 \end{pmatrix}$$

The transformation represented by **A** maps triangle T onto triangle T'

Given that the area of triangle T is 23 cm^2

(a) determine the area of triangle T'

(2)

The point P has coordinates $(3p + 2, 2p - 1)$ where p is a constant. The transformation represented by **A** maps P onto the point P' with coordinates $(17, -18)$

(b) Determine the value of p.

(2)

Given that

$$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(c) describe fully the single geometrical transformation represented by matrix **B**

(2)

The transformation represented by matrix **A** followed by the transformation represented by matrix **C** is equivalent to the transformation represented by matrix **B**

(d) Determine **C**

(3)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 9 marks)

Leave blank

8. The hyperbola H has Cartesian equation $xy = 25$

The parabola P has parametric equations $x = 10t^2$, $y = 20t$

The hyperbola H intersects the parabola P at the point A

(a) Use algebra to determine the coordinates of A

(3)

The point B with coordinates $(10, 20)$ lies on P

(b) Find an equation for the normal to P at B

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers to be determined.

(5)

(c) Use algebra to determine, in simplest form, the exact coordinates of the points where this normal intersects the hyperbola H

(6)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 14 marks)

Leave blank

9. (i) A sequence of numbers $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{3}(2u_n - 1) \qquad u_1 = 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3\left(\frac{2}{3}\right)^n - 1$$

(6)

(ii) $f(n) = 2^{n+2} + 3^{2n+1}$

Prove by induction that, for $n \in \mathbb{Z}^+$, $f(n)$ is a multiple of 7

(6)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

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- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Leave blank

1. (i) $$f(x) = x^3 + 4x - 6$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[1, 1.5]$

(2)

(b) Taking 1.5 as a first approximation, apply the Newton Raphson process twice to $f(x)$ to obtain an approximate value of α. Give your answer to 3 decimal places. Show your working clearly.

(4)

(ii) $$g(x) = 4x^2 + x - \tan x$$

where x is measured in radians.

The equation $g(x) = 0$ has a single root β in the interval $[1.4, 1.5]$

Use linear interpolation on the values at the end points of this interval to obtain an approximation to β. Give your answer to 3 decimal places.

(4)

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Leave blank

Question 1 continued

Q1

(Total 10 marks)

Leave blank

2. The complex numbers z_1, z_2 and z_3 are given by

$$z_1 = 2 - \mathrm{i} \qquad z_2 = p - \mathrm{i} \qquad z_3 = p + \mathrm{i}$$

where p is a real number.

(a) Find $\dfrac{z_2 z_3}{z_1}$ in the form $a + b\mathrm{i}$ where a and b are real. Give your answer in its simplest form in terms of p.

(3)

Given that $\left|\dfrac{z_2 z_3}{z_1}\right| = 2\sqrt{5}$

(b) find the possible values of p.

(4)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

(Total 7 marks)

Q2

Leave blank

3. The triangle T has vertices $A(2, 1)$, $B(2, 3)$ and $C(0, 1)$.

The triangle T' is the image of T under the transformation represented by the matrix

$$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(a) Find the coordinates of the vertices of T'

(2)

(b) Describe fully the transformation represented by **P**

(2)

The 2×2 matrix **Q** represents a reflection in the x-axis and the 2×2 matrix **R** represents a rotation through 90° anticlockwise about the origin.

(c) Write down the matrix **Q** and the matrix **R**

(2)

(d) Find the matrix **RQ**

(2)

(e) Give a full geometrical description of the single transformation represented by the answer to part (d).

(2)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 10 marks)

Leave blank

4. A rectangular hyperbola H has equation $xy = 25$

The point $P\left(5t, \frac{5}{t}\right)$, $t \neq 0$, is a general point on H.

(a) Show that the equation of the tangent to H at P is $t^2y + x = 10t$

(4)

The distinct points Q and R lie on H. The tangent to H at the point Q and the tangent to H at the point R meet at the point $(15, -5)$.

(b) Find the coordinates of the points Q and R.

(4)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 8 marks)

Leave blank

5. $$f(x) = (9x^2 + d)\left(x^2 - 8x + (10d + 1)\right)$$

where d is a positive constant.

(a) Find the four roots of $f(x)$ giving your answers in terms of d.

(3)

Given $d = 4$

(b) Express these four roots in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{R}$.

(2)

(c) Show these four roots on a single Argand diagram.

(2)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 7 marks)

Leave blank

6. The parabola C has Cartesian equation $y^2 = 8x$

The point $P(2p^2, 4p)$ and the point $Q(2q^2, 4q)$, where $p, q \neq 0$, $p \neq q$, are points on C.

(a) Show that an equation of the normal to C at P is

$$y + px = 2p^3 + 4p$$

(5)

(b) Write down an equation of the normal to C at Q

(1)

The normal to C at P and the normal to C at Q meet at the point N

(c) Show that N has coordinates

$$\left(2(p^2 + pq + q^2 + 2), -2pq(p + q)\right)$$

(5)

The line ON, where O is the origin, is perpendicular to the line PQ

(d) Find the value of $(p + q)^2 - 3pq$

(5)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 16 marks)

Leave blank

7. (a) Prove by induction that for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$$

(5)

(b) Hence show that

$$\sum_{r=1}^{n} (r^2 + 2) = \frac{n}{6}(an^2 + bn + c)$$

where a, b and c are integers to be found.

(4)

(c) Using your answers to part (b), find the value of

$$\sum_{r=10}^{25} (r^2 + 2)$$

(2)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 11 marks)

Leave blank

8. Prove by induction that $4^{n+2} + 5^{2n+1}$ is divisible by 21 for all positive integers n.

(6)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 6 marks)

TOTAL FOR PAPER: 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- Inexact answers should be given to three significant figures unless otherwise stated.

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- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Leave blank

1.

$$\mathbf{A} = \begin{pmatrix} 3 & a \\ -2 & -2 \end{pmatrix}$$

where a is a non-zero constant and $a \neq 3$

(a) Determine $\mathbf{A}^{-1}$ giving your answer in terms of a.

(2)

Given that $\mathbf{A} + \mathbf{A}^{-1} = \mathbf{I}$ where $\mathbf{I}$ is the 2×2 identity matrix,

(b) determine the value of a.

(3)

Leave blank

Question 1 continued

Q1

(Total 5 marks)

Leave blank

2. $$f(x) = 7\sqrt{x} - \frac{1}{2}x^3 - \frac{5}{3x} \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval [2.8, 2.9]

(2)

(b) (i) Find $f'(x)$.

(ii) Hence, using $x_0 = 2.8$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to calculate a second approximation to α, giving your answer to 3 decimal places.

(4)

(c) Use linear interpolation once on the interval [2.8, 2.9] to find another approximation to α. Give your answer to 3 decimal places.

(3)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 9 marks)

Leave blank

3. The quadratic equation

$$2x^2 - 5x + 7 = 0$$

has roots α and β

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(1)

(b) determine, giving each answer as a simplified fraction, the value of

(i) $\alpha^2 + \beta^2$

(ii) $\alpha^3 + \beta^3$

(4)

(c) find a quadratic equation that has roots

$$\frac{1}{\alpha^2 + \beta} \text{ and } \frac{1}{\beta^2 + \alpha}$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers to be determined.

(4)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 9 marks)

Leave blank

4. $$f(z) = 2z^3 - z^2 + az + b$$

where a and b are integers.

The complex number $-1 - 3i$ is a root of the equation $f(z) = 0$

(a) Write down another complex root of this equation.

(1)

(b) Determine the value of a and the value of b.

(4)

(c) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram.

(2)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 7 marks)

Leave blank

5. (a) Use the standard results for $\sum_{r=1}^{n} r^3$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that for all positive integers n,

$$\sum_{r=1}^{n} r(r-1)(r-3) = \frac{1}{12}n(n+1)(n-1)(3n-10)$$

(5)

(b) Hence show that

$$\sum_{r=n+1}^{2n+1} r(r-1)(r-3) = \frac{1}{12}n(n+1)(an^2+bn+c)$$

where a, b and c are integers to be determined.

(3)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 8 marks)

Leave blank

6. The curve H has equation

$$xy = a^2 \qquad x > 0$$

where a is a positive constant.

The line with equation $y = kx$, where k is a positive constant, intersects H at the point P

(a) Use calculus to determine, in terms of a and k, an equation for the tangent to H at P
(4)

The tangent to H at P meets the x-axis at the point A and meets the y-axis at the point B

(b) Determine the coordinates of A and the coordinates of B, giving your answers in terms of a and k
(2)

(c) Hence show that the area of triangle AOB, where O is the origin, is independent of k
(2)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

7. In part (i), the elements of each matrix should be expressed in exact numerical form.

(i) (a) Write down the 2×2 matrix that represents a rotation of 210° anticlockwise about the origin.

(1)

(b) Write down the 2×2 matrix that represents a stretch parallel to the y-axis with scale factor 5

(1)

The transformation T is a rotation of 210° anticlockwise about the origin followed by a stretch parallel to the y-axis with scale factor 5

(c) Determine the 2×2 matrix that represents T

(2)

(ii)

$$\mathbf{M} = \begin{pmatrix} k & k+3 \\ -5 & 1-k \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

(a) Find det $\mathbf{M}$, giving your answer in simplest form in terms of k.

(2)

A closed shape R is transformed to a closed shape R' by the transformation represented by the matrix $\mathbf{M}$.

Given that the area of R is 2 square units and that the area of R' is $16k$ square units,

(b) determine the possible values of k.

(3)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 9 marks)

Leave blank

8. The parabola C has equation $y^2 = 20x$

The point P on C has coordinates $(5p^2, 10p)$ where p is a non-zero constant.

(a) Use calculus to show that the tangent to C at P has equation

$$py - x = 5p^2$$

(3)

The tangent to C at P meets the y-axis at the point A.

(b) Write down the coordinates of A.

(1)

The point S is the focus of C.

(c) Write down the coordinates of S.

(1)

The straight line l_1 passes through A and S.

The straight line l_2 passes through O and P, where O is the origin.

Given that l_1 and l_2 intersect at the point B,

(d) show that the coordinates of B satisfy the equation

$$2x^2 + y^2 = 10x$$

(5)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 10 marks)

Leave blank

9. (i) A sequence of numbers is defined by

$$u_1 = 0 \qquad u_2 = -6$$

$$u_{n+2} = 5u_{n+1} - 6u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3 \times 2^n - 2 \times 3^n$$

(5)

(ii) Prove by induction that, for all positive integers n,

$$f(n) = 3^{3n-2} + 2^{4n-1}$$

is divisible by 11

(5)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

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- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Leave blank

1.

$$\mathbf{M} = \begin{pmatrix} 3x & 7 \\ 4x + 1 & 2 - x \end{pmatrix}$$

Find the range of values of x for which the determinant of the matrix **M** is positive.

(5)

Leave blank

Question 1 continued

Q1

(Total 5 marks)

Leave blank

2. The complex numbers z_1 and z_2 are given by

$$z_1 = 3 + 5\text{i} \quad \text{and} \quad z_2 = -2 + 6\text{i}$$

(a) Show z_1 and z_2 on a single Argand diagram.

(2)

(b) **Without using your calculator and showing all stages of your working,**

(i) determine the value of $|z_1|$

(1)

(ii) express $\frac{z_1}{z_2}$ in the form $a + b\text{i}$, where a and b are fully simplified fractions.

(3)

(c) Hence determine the value of $\arg\frac{z_1}{z_2}$

Give your answer in radians to 2 decimal places.

(2)

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Leave blank

Question 2 continued

Q2

(Total 8 marks)

Leave blank

3. The parabola C has equation $y^2 = 18x$

The point S is the focus of C

(a) Write down the coordinates of S

(1)

The point P, with $y > 0$, lies on C

The shortest distance from P to the directrix of C is 9 units.

(b) Determine the exact perimeter of the triangle OPS, where O is the origin.

Give your answer in simplest form.

(4)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

(Total 5 marks)

Q3

Leave blank

4. The equation

$$x^4 + Ax^3 + Bx^2 + Cx + 225 = 0$$

where A, B and C are real constants, has

- a complex root $4 + 3i$
- a repeated positive real root

(a) Write down the other complex root of this equation.

(1)

(b) Hence determine a quadratic factor of $x^4 + Ax^3 + Bx^2 + Cx + 225$

(2)

(c) Deduce the real root of the equation.

(2)

(d) Hence determine the value of each of the constants A, B and C

(3)

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Leave blank

Question 4 continued

(Total 8 marks)

Q4

Leave blank

5. $$\mathbf{P} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

The matrix **P** represents the transformation U

(a) Give a full description of U as a single geometrical transformation.

(2)

The transformation V, represented by the 2×2 matrix **Q**, is a reflection in the line $y = -x$

(b) Write down the matrix **Q**

(1)

The transformation U followed by the transformation V is represented by the matrix **R**

(c) Determine the matrix **R**

(2)

The transformation W is represented by the matrix 3**R**

The transformation W maps a triangle T to a triangle T'

The transformation W' maps the triangle T' back to the original triangle T

(d) Determine the matrix that represents W'

(3)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 8 marks)

Leave blank

6. The quadratic equation

$$Ax^2 + 5x - 12 = 0$$

where A is a constant, has roots α and β

(a) Write down an expression in terms of A for

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(2)

The equation

$$4x^2 - 5x + B = 0$$

where B is a constant, has roots $\alpha - \dfrac{3}{\beta}$ and $\beta - \dfrac{3}{\alpha}$

(b) Determine the value of A

(3)

(c) Determine the value of B

(3)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 8 marks)

Leave blank

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The rectangular hyperbola H has equation $xy = 36$

The point $P(4, 9)$ lies on H

(a) Show, using calculus, that the normal to H at P has equation

$$4x - 9y + 65 = 0$$

(4)

The normal to H at P crosses H again at the point Q

(b) Determine an equation for the tangent to H at Q, giving your answer in the form $y = mx + c$ where m and c are rational constants.

(5)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 9 marks)

Leave blank

8. $$f(x) = 2x^{-\frac{2}{3}} + \frac{1}{2}x - \frac{1}{3x-5} - \frac{5}{2} \qquad x \neq \frac{5}{3}$$

The table below shows values of f(x) for some values of x, with values of f(x) given to 4 decimal places where appropriate.

x	1	2	3	4	5
f(x)	0.5		–0.2885		0.5834

(a) Complete the table giving the values to 4 decimal places.

(2)

The equation f(x) = 0 has exactly one positive root, α.

Using the values in the completed table and explaining your reasoning,

(b) determine an interval of width one that contains α.

(2)

(c) Hence use interval bisection twice to obtain an interval of width 0.25 that contains α.

(3)

Given also that the equation f(x) = 0 has a negative root, β, in the interval [–1, –0.5]

(d) use linear interpolation once on this interval to find an approximation for β.

Give your answer to 3 significant figures.

(3)

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 10 marks)

Leave blank

9. (a) Prove by induction that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

(5)

(b) Using the standard summation formulae, show that

$$\sum_{r=1}^{n} r(r+1)(r-1) = \frac{1}{4}n(n+A)(n+B)(n+C)$$

where A, B and C are constants to be determined.

(4)

(c) Determine the value of n for which

$$3\sum_{r=1}^{n} r(r+1)(r-1) = 17\sum_{r=n}^{2n} r^2$$

(5)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

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- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. $z_1 = 3 + 3\text{i} \qquad z_2 = p + q\text{i} \qquad p, q \in \mathbb{R}$

Given that $|z_1 z_2| = 15\sqrt{2}$

(a) determine $|z_2|$

(2)

Given also that $p = -4$

(b) determine the possible values of q

(2)

(c) Show z_1 and the possible positions for z_2 on the same Argand diagram.

(2)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 6 marks)

2. $$f(x) = 10 - 2x - \frac{1}{2\sqrt{x}} - \frac{1}{x^3} \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [0.4, 0.5]

(2)

(b) Determine $f'(x)$.

(3)

(c) Using $x_0 = 0.5$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places.

(2)

The equation $f(x) = 0$ has another root β in the interval [4.8, 4.9]

(d) Use linear interpolation once on the interval [4.8, 4.9] to find an approximation to β, giving your answer to 3 decimal places.

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3. $$\mathbf{M} = \begin{pmatrix} k & k \\ 3 & 5 \end{pmatrix} \quad \text{where } k \text{ is a non-zero constant}$$

(a) Determine $\mathbf{M}^{-1}$, giving your answer in simplest form in terms of k.

(2)

Hence, given that $\mathbf{N}^{-1} = \begin{pmatrix} k & k \\ 4 & -1 \end{pmatrix}$

(b) determine $(\mathbf{MN})^{-1}$, giving your answer in simplest form in terms of k.

(2)

Question 3 continued

(Total for Question 3 is 4 marks)

4. $$f(z) = 2z^4 - 19z^3 + Az^2 + Bz - 156$$

where A and B are constants.

The complex number $5 - i$ is a root of the equation $f(z) = 0$

(a) Write down another complex root of this equation.

(1)

(b) Solve the equation $f(z) = 0$ completely.

(5)

(c) Determine the value of A and the value of B.

(2)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

5. The quadratic equation

$$2x^2 - 3x + 5 = 0$$

has roots α and β

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(1)

(b) determine the value of

(i) $\alpha^2 + \beta^2$

(ii) $\alpha^3 + \beta^3$

(4)

(c) find a quadratic equation which has roots

$$(\alpha^3 - \beta) \text{ and } (\beta^3 - \alpha)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers to be determined.

(5)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6. The parabola C has equation $y^2 = 36x$

The point $P(9t^2, 18t)$, where $t \neq 0$, lies on C

(a) Use calculus to show that the normal to C at P has equation

$$y + tx = 9t^3 + 18t$$

(4)

(b) Hence find the equations of the two normals to C which pass through the point (54, 0), giving your answers in the form $y = px + q$ where p and q are constants to be determined.

(4)

Given that

- the normals found in part (b) intersect the directrix of C at the points A and B
- the point F is the focus of C

(c) determine the area of triangle AFB

(3)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 11 marks)

7.

$$\mathbf{A} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$

(a) Determine the matrix $\mathbf{A}^2$

(1)

(b) Describe fully the single geometrical transformation represented by the matrix $\mathbf{A}^2$

(2)

(c) Hence determine the smallest positive integer value of n for which $\mathbf{A}^n = \mathbf{I}$

(1)

The matrix **B** represents a stretch scale factor 4 parallel to the x-axis.

(d) Write down the matrix **B**

(1)

The transformation represented by matrix **A** followed by the transformation represented by matrix **B** is represented by the matrix **C**

(e) Determine the matrix **C**

(2)

The parallelogram P is transformed onto the parallelogram P' by the matrix **C**

(f) Given that the area of parallelogram P' is 20 square units, determine the area of parallelogram P

(2)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)

8. (a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that for all positive integers n

$$\sum_{r=0}^{n} (r+1)(r+2) = \frac{1}{3}(n+1)(n+2)(n+3)$$

(5)

(b) Hence determine the value of

$$10 \times 11 + 11 \times 12 + 12 \times 13 + \dots + 100 \times 101$$

(3)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 8 marks)

9. (i) A sequence of numbers is defined by

$$u_1 = 3$$

$$u_{n+1} = 2u_n - 2^{n+1} \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{N}$

$$u_n = 5 \times 2^{n-1} - n \times 2^n$$

(5)

(ii) Prove by induction that, for $n \in \mathbb{N}$

$$\mathrm{f}(n) = 5^{n+2} - 4n - 9$$

is divisible by 16

(5)

Question 9 continued

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 10 marks)

TOTAL FOR PAPER: 75 MARKS

END

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Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistics Tables (Yellow), calculator

Total Marks

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- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Given that

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 3 \\ -2 & 3 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 1 & k \\ 0 & -3 \\ 2k & 2 \end{pmatrix}$$

where k is a non-zero constant,

(a) determine the matrix **AB**

(2)

(b) determine the value of k for which $\det(\mathbf{AB}) = 0$

(3)

Question 1 continued

(Total for Question 1 is 5 marks)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that for all positive integers n

$$\sum_{r=1}^{n}(7r-5)^2 = \frac{n}{6}(7n+1)(An+B)$$

where A and B are integers to be determined.

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(z) = 4z^3 + pz^2 - 24z + 108$$

where p is a constant.

Given that -3 is a root of the equation $f(z) = 0$

(a) determine the value of p

(2)

(b) using algebra, solve $f(z) = 0$ completely, giving the roots in simplest form,

(4)

(c) determine the modulus of the complex roots of $f(z) = 0$

(2)

(d) show the roots of $f(z) = 0$ on a single Argand diagram.

(2)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4.

$$f(x) = 1 - \frac{1}{8x^4} + \frac{2}{7\sqrt{x^7}} \qquad x > 0$$

The equation f(x) = 0 has a single root, α, that lies in the interval $[0.15, 0.25]$

(a) (i) Determine $f'(x)$

(ii) Explain why 0.25 cannot be used as an initial approximation for α in the Newton-Raphson process.

(iii) Taking 0.15 as a first approximation to α apply the Newton-Raphson process once to f(x) to obtain a second approximation to α
Give your answer to 3 decimal places.

(5)

(b) Use linear interpolation once on the interval $[0.15, 0.25]$ to find another approximation to α
Give your answer to 3 decimal places.

(3)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

5. The quadratic equation

$$4x^2 + 3x + k = 0$$

where k is an integer, has roots α and β

(a) Write down, in terms of k where appropriate, the value of $\alpha + \beta$ and the value of $\alpha\beta$

(2)

(b) Determine, in simplest form in terms of k, the value of $\dfrac{\alpha}{\beta^2} + \dfrac{\beta}{\alpha^2}$

(4)

(c) Determine a quadratic equation which has roots

$$\frac{\alpha}{\beta^2} \text{ and } \frac{\beta}{\alpha^2}$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integer values in terms of k

(3)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The rectangular hyperbola H has equation $xy = 20$

The point $P\left(2t\sqrt{a}, \dfrac{2\sqrt{a}}{t}\right)$, $t \neq 0$, where a is a constant, is a general point on H

(a) State the value of a

(1)

(b) Show that the normal to H at the point P has equation

$$ty - t^3x - 2\sqrt{5}\left(1 - t^4\right) = 0$$

(4)

The points A and B lie on H

The point A has parameter $t = c$ and the point B has parameter $t = -\dfrac{1}{2c}$, where c is a constant.

The normal to H at A meets H again at B

(c) Determine the possible values of c

(4)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. (i)

$$\mathbf{P} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

The matrix **P** represents a geometrical transformation U

(a) Describe U fully as a single geometrical transformation.

(2)

The transformation V, represented by the 2×2 matrix **Q**, is a rotation through 240° anticlockwise about the origin followed by an enlargement about (0, 0) with scale factor 6

(b) Determine the matrix **Q**, giving each entry in exact numerical form.

(2)

Given that U followed by V is the transformation T, which is represented by the matrix **R**

(c) determine the matrix **R**

(2)

(ii) The transformation W is represented by the matrix

$$\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix}$$

Show that there is a real number λ for which W maps the point $(\lambda, 1)$ onto the point $(4\lambda, 4)$, giving the exact value of λ

(5)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8. A parabola C has equation $y^2 = 4ax$ where a is a positive constant.

The point S is the focus of C

The line l_1 with equation $y = k$ where k is a positive constant, intersects C at the point P

(a) Show that

$$PS = \frac{k^2 + 4a^2}{4a}$$

(3)

The line l_2 passes through P and intersects the directrix of C on the x-axis.

The line l_2 intersects the y-axis at the point A

(b) Show that the y coordinate of A is $\dfrac{4a^2k}{k^2 + 4a^2}$

(3)

The line l_1 intersects the directrix of C at the point B

Given that the areas of triangles BPA and OSP, where O is the origin, satisfy the ratio

$$\text{area } BPA : \text{area } OSP = 4k^2 : 1$$

(c) determine the exact value of a

(5)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 11 marks)

9. Prove by induction that for all positive integers n

$$\sum_{r=1}^{n} \log(2r-1) = \log\left(\frac{(2n)!}{2^n n!}\right)$$

(6)

Question 9 continued

Question 9 continued

(Total for Question 9 is 6 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 30 May 2023

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n

$$\sum_{r=1}^{n} r^2 (r+2) = \frac{1}{12} n(n+1)(an^2 + bn + c)$$

where a, b and c are integers to be determined.

(4)

Question 1 continued

(Total for Question 1 is 4 marks)

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given that $x = 2 + 3i$ is a root of the equation

$$2x^4 - 8x^3 + 29x^2 - 12x + 39 = 0$$

(a) write down another complex root of this equation.

(1)

(b) Use algebra to determine the other 2 roots of the equation.

(4)

(c) Show all 4 roots on a single Argand diagram.

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 7 marks)

3. The rectangular hyperbola H has Cartesian equation $xy = 9$

The point P with coordinates $\left(3t, \frac{3}{t}\right)$, where $t \neq 0$, lies on H

(a) Use calculus to determine an equation for the normal to H at the point P

Give your answer in the form $ty - t^3x = \mathrm{f}(t)$

(4)

Given that $t = 2$

(b) determine the coordinates of the point where the normal meets H again.

Give your answer in simplest form.

(3)

Question 3 continued

(Total for Question 3 is 7 marks)

4. (i) $$\mathbf{A} = \begin{pmatrix} -3 & 8 \\ -3 & k \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

The transformation represented by **A** transforms triangle T to triangle T'

The area of triangle T' is three times the area of triangle T

Determine the possible values of k

(4)

(ii) $$\mathbf{B} = \begin{pmatrix} a & -4 \\ 2 & 3 \end{pmatrix} \text{ and } \mathbf{BC} = \begin{pmatrix} 2 & 5 & 1 \\ 1 & 4 & 2 \end{pmatrix} \text{ where } a \text{ is a constant}$$

Determine, in terms of a, the matrix **C**

(4)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

5.
$$f(x) = x^2 - 6x + 3$$

The equation $f(x) = 0$ has roots α and β

Without solving the equation,

(a) determine the value of

$$(\alpha^2 + 1)(\beta^2 + 1)$$

(4)

(b) find a quadratic equation which has roots

$$\frac{\alpha}{(\alpha^2 + 1)} \text{ and } \frac{\beta}{(\beta^2 + 1)}$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers to be determined.

(6)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$z_1 = 3 + 2\text{i} \qquad z_2 = 2 + 3\text{i} \qquad z_3 = a + b\text{i} \qquad a, b \in \mathbb{R}$$

(a) Determine the exact value of $|z_1 + z_2|$

(2)

Given that $w = \frac{z_2 z_3}{z_1}$

(b) determine w in terms of a and b, giving your answer in the form $x + \text{i}y$, where $x, y \in \mathbb{R}$

(4)

Given also that $w = \frac{4}{13} + \frac{58}{13}\text{i}$

(c) determine the value of a and the value of b

(2)

(d) determine arg w, giving your answer in radians to 4 significant figures.

(2)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 10 marks)

7.

$$f(x) = x^{\frac{3}{2}} + x - 3$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval [1, 2]

(2)

(b) Starting with the interval [1, 2], use interval bisection twice to show that α lies in the interval [1.25, 1.5]

(3)

(c) (i) Determine $f'(x)$

(ii) Using 1.375 as a first approximation for α, apply the Newton-Raphson process once to $f(x)$ to determine a second approximation for α, giving your answer to 3 decimal places.

(3)

(d) Use linear interpolation once on the interval [1.25, 1.5] to obtain a different approximation for α, giving your answer to 3 decimal places.

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8. The point $P(2p^2, 4p)$ lies on the parabola with equation $y^2 = 8x$

(a) Show that the point $Q\left(\dfrac{2}{p^2}, \dfrac{-4}{p}\right)$, where $p \neq 0$, lies on the parabola.

(1)

(b) Show that the chord PQ passes through the focus of the parabola.

(4)

The tangent to the parabola at P and the tangent to the parabola at Q meet at the point R

(c) Determine, in simplest form, the coordinates of R

(8)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 13 marks)

9. Prove, by induction, that for $n \in \mathbb{Z}, n \geqslant 2$

$$4^n + 6n - 10$$

is divisible by 18

(5)

Question 9 continued

Question 9 continued

(Total for Question 9 is 5 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Friday 12 January 2024

Morning (Time: 1 hour 30 minutes) | Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1.

$$\mathbf{M} = \begin{pmatrix} 2k+1 & k \\ k+7 & k+4 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

(a) Show that **M** is non-singular for all real values of k.

(3)

(b) Determine $\mathbf{M}^{-1}$ in terms of k.

(2)

Question 1 continued

(Total for Question 1 is 5 marks)

2.

$$f(z) = 2z^3 + pz^2 + qz - 41$$

where p and q are integers.

The complex number $5 - 4i$ is a root of the equation $f(z) = 0$

(a) Write down another complex root of this equation.

(1)

(b) Solve the equation $f(z) = 0$ completely.

(4)

(c) Determine the value of p and the value of q.

(2)

When plotted on an Argand diagram, the points representing the roots of the equation $f(z) = 0$ form the vertices of a triangle.

(d) Determine the area of this triangle.

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3. The hyperbola H has equation $xy = c^2$ where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, where $t > 0$, lies on H.

The tangent to H at P meets the x-axis at the point A and meets the y-axis at the point B.

(a) Determine, in terms of c and t,

(i) the coordinates of A,

(ii) the coordinates of B.

(4)

Given that the area of triangle AOB, where O is the origin, is 90 square units,

(b) determine the value of c, giving your answer as a simplified surd.

(2)

Question 3 continued

(Total for Question 3 is 6 marks)

4.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

(a) Describe the single geometrical transformation represented by the matrix **A**.

(2)

The matrix **B** represents a rotation of 210° anticlockwise about centre (0, 0).

(b) Write down the matrix **B**, giving each element in exact form.

(1)

The transformation represented by matrix **A** followed by the transformation represented by matrix **B** is represented by the matrix **C**.

(c) Find **C**.

(2)

The hexagon H is transformed onto the hexagon H' by the matrix **C**.

(d) Given that the area of hexagon H is 5 square units, determine the area of hexagon H'

(2)

Question 4 continued

(Total for Question 4 is 7 marks)

5. The quadratic equation

$$2x^2 - 3x + 7 = 0$$

has roots α and β

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(1)

(b) determine the value of $\alpha^2 + \beta^2$

(2)

(c) find a quadratic equation which has roots

$$\left(\alpha - \frac{1}{\beta^2}\right) \text{ and } \left(\beta - \frac{1}{\alpha^2}\right)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers to be determined.

(6)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6. (i)

$$f(x) = x - 4 - \cos\left(5\sqrt{x}\right) \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [2.5, 3.5]

(2)

(b) Use linear interpolation once on the interval [2.5, 3.5] to find an approximation to α, giving your answer to 2 decimal places.

(2)

(ii)

$$g(x) = \frac{1}{10}x^2 - \frac{1}{2x^2} + x - 11 \qquad x > 0$$

(a) Determine $g'(x)$.

(2)

The equation $g(x) = 0$ has a root β in the interval [6, 7]

(b) Using $x_0 = 6$ as a first approximation to β, apply the Newton–Raphson procedure once to $g(x)$ to find a second approximation to β, giving your answer to 3 decimal places.

(2)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 8 marks)

7. The parabola C has equation $y^2 = \frac{4}{3}x$

The point $P\left(\frac{1}{3}t^2, \frac{2}{3}t\right)$, where $t \neq 0$, lies on C.

(a) Use calculus to show that the normal to C at P has equation

$$3tx + 3y = t^3 + 2t$$

(3)

The normal to C at the point where $t = 9$ meets C again at the point Q.

(b) Determine the exact coordinates of Q.

(4)

Question 7 continued

(Total for Question 7 is 7 marks)

8. (a) Use the standard results for summations to show that, for all positive integers n,

$$\sum_{r=1}^{n} r\left(2r^2 - 3r - 1\right) = \frac{1}{2}n(n+1)^2(n-2)$$

(4)

(b) Hence show that, for all positive integers n,

$$\sum_{r=n}^{2n} r\left(2r^2 - 3r - 1\right) = \frac{1}{2}n(n-1)(an+b)(cn+d)$$

where a, b, c and d are integers to be determined.

(4)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 8 marks)

9. Given that

$$\frac{3z - 1}{2} = \frac{\lambda + 5\text{i}}{\lambda - 4\text{i}}$$

where λ is a real constant,

(a) determine z, giving your answer in the form $x + y\text{i}$, where x and y are real and in terms of λ.

(4)

Given also that $\arg z = \frac{\pi}{4}$

(b) find the possible values of λ.

(2)

Question 9 continued

(Total for Question 9 is 6 marks)

10. (i) Prove by induction that for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}^n = 3^{n-1}\begin{pmatrix} 2n+3 & -n \\ 4n & 3-2n \end{pmatrix}$$

(5)

(ii) Prove by induction that for $n \in \mathbb{Z}^+$

$$\mathrm{f}(n) = 8^{2n+1} + 6^{2n-1}$$

is divisible by 7

(5)

Question 10 continued

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Thursday 23 May 2024

Morning (Time: 1 hour 30 minutes) | **Paper reference** **WFM01/01**

Mathematics

International Advanced Subsidiary/ Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

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- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
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- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. (i) The matrix **A** is defined by

$$\mathbf{A} = \begin{pmatrix} 3k & 4k-1 \\ 2 & 6 \end{pmatrix}$$

where k is a constant.

(a) Determine the value of k for which **A** is singular.

(2)

Given that **A** is non-singular,

(b) determine $\mathbf{A}^{-1}$ in terms of k, giving your answer in simplest form.

(2)

(ii) The matrix **B** is defined by

$$\mathbf{B} = \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}$$

where p and q are integers.

State the value of p and the value of q when **B** represents

(a) an enlargement about the origin with scale factor –2

(b) a reflection in the y–axis.

(2)

Question 1 continued

(Total for Question 1 is 6 marks)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(z) = z^3 - 13z^2 + 59z + p \qquad p \in \mathbb{Z}$$

Given that $z = 3$ is a root of the equation $f(z) = 0$

(a) show that $p = -87$

(2)

(b) Use algebra to determine the other roots of $f(z) = 0$, giving your answers in simplest form.

(4)

On an Argand diagram

- the root $z = 3$ is represented by the point P
- the other roots of $f(z) = 0$ are represented by the points Q and R
- the number $z = -9$ is represented by the point S

(c) Show on a single Argand diagram the positions of P, Q, R and S

(1)

(d) Determine the perimeter of the quadrilateral $PQSR$, giving your answer as a simplified surd.

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3. $$f(x) = x^3 - 5\sqrt{x} - 4x + 7 \qquad x \geqslant 0$$

The equation $f(x) = 0$ has a root α in the interval [0.25, 1]

(a) Use linear interpolation once on the interval [0.25, 1] to determine an approximation to α, giving your answer to 3 decimal places.

(3)

The equation $f(x) = 0$ has another root β in the interval [1.5, 2.5]

(b) Determine $f'(x)$

(2)

(c) Hence, using $x_0 = 1.75$ as a first approximation to β, apply the Newton–Raphson process once to $f(x)$ to determine a second approximation to β, giving your answer to 3 decimal places.

(2)

Question 3 continued

(Total for Question 3 is 7 marks)

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The complex number z is defined by

$$z = -3 + 4\text{i}$$

(a) Determine $\left|z^2 - 3\right|$

(3)

(b) Express $\frac{50}{z^*}$ in the form kz, where k is a positive integer.

(3)

(c) Hence find the value of $\arg\left(\frac{50}{z^*}\right)$

Give your answer in radians to 3 significant figures.

(2)

Question 4 continued

(Total for Question 4 is 8 marks)

5. The equation $5x^2 - 4x + 2 = 0$ has roots $\frac{1}{p}$ and $\frac{1}{q}$

(a) Without solving the equation,

(i) show that $pq = \frac{5}{2}$

(ii) determine the value of $p + q$

(4)

(b) Hence, without finding the values of p and q, determine a quadratic equation with roots

$$\frac{p}{p^2 + 1} \text{ and } \frac{q}{q^2 + 1}$$

giving your answer in the form $ax^2 + bx + c = 0$ where a, b and c are integers.

(5)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6. (a) Prove by induction that for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} 1 & r \\ 0 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & (2^n - 1)r \\ 0 & 2^n \end{pmatrix}$$

where r is a constant.

(4)

$$\mathbf{M} = \begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix} \qquad \mathbf{N} = \begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix}^4$$

The transformation represented by matrix **M** followed by the transformation represented by matrix **N** is represented by the matrix **B**

(b) (i) Determine **N** in the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c and d are integers.

(ii) Determine **B**

(3)

Hexagon S is transformed onto hexagon S' by matrix **B**

(c) Given that the area of S' is 720 square units, determine the area of S

(2)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. **In this question use the standard results for summations.**

(a) Show that for all positive integers n

$$\sum_{r=1}^{n}(12r^2 + 2r - 3) = An^3 + Bn^2$$

where A and B are integers to be determined.

(4)

(b) Hence determine the value of n for which

$$\sum_{r=1}^{2n} r^3 - \sum_{r=1}^{n}(12r^2 + 2r - 3) = 270$$

(4)

Question 7 continued

(Total for Question 7 is 8 marks)

8. Prove by induction that for $n \in \mathbb{Z}^+$

$$f(n) = 7^{n-1} + 8^{2n+1}$$

is divisible by 57

(6)

Question 8 continued

(Total for Question 8 is 6 marks)

9. The rectangular hyperbola H has equation $xy = c^2$ where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, where $t > 0$, lies on H

(a) Use calculus to show that an equation of the normal to H at P is

$$t^3x - ty = c(t^4 - 1)$$

(4)

The parabola C has equation $y^2 = 6x$

The normal to H at the point with coordinates (8, 2) meets C at the point Q where $y > 0$

(b) Determine the exact coordinates of Q

(4)

Given that

- the point R is the focus of C
- the line l is the directrix of C
- the line through Q and R meets l at the point S

(c) determine the exact length of QS

(5)

Question 9 continued

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS