Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Tuesday 30 May 2023

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n

$$\sum_{r=1}^{n} r^2(r+2) = \frac{1}{12}n(n+1)(an^2+bn+c)$$

where a, b and c are integers to be determined.

(4)

$$= \sum_{r=1}^{n} r^3 + 2\sum_{r=1}^{n} r^2$$

$$= \tfrac{1}{4}n^2(n+1)^2 + 2 \cdot \tfrac{1}{6}n(n+1)(2n+1)$$

$$= \tfrac{1}{12}n(n+1)\left[3n(n+1) + 4(2n+1)\right]$$

$$= \tfrac{1}{12}n(n+1)\left[3n^2 + 11n + 4\right]$$

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given that $x = 2 + 3\text{i}$ is a root of the equation

$$2x^4 - 8x^3 + 29x^2 - 12x + 39 = 0$$

(a) write down another complex root of this equation.

$2 - 3i$

(1)

(b) Use algebra to determine the other 2 roots of the equation.

(4)

(c) Show all 4 roots on a single Argand diagram.

(2)

(b) Sum $= 4$ prod $= 4 + 9 = 13$

$$\begin{array}{r|l} & 2x^2 \qquad\qquad + 3 \\ x^2 - 4x + 13 & 2x^4 - 8x^3 + 29x^2 - 12x + 39 \\ & 2x^4 - 8x^3 + 26x^2 \\ \hline & \qquad\qquad 3x^2 - 12x + 39 \\ & \qquad\qquad 3x^2 - 12x + 39 \\ \hline & \qquad\qquad\qquad\qquad 0 \end{array}$$

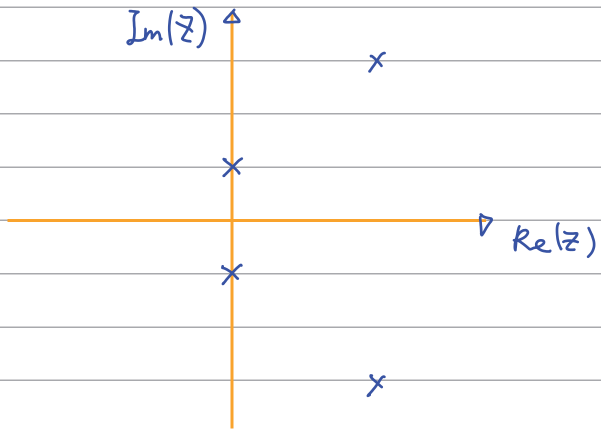
$2x^2 + 3 = 0$

$2x^2 = -3$

$x^2 = -3/2$

$x = \pm\sqrt{3/2}\, i$

(c)



3. The rectangular hyperbola H has Cartesian equation $xy = 9$

The point P with coordinates $\left(3t, \frac{3}{t}\right)$, where $t \neq 0$, lies on H

(a) Use calculus to determine an equation for the normal to H at the point P

Give your answer in the form $ty - t^3x = \mathrm{f}(t)$

(4)

Given that $t = 2$

(b) determine the coordinates of the point where the normal meets H again.

Give your answer in simplest form.

(3)

(a) $\frac{dx}{dt} = 3$ $\quad \frac{dy}{dt} = -\frac{3}{t^2}$ $\quad \frac{dy}{dx} = -\frac{1}{t^2}$

NORMAL: $y - 3/t = t^2(x - 3t)$

$ty - 3 = t^3x - 3t^4$

$ty - t^3x = 3 - 3t^4$

(b) $2y - 8x = 3 - 3\cdot 16 = -45$

$2y - 8\frac{9}{y} = -45$

$2y^2 + 45y - 72 = 0$ $\quad$ 2 -3, 1 $+24$

$(2y-3)(y+24) = 0$

$y = 3/2$ $\quad t = 2$

$y = -24$ $\quad t = -1/8$ $\quad x = -3/8$

$(-3/8, -24)$

4. (i) $\mathbf{A} = \begin{pmatrix} -3 & 8 \\ -3 & k \end{pmatrix}$ where k is a constant

The transformation represented by **A** transforms triangle T to triangle T'

The area of triangle T' is three times the area of triangle T

Determine the possible values of k

(4)

(ii) $\mathbf{B} = \begin{pmatrix} a & -4 \\ 2 & 3 \end{pmatrix}$ and $\mathbf{BC} = \begin{pmatrix} 2 & 5 & 1 \\ 1 & 4 & 2 \end{pmatrix}$ where a is a constant

Determine, in terms of a, the matrix **C**

(4)

(i) $\det(A) = -3k + 24 = 3$ or $-3k + 24 = -3$

$-3k = -21$ $\qquad$ $-3k = -27$

$k = 7$ $\qquad$ $k = 9$

(ii) $\det(B) = 3a + 8$

$B^{-1} = \frac{1}{3a+8}\begin{pmatrix} 3 & 4 \\ -2 & a \end{pmatrix}$

$C = \frac{1}{3a+8}\begin{pmatrix} 3 & 4 \\ -2 & a \end{pmatrix}\begin{pmatrix} 2 & 5 & 1 \\ 1 & 4 & 2 \end{pmatrix}$

$= \frac{1}{3a+8}\begin{pmatrix} 10 & 31 & 11 \\ -4+a & 4a-10 & 2a-2 \end{pmatrix}$

5.

$$f(x) = x^2 - 6x + 3$$

The equation $f(x) = 0$ has roots α and β

Without solving the equation,

(a) determine the value of

$$(\alpha^2 + 1)(\beta^2 + 1)$$

(4)

(b) find a quadratic equation which has roots

$$\frac{\alpha}{(\alpha^2 + 1)} \text{ and } \frac{\beta}{(\beta^2 + 1)}$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers to be determined.

(6)

(a) $\alpha+\beta = 6 \quad \alpha\beta = 3$

$$(\alpha^2+1)(\beta^2+1) = (\alpha\beta)^2 + \alpha^2+\beta^2 + 1$$
$$= 3^2 + (\alpha+\beta)^2 - 2\alpha\beta + 1$$
$$= 10 + 36 - 6$$
$$= 40$$

(b) $\text{sum} = \dfrac{\alpha\beta^2 + \alpha + \alpha^2\beta + \beta}{(\alpha^2+1)(\beta^2+1)} = \dfrac{\alpha\beta(\alpha+\beta) + 6}{40} = \dfrac{24}{40} = \dfrac{3}{5}$

$\text{prod} = \dfrac{\alpha\beta}{(\alpha^2+1)(\beta^2+1)} = \dfrac{3}{40}$

$$x^2 - 3/5\,x + 3/40 = 0$$

$$40x^2 - 24x + 3 = 0$$

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$z_1 = 3 + 2\text{i} \qquad z_2 = 2 + 3\text{i} \qquad z_3 = a + b\text{i} \qquad a,b \in \mathbb{R}$$

(a) Determine the exact value of $|z_1 + z_2|$

(2)

Given that $w = \dfrac{z_2 z_3}{z_1}$

(b) determine w in terms of a and b, giving your answer in the form $x + \text{i}y$, where $x, y \in \mathbb{R}$

(4)

Given also that $w = \dfrac{4}{13} + \dfrac{58}{13}\text{i}$

(c) determine the value of a and the value of b

(2)

(d) determine arg w, giving your answer in radians to 4 significant figures.

(2)

(a) $|z_1 + z_2| = |5 + 5i| = 5\sqrt{2}$

(b)
$$W = \frac{(2+3i)(a+bi)}{3+2i} = \frac{2a + 3bi^2 + (3a+2b)i}{3+2i} = \frac{2a - 3b + (3a+2b)i}{3+2i}$$
$$= \frac{(2a-3b) + (3a-2b)i}{3+2i} \times \frac{3-2i}{3-2i} = \frac{6a - 9b + 6a + 4b + (-4a + 6b + 9a + 6b)i}{13}$$
$$= \frac{12a - 5b}{13} + \frac{5a + 12b}{13}i$$

(c)
$$\begin{cases} 12a - 5b = 4 \\ 5a + 12b = 58 \end{cases} \qquad \begin{cases} 60a - 25b = 20 \\ 60a + 144b = 696 \end{cases}$$
$$169b = 676$$
$$\boxed{\begin{aligned} b &= 4 \\ a &= 2 \end{aligned}}$$

(d)
$$\arg(w) = \tan^{-1}\left(\frac{58}{4}\right) = 1.501939837$$
$$= 1.502$$

7. $$f(x) = x^{\frac{3}{2}} + x - 3$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval [1, 2]
(2)

(b) Starting with the interval [1, 2], use interval bisection twice to show that α lies in the interval [1.25, 1.5]
(3)

(c) (i) Determine $f'(x)$

(ii) Using 1.375 as a first approximation for α, apply the Newton-Raphson process once to $f(x)$ to determine a second approximation for α, giving your answer to 3 decimal places.
(3)

(d) Use linear interpolation once on the interval [1.25, 1.5] to obtain a different approximation for α, giving your answer to 3 decimal places.
(3)

(a)

1	1	$\text{Ans}^{1.5}+\text{Ans}-3$	-1
2	2	$\text{Ans}^{1.5}+\text{Ans}-3$	1.828427125

(b)

1.5	$\frac{3}{2}$	$\text{Ans}^{1.5}+\text{Ans}-3\rightarrow E$	0.3371173071
1.25	$\frac{5}{4}$	$\text{Ans}^{1.5}+\text{Ans}-3\rightarrow D$	-0.3524575141

$1.25 \le \alpha \le 1.5$

Question 7 continued

(c) (i) $f(x) = x^{3/2} + x - 3$

$f'(x) = \frac{3}{2}x^{1/2} + 1$

(ii) $\alpha = 1.375 - \dfrac{f(1.375)}{f'(1.375)} \approx 1.380$

$1.375 - \dfrac{A}{B}$

1.379592249

1.375

$\frac{11}{8}$

$\text{Ans}^{1.5} + \text{Ans} - 3 \rightarrow A$

-0.01266958256

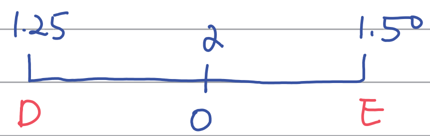
1.375

$\frac{11}{8}$

$1.5\text{Ans}^{.5} + 1 \rightarrow B$

2.75890591

(d)



$\dfrac{\alpha - 1.25}{0 - D} = \dfrac{1.50 - 1.25}{E - D}$

$\dfrac{1.5 - 1.25}{E - D}$

0.3625422396

$\text{Ans} \times -D$

0.1277807365

$\text{Ans} + 1.25$

1.377780736

$\alpha = 1.378$

8. The point $P(2p^2, 4p)$ lies on the parabola with equation $y^2 = 8x$ $= 4(2)x$

(a) Show that the point $Q\left(\frac{2}{p^2}, \frac{-4}{p}\right)$, where $p \neq 0$, lies on the parabola.

(1)

(b) Show that the chord PQ passes through the focus of the parabola.

(4)

The tangent to the parabola at P and the tangent to the parabola at Q meet at the point R

(c) Determine, in simplest form, the coordinates of R

(8)

(a) $\left(\frac{-4}{p}\right)^2 = \frac{16}{p^2}$ $\quad 8\left(\frac{2}{p^2}\right) = \frac{16}{p^2}$

$y^2 = 8x$ $\quad$ Q on the parabola

(b) PQ: $\frac{y - {-4/p}}{x - 2/p^2} = \frac{y - 4p}{x - 2p^2}$ $\quad (2, 0)$ Focus

$\frac{4/p}{2 - 2/p^2} = \frac{-4p}{2 - 2p^2}$

$\frac{4p}{2p^2 - 2} = \frac{4p}{2p^2 - 2}$ $\quad$ So Focus Is on PQ.

(c) $\frac{dx}{dt} = \frac{-4}{p^3}$ $\quad \frac{dy}{dt} = \frac{4}{p^2}$ $\quad \frac{dy}{dx} = \frac{4/p^2}{-4/p^3} = -p$

TANGENT AT $Q(2/p^2, -4/p)$: $\quad y + 4/p = -p(x - 2/p^2)$

$\frac{dx}{dt} = 4p$ $\quad \frac{dy}{dt} = 4$ $\quad \frac{dy}{dx} = \frac{4}{4p} = \frac{1}{p}$

TANGENT AT $P(2p^2, 4p)$ $\quad y - 4p = 1/p(x - 2p^2)$

$-px + 2/p - 4/p = x/p - 2p + 4p$

$-p^2x + 2 - 4 = x + 2p^2$

$-(2 + 2p^2) = (1 + p^2)x$

$x = -2$

$R\left(-2, 2p - 2/p\right)$

9. Prove, by induction, that for $n \in \mathbb{Z}, n \geqslant 2$

$$4^n + 6n - 10$$

is divisible by 18

(5)

$P(2)$ $\quad 4^2 + 6(2) - 10 = 16 + 12 - 10 = 18$

$P(2)$ TRUE, ASSUME $P(n)$

$$4^n + 6n - 10 = 18k \qquad k \in \mathbb{Z}.$$

NOW $\quad P(n+1) - P(n) = 4^{n+1} + 6n + 6 - 10 - (4^n + 6n - 10)$

$$= 4 \cdot 4^n + 6 - 4^n$$
$$= 3 \cdot 4^n + 6$$
$$= 3[18k + 10 - 6n] + 6$$
$$= 3 \cdot 18k + 36 - 18n$$
$$= 18[3k + 2 - n] \quad \text{multiple of } 18$$

$P(n)$ is a multiple of 18, so
$P(n+1)$ is a multiple of 18

$P(n) \longrightarrow P(n+1)$ by induction.

$P(2)$ TRUE, So $P(n)$ TRUE FOR $n \geqslant 2$