Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. $z_1 = 3 + 3\text{i} \qquad z_2 = p + q\text{i} \qquad p, q \in \mathbb{R}$

Given that $|z_1 z_2| = 15\sqrt{2}$

(a) determine $|z_2|$

(2)

Given also that $p = -4$

(b) determine the possible values of q

(2)

(c) Show z_1 and the possible positions for z_2 on the same Argand diagram.

(2)

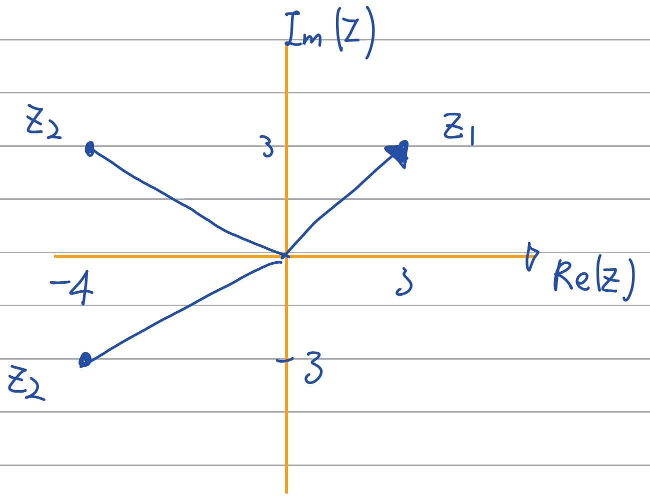
(a) $|z_1| = \sqrt{3^2+3^2} = 3\sqrt{2}$

$|z_2| = \dfrac{15\sqrt{2}}{3\sqrt{2}} = 5$

(b) $|z_2| = 5^2 = p^2 + q^2 = 16 + q^2$ $\qquad q^2 = 9$

$q = \pm 3$

(c)



2. $$f(x) = 10 - 2x - \frac{1}{2\sqrt{x}} - \frac{1}{x^3} \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [0.4, 0.5]

(2)

(b) Determine $f'(x)$.

(3)

(c) Using $x_0 = 0.5$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places.

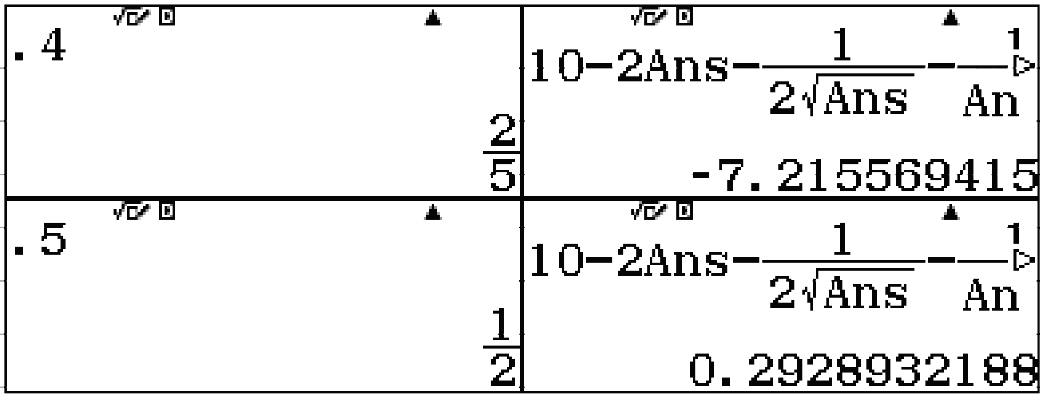
(2)

The equation $f(x) = 0$ has another root β in the interval [4.8, 4.9]

(d) Use linear interpolation once on the interval [4.8, 4.9] to find an approximation to β, giving your answer to 3 decimal places.

(2)

(a)



< 0

> 0

f(x) is continuous and there is a sign change.

Hence there is a root between 0.4 and 0.5.

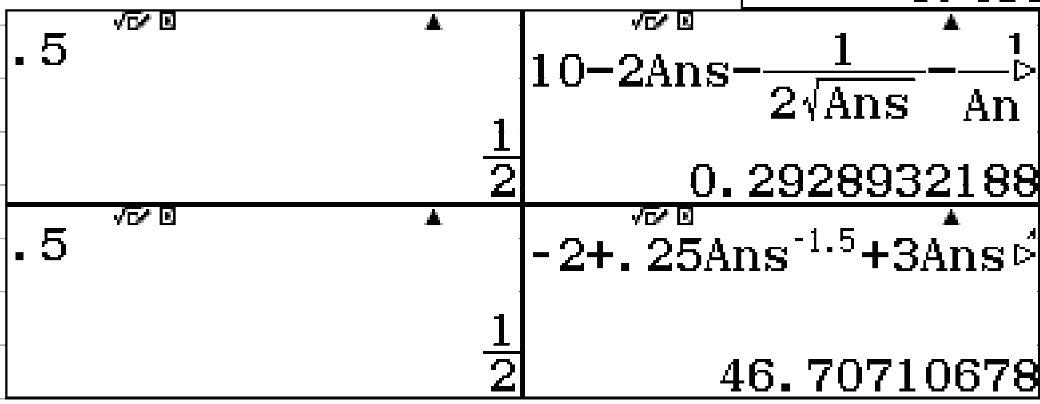
(b)

$f(x) = 10 - 2x - \frac{1}{2}x^{-1/2} - x^{-3}$

$f'(x) = -2 + \frac{1}{4}x^{-3/2} + 3x^{-4}$

(c)

$\alpha = 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.494$



A

B

Question 2 continued

(d)

4.8 $\qquad$ β $\qquad$ 4.9

$f(4.8)$ $\qquad$ 0 $\qquad$ $f(4.9)$

A $\qquad\qquad$ B

4.8

$\frac{24}{5}$

$10-2\text{Ans}-\frac{1}{2\sqrt{\text{Ans}}}-\frac{1}{\text{An}}$ ▷

0.1627400223

4.9

$\frac{49}{10}$

$10-2\text{Ans}-\frac{1}{2\sqrt{\text{Ans}}}-\frac{1}{\text{An}}$ ▷

-0.03437683548

$$\frac{\beta-4.8}{-A} = \frac{4.9-4.8}{B-A}$$

$\frac{.1}{\text{B}-\text{A}}$

-0.5073132816

Ans×-A

0.08256017478

Ans+4.8

4.882560175

$\beta = 4.883$

3. $\mathbf{M} = \begin{pmatrix} k & k \\ 3 & 5 \end{pmatrix}$ where k is a non-zero constant

(a) Determine $\mathbf{M}^{-1}$, giving your answer in simplest form in terms of k.

(2)

Hence, given that $\mathbf{N}^{-1} = \begin{pmatrix} k & k \\ 4 & -1 \end{pmatrix}$

(b) determine $(\mathbf{MN})^{-1}$, giving your answer in simplest form in terms of k.

(2)

(a) $M^{-1} = \dfrac{1}{5k-3k}\begin{pmatrix} 5 & -k \\ -3 & k \end{pmatrix} = \dfrac{1}{2k}\begin{pmatrix} 5 & -k \\ -3 & k \end{pmatrix}$

(b) $(MN)^{-1} = N^{-1} \cdot M^{-1}$

$= \begin{pmatrix} k & k \\ 4 & -1 \end{pmatrix} \cdot \dfrac{1}{2k}\begin{pmatrix} 5 & -k \\ -3 & k \end{pmatrix}$

$= \dfrac{1}{2k}\begin{pmatrix} 2k & -k^2+k^2 \\ 20+3 & -4k-k \end{pmatrix}$

$= \dfrac{1}{2k}\begin{pmatrix} 2k & 0 \\ 23 & -5k \end{pmatrix}$

4. $$f(z) = 2z^4 - 19z^3 + Az^2 + Bz - 156$$

where A and B are constants.

The complex number $5 - i$ is a root of the equation $f(z) = 0$

(a) Write down another complex root of this equation. **(1)**

$5+i$

(b) Solve the equation $f(z) = 0$ completely. **(5)**

(c) Determine the value of A and the value of B. **(2)**

(b)

sum $= 10$

prod $= 25 + 1 = 26$

$f(z) = (z^2 - 10z + 26)(2z^2 + pz - 6)$

$p = 1$

$2z^2 + z - 6 = 0$

$z = \dfrac{-1 \pm \sqrt{1+48}}{4} = \dfrac{-1 \pm 7}{4}$

$= -2, \ 3/2$

$f(z) = 0$: $z = -2, \ 3/2, \ 5 \pm i$

(c) $f(z) = (z^2 - 10z + 26)(2z^2 + z - 6)$

$A = 36$

$B = 86$

5. The quadratic equation

$$2x^2 - 3x + 5 = 0$$

has roots α and β

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$ **(1)**

$\frac{3}{2}$ $\frac{5}{2}$

(b) determine the value of

(i) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \frac{9}{4} - 5 = -\frac{11}{4}$

(ii) $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = \frac{27}{8} - 3\cdot\left(\frac{5}{2}\right)\cdot\frac{3}{2} = -\frac{63}{8}$ **(4)**

(c) find a quadratic equation which has roots

$$(\alpha^3 - \beta) \text{ and } (\beta^3 - \alpha)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers to be determined. **(5)**

(c) Sum $= \alpha^3 + \beta^3 - (\alpha+\beta) = -\frac{63}{8} - \frac{3}{2} = -\frac{75}{8}$

prod $= (\alpha\beta)^3 - (\alpha^4 + \beta^4) + \alpha\beta$

$= \frac{125}{8} - \left[(\alpha^2+\beta^2)^2 - 2(\alpha\beta)^2\right] + \frac{5}{2}$

$= \frac{125}{8} - \left[\frac{121}{16} - 2\cdot\frac{25}{4}\right] + \frac{5}{2} = \frac{369}{16}$

$x^2 + \frac{75}{8}x + \frac{369}{16} = 0$

$16x^2 + 150x + 369 = 0$

6. The parabola C has equation $y^2 = 36x$ $= 4 \cdot a \; x$ $\quad a = 9$

The point $P(9t^2, 18t)$, where $t \neq 0$, lies on C

(a) Use calculus to show that the normal to C at P has equation

$$y + tx = 9t^3 + 18t$$

(4)

(b) Hence find the equations of the two normals to C which pass through the point $(54, 0)$, giving your answers in the form $y = px + q$ where p and q are constants to be determined.

(4)

Given that

- the normals found in part (b) intersect the directrix of C at the points A and B
- the point F is the focus of C $\quad F(9, 0) \quad x = -9$

(c) determine the area of triangle AFB

(3)

(a) $\dfrac{dx}{dt} = 18t \quad \dfrac{dy}{dt} = 18 \quad \dfrac{dy}{dx} = \dfrac{18}{18t} = \dfrac{1}{t}$

NORMAL: $y - 18t = -t(x - 9t^2)$

$tx + y = 9t^3 + 18t$

(b) $x = 54$, $y = 0$

$54t = 9t^3 + 18t$

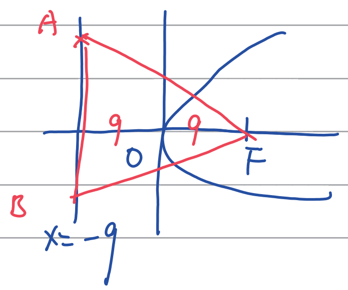
$9t^2 = 36$

$t = \pm 2$

$t = 2$: $2x + y = 108$, $y = -2x + 108$

$t = -2$: $-2x + y = -108$, $y = 2x - 108$

(c)



$x = -9$: $y = 126 \quad y = -126$

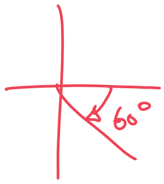
AREA $= \frac{1}{2} \times (126 + 126) \times 18$

$= 2268$

7.

$$\mathbf{A} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$

(a) Determine the matrix $\mathbf{A}^2$ $= \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$ **(1)**



(b) Describe fully the single geometrical transformation represented by the matrix $\mathbf{A}^2$ **(2)**

Rotation 60° clockwise about the origin

(c) Hence determine the smallest positive integer value of n for which $\mathbf{A}^n = \mathbf{I}$ **(1)**

$(A^2)^6 \rightarrow A^{12} \rightarrow n = 12$

The matrix **B** represents a stretch scale factor 4 parallel to the x-axis.

(d) Write down the matrix **B** $= \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$ **(1)**

The transformation represented by matrix **A** followed by the transformation represented by matrix **B** is represented by the matrix **C**

(e) Determine the matrix **C** $= BA = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{2}\begin{pmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{3} \end{pmatrix} = \begin{pmatrix} -2\sqrt{3} & -2 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$ **(2)**

The parallelogram P is transformed onto the parallelogram P' by the matrix **C**

(f) Given that the area of parallelogram P' is 20 square units, determine the area of parallelogram P **(2)**

(f) $\det(C) = 3 + 1 = 4$

AREA of $P = 20/4 = 5$

8. (a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that for all positive integers n

$$\sum_{r=0}^{n} (r+1)(r+2) = \frac{1}{3}(n+1)(n+2)(n+3)$$

(5)

(b) Hence determine the value of

$$10 \times 11 + 11 \times 12 + 12 \times 13 + \ldots + 100 \times 101$$

(3)

(a)

$$\sum_{r=0}^{n} r^2+3r+2 = \sum_{r=0}^{n} r^2 + 3\sum_{r=0}^{n} r + 2(n+1)$$

$$= \frac{1}{6}n(n+1)(2n+1) + 3\frac{n}{2}(1+n) + 2(n+1)$$

$$= (n+1)\left[\frac{n}{6}(2n+1) + \frac{3n}{2} + 2\right]$$

$$= (n+1)\left[\frac{1}{3}n^2 + \frac{5}{3}n + 2\right]$$

$$= \frac{1}{3}(n+1)\left[n^2+5n+6\right]$$

$$= \frac{1}{3}(n+1)(n+2)(n+3)$$

(b)

$$\sum_{r=9}^{99}(r+1)(r+2) = \sum_{r=0}^{99}(\ )(\ ) - \sum_{r=0}^{8}(\ )(\ )$$

$$\frac{100\times101\times102}{3} \qquad 343400$$

$$\frac{9\times10\times11}{3} \qquad 330$$

$$343400-\text{Ans} \qquad 343070$$

9. (i) A sequence of numbers is defined by

$$u_1 = 3$$

$$u_{n+1} = 2u_n - 2^{n+1} \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{N}$

$$u_n = 5 \times 2^{n-1} - n \times 2^n$$

(5)

(ii) Prove by induction that, for $n \in \mathbb{N}$

$$f(n) = 5^{n+2} - 4n - 9$$

is divisible by 16

(5)

(i) $P(1)$: $U_1 = 5 \times 2^0 - 1 \times 2 = 5 - 2 = 3$ TRUE.

ASSUME $P(n)$ TRUE $U_n = 5 \times 2^{n-1} - n \cdot 2^n$

NOW $U_{n+1} = 2\left[5 \times 2^{n-1} - n \cdot 2^n\right] - 2^{n+1}$
$= 5 \cdot 2^n - n \cdot 2^{n+1} - 2^{n+1}$
$= 5 \cdot 2^n - (n+1) \cdot 2^{n+1}$

$P(n) \longrightarrow P(n+1)$ by induction.
$P(1)$ TRUE. SO $P(n)$ TRUE FOR $n \geq 1$

(ii) $P(1)$ $f(1) = 5^3 - 4 - 9 = 125 - 13 = 112 = 16 \times 7$. TRUE.
ASSUME $P(n)$ TRUE. $f(n) = 5^{n+2} - 4n - 9 = 16k$ $k \in \mathbb{Z}$.
$f(n+1) - f(n) = 5^{n+3} - 4(n+1) - 9 - 5^{n+2} + 4n + 9$
$= 5 \cdot 5^{n+2} - 4n - 13 - 5^{n+2} + 4n + 9$
$= 4 \cdot 5^{n+2} - 4$
$= 4 \cdot [16k + 4n + 9] - 4$
$= 64k + 16n + 32$
$= 16[4k + n + 2]$ multiple of 16

So $f(n+1)$ is a multiple of 16

$P(n) \longrightarrow P(n+1)$ by induction. $P(1)$ TRUE.
SO $P(n)$ TRUE FOR $n \geq 1$