Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Leave blank

1. $$\mathbf{M} = \begin{pmatrix} 3x & 7 \\ 4x+1 & 2-x \end{pmatrix}$$

Find the range of values of x for which the determinant of the matrix $\mathbf{M}$ is positive.

(5)

$\det(M) = 3x(2-x) - 7(4x+1) > 0$

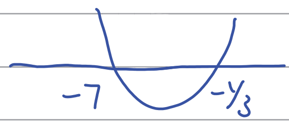
$-3x^2 + 6x - 28x - 7 > 0$

$-3x^2 - 22x - 7 > 0$

$3x^2 + 22x + 7 < 0$

$3 \quad +1$

$1 \quad +7$ $\qquad (3x+1)(x+7) < 0$



$-7 < x < -\frac{1}{3}$

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2. The complex numbers z_1 and z_2 are given by

$$z_1 = 3 + 5\mathrm{i} \quad \text{and} \quad z_2 = -2 + 6\mathrm{i}$$

(a) Show z_1 and z_2 on a single Argand diagram.

(2)

(b) **Without using your calculator and showing all stages of your working,**

(i) determine the value of $|z_1|$ $= \sqrt{3^2+5^2} = \sqrt{34}$

(1)

(ii) express $\dfrac{z_1}{z_2}$ in the form $a + b\mathrm{i}$, where a and b are fully simplified fractions.

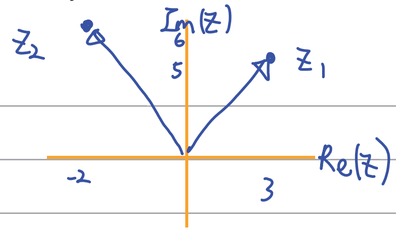
(3)

(c) Hence determine the value of $\arg\dfrac{z_1}{z_2}$

Give your answer in radians to 2 decimal places.

(2)

2(a)



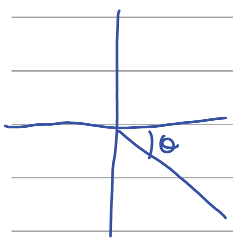
(b) (ii) $\dfrac{3+5i}{-2+6i} \times \dfrac{-2-6i}{-2-6i}$

$= \dfrac{-6-18i-10i-30i^2}{4+36} = \dfrac{24-28i}{40}$

$= \dfrac{3}{5} - \dfrac{7}{10}i$

(c) $\arg\left(\dfrac{z_1}{z_2}\right) = -\tan^{-1}\left(\dfrac{7/10}{3/5}\right)$

$= -0.86$

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-tan⁻¹(7÷10÷(3÷5))
                     -0.8621700547
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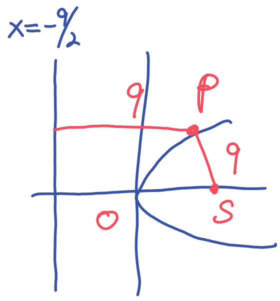
Leave blank

3. The parabola C has equation $y^2 = 18x$ $= 4 \cdot \frac{9}{2} x = 4ax$

The point S is the focus of C

(a) Write down the coordinates of S $\left(\frac{9}{2}, 0\right)$



(1)

The point P, with $y > 0$, lies on C

The shortest distance from P to the directrix of C is 9 units.

(b) Determine the exact perimeter of the triangle OPS, where O is the origin.

Give your answer in simplest form.

(4)

(b)

$\frac{9}{2} + x = 9$

$x = \frac{9}{2}$ $\qquad y = \sqrt{18 \cdot \frac{9}{2}} = 9$

$OP = \sqrt{\left(\frac{9}{2}\right)^2 + 81} = \frac{9\sqrt{5}}{2}$

Perimeter $= \frac{9\sqrt{5}}{2} + 9 + \frac{9}{2} = \boxed{\frac{9\sqrt{5}}{2} + \frac{27}{2}}$

Leave blank

4. The equation

$$x^4 + Ax^3 + Bx^2 + Cx + 225 = 0$$

where A, B and C are real constants, has

- a complex root $4 + 3i$
- a repeated positive real root

(a) Write down the other complex root of this equation.
(1)

(b) Hence determine a quadratic factor of $x^4 + Ax^3 + Bx^2 + Cx + 225$
(2)

(c) Deduce the real root of the equation.
(2)

(d) Hence determine the value of each of the constants A, B and C
(3)

(a) $4-3i$

(b) Sum: 8 $\quad$ $x^2 - 8x + 25$

product: $16+9=25$

(c)

$x^4 + Ax^3 + Bx^2 + Cx + 225 = (x^2 - 8x + 25)(x^2 + px + q)$

$q = 225/25 = 9$

repeated real root: $p = -6$

so $x^2 - 6x + 9 = (x-3)^2$ $\quad$ $x = 3$

(d)

$(x^2 - 8x + 25)(x^2 - 6x + 9)$

$= x^4 - 6x^3 + 9x^2 - 8x^3 + 48x^2 - 72x + 25x^2 - 150x + 225$

$= x^4 - 14x^3 + 82x^2 - 222x + 225$

$\boxed{A = -14, \quad B = 82, \quad C = -222}$

Leave blank

5.

$$\mathbf{P} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

The matrix **P** represents the transformation U

(a) Give a full description of U as a single geometrical transformation. **(2)**

Rotation about O, $60°$ anticlockwise

The transformation V, represented by the 2×2 matrix **Q**, is a reflection in the line $y = -x$

(b) Write down the matrix **Q** $= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ **(1)**

The transformation U followed by the transformation V is represented by the matrix **R**

(c) Determine the matrix **R** $= V \cdot U = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -\sqrt{3} & -1 \\ -1 & \sqrt{3} \end{pmatrix}$ **(2)**

The transformation W is represented by the matrix 3**R**

The transformation W maps a triangle T to a triangle T'

The transformation W' maps the triangle T' back to the original triangle T

(d) Determine the matrix that represents W' **(3)**

(d) W: $T \to T'$ $\quad 3R = \frac{3}{2}\begin{pmatrix} -\sqrt{3} & -1 \\ -1 & \sqrt{3} \end{pmatrix} = \begin{pmatrix} -\frac{3\sqrt{3}}{2} & -\frac{3}{2} \\ -\frac{3}{2} & \frac{3\sqrt{3}}{2} \end{pmatrix}$

$\det(3R) = -\frac{27}{4} - \frac{9}{4} = -9$

W' :

$$(3R)^{-1} = \frac{1}{-9}\begin{pmatrix} \frac{3\sqrt{3}}{2} & \frac{3}{2} \\ \frac{3}{2} & -\frac{3\sqrt{3}}{2} \end{pmatrix}$$

$$= -\frac{1}{9} \cdot \frac{3}{2}\begin{pmatrix} \sqrt{3} & 1 \\ 1 & -\sqrt{3} \end{pmatrix}$$

$$= -\frac{1}{6}\begin{pmatrix} \sqrt{3} & 1 \\ 1 & -\sqrt{3} \end{pmatrix}$$

Leave blank

6. The quadratic equation

$$Ax^2 + 5x - 12 = 0$$

where A is a constant, has roots α and β

(a) Write down an expression in terms of A for

(i) $\alpha + \beta$ $= -\frac{5}{A}$

(ii) $\alpha\beta$ $= \frac{-12}{A}$

(2)

The equation

$$4x^2 - 5x + B = 0$$

where B is a constant, has roots $\alpha - \dfrac{3}{\beta}$ and $\beta - \dfrac{3}{\alpha}$

(b) Determine the value of A

(3)

(c) Determine the value of B

(3)

(b) sum $= \frac{5}{4} = \alpha + \beta - 3\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = -\frac{5}{A} - 3\left(\frac{\alpha+\beta}{\alpha\beta}\right)$

$\frac{5}{4} = -\frac{5}{A} - 3\left(\frac{-5/A}{-12/A}\right)$

$\frac{5}{4} = -\frac{5}{A} - \frac{5}{4}$

$\frac{5}{2} = -\frac{5}{A}$

$A = -2$

(c) prod: $\frac{B}{4} = \alpha\beta - 3 - 3 + \frac{9}{\alpha\beta}$

$B = 4\left(\frac{-12}{-2} - 6 + \frac{9}{6}\right)$

$= 4\left(6 - 6 + \frac{9}{6}\right)$

$= 6$

Leave blank

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The rectangular hyperbola H has equation $xy = 36$

The point $P(4, 9)$ lies on H

(a) Show, using calculus, that the normal to H at P has equation

$$4x - 9y + 65 = 0$$

(4)

The normal to H at P crosses H again at the point Q

(b) Determine an equation for the tangent to H at Q, giving your answer in the form $y = mx + c$ where m and c are rational constants.

(5)

1a) $y = 36x^{-1}$ $\quad \frac{dy}{dx} = -\frac{36}{x^2}$ $\quad$ AT $P(4,9)$

$= -\frac{36}{16} = -\frac{9}{4}$

NORMAL: $y - 9 = 4/9\,(x-4)$

$9y - 81 = 4x - 16$

$4x - 9y + 65 = 0$

(b) $4x - 9\left(\frac{36}{x}\right) + 65 = 0$

$4x^2 + 65x - 324 = 0$

$(4x+81)(x-4) = 0$

$x = -81/4 \quad y = -16/9$

$Q(-81/4, -16/9)$ $\quad \frac{dy}{dx} = \frac{-36}{(81/4)^2} = -\frac{64}{729}$

TANGENT::

$y + \frac{16}{9} = -\frac{64}{729}\left(x + \frac{81}{4}\right)$

$y = -\frac{64}{729}x - \frac{32}{9}$

Leave blank

8. $$f(x) = 2x^{-\frac{2}{3}} + \frac{1}{2}x - \frac{1}{3x-5} - \frac{5}{2} \qquad x \neq \frac{5}{3}$$

The table below shows values of f(x) for some values of x, with values of f(x) given to 4 decimal places where appropriate.

x	1	2	3	4	5
f(x)	0.5	−1.2401	−0.2885	0.1508	0.5834

(a) Complete the table giving the values to 4 decimal places.

(2)

The equation f(x) = 0 has exactly one positive root, α.

Using the values in the completed table and explaining your reasoning,

(b) determine an interval of width one that contains α.

(2)

(c) Hence use interval bisection twice to obtain an interval of width 0.25 that contains α.

(3)

Given also that the equation f(x) = 0 has a negative root, β, in the interval [–1, –0.5]

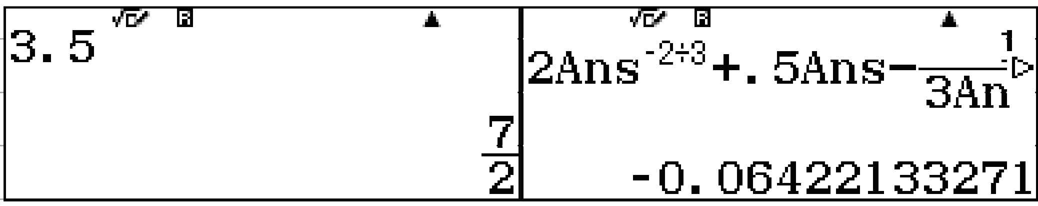
(d) use linear interpolation once on this interval to find an approximation for β.

Give your answer to 3 significant figures.

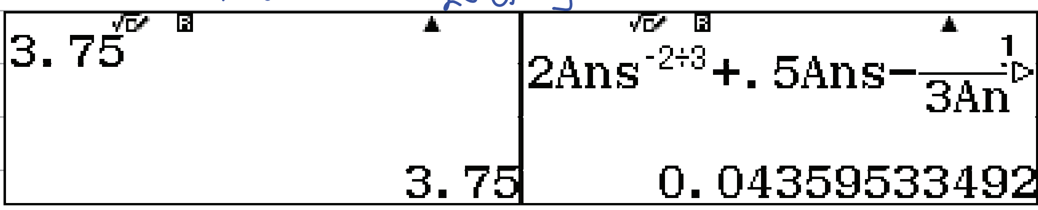
(3)

1b) f(x) is CONTINUOUS ON [3, 4], NOT ON [1, 2],
ROOT IN [3, 4] AS THERE IS A SIGN CHANGE.

(c)



ROOT IN [3.5, 4]



ROOT IN [3.75, 4]

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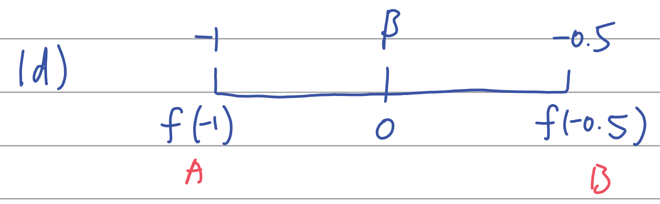
Question 8 continued

(d)




```
-1
                                  -1

2Ans^(-2÷3)+.5Ans-1/(3An▷
                                  -7/8

-.5
                                  -1/2

2Ans^(-2÷3)+.5Ans-1/(3An▷
                          0.5786482578
```

$$\frac{\beta-(-1)}{0-f(-1)}=\frac{-0.5-(-1)}{f(-0.5)-f(-1)}$$

```
.5/(B-A)
                          0.3439621637

Ans×(-A)
                          0.3009668932

Ans-1
                         -0.6990331068
```

$\beta = -0.699$

Leave blank

9. (a) Prove by induction that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

(5)

(b) Using the standard summation formulae, show that

$$\sum_{r=1}^{n} r(r+1)(r-1) = \frac{1}{4}n(n+A)(n+B)(n+C)$$

where A, B and C are constants to be determined.

(4)

(c) Determine the value of n for which

$$3\sum_{r=1}^{n} r(r+1)(r-1) = 17\sum_{r=n}^{2n} r^2$$

(5)

(a) P(1) $n=1$: $1^3 = 1$

$\frac{1}{4} \times 1 \times (1+1)^2 = 1$ P(1) TRUE.

ASSUME P(n) TRUE.

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

NOW $\displaystyle\sum_{r=1}^{n+1} r^3 = \frac{1}{4}n^2(n+1)^2 + (n+1)^3$

$= \frac{1}{4}(n+1)^2\left[n^2 + 4(n+1)\right]$

$= \frac{1}{4}(n+1)^2\left[n^2+4n+4\right]$

$= \frac{1}{4}(n+1)^2(n+2)^2$

$P(n) \longrightarrow P(n+1)$ by induction.

P(1) TRUE, SO P(n) TRUE FOR ALL $n \geq 1$

Leave blank

Question 9 continued

(b) $\sum_{r=1}^{n} r(r^2-1) = \sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} r$

$= \frac{1}{4}n^2(n+1)^2 - \frac{n}{2}(1+n)$

$= \frac{1}{4}n(n+1)\left[n(n+1)-2\right] = \frac{1}{4}n(n+1)(n^2+n-2)$

$= \frac{1}{4}n(n+1)(n-1)(n+2)$

(c) $\frac{3}{4}n(n+1)(n-1)(n+2) = 17\left[\frac{1}{6}(\overset{2n}{2n})(2n+1)(4n+1) - \right.$

$\left. \quad n-1 \quad \frac{1}{6}(n-1)n(2n-1) \right]$

$\frac{3}{4}n(n+1)(n-1)(n+2) = \frac{17}{6}n\left[2(8n^2+6n+1)-(2n^2-3n+1)\right]$

$= \frac{17}{6}n\left[14n^2+15n+1\right]$

$3/4\, n(n+1)(n-1)(n+2) = \frac{17}{6}n(14n+1)(n+1)$

$9(n^2+n-2) = 34(14n+1)$

$9n^2+9n-18 = 476n+34$

$9n^2-467n-52=0$

$n = \frac{467 \pm 469}{18} = 52$