Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number

Candidate Number

Friday 8 January 2021

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
  Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

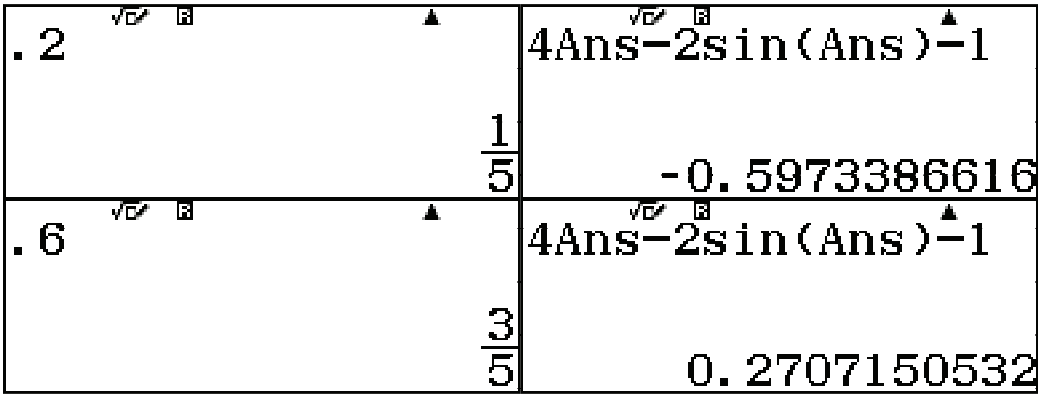
Leave blank

1. (a) Show that the equation $4x - 2\sin x - 1 = 0$, where x is in radians, has a root α in the interval [0.2, 0.6]

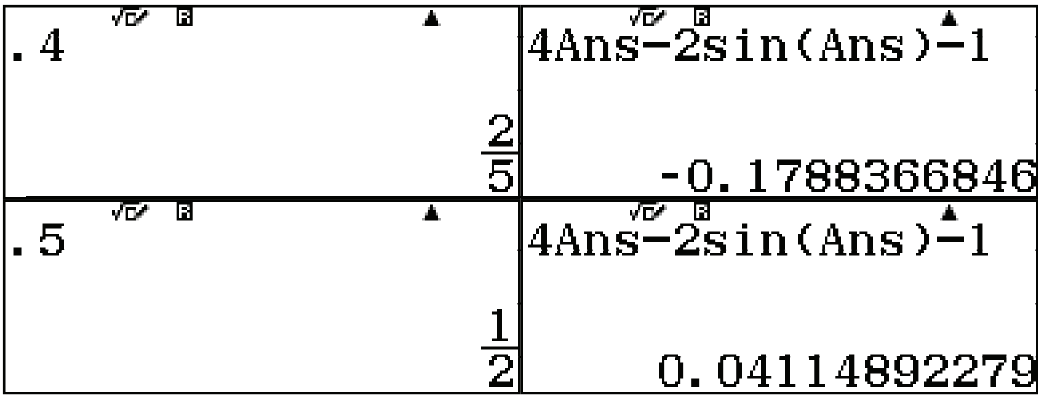
(2)

(b) Starting with the interval [0.2, 0.6], use interval bisection twice to find an interval of width 0.1 in which α lies.

(3)



(a) Continuous function with change of sign so root (in given interval)



(b) $0.4 \leqslant \alpha \leqslant 0.5$

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2. Given that $x = \frac{3}{8} + \frac{\sqrt{71}}{8}\mathrm{i}$ is a root of the equation

$$4x^3 - 19x^2 + px + q = 0$$

(a) write down the other complex root of the equation.

(1)

Given that $x = 4$ is also a root of the equation,

(b) find the value of p and the value of q.

(4)

(a) $x = \frac{3}{8} - \frac{\sqrt{71}}{8}i$

(b)

$\left(\frac{3}{8} + \frac{\sqrt{71}}{8}i\right)\left(\frac{3}{8} - \frac{\sqrt{71}}{8}i\right) = \frac{5}{4}$

$\frac{3}{8} + \frac{\sqrt{71}}{8}i + \frac{3}{8} - \frac{\sqrt{71}}{8}i = \frac{3}{4}$

$(x^2 - 3/4\, x + 5/4)(x - 4) = 0$

$(4x^2 - 3x + 5)(x - 4) = 0$

$p = 17, \quad q = -20$

Leave blank

3. The matrix **M** is defined by

$$\mathbf{M} = \begin{pmatrix} k+5 & -2 \\ -3 & k \end{pmatrix}$$

(a) Determine the values of k for which **M** is singular.

(2)

Given that **M** is non-singular,

(b) find $\mathbf{M}^{-1}$ in terms of k.

(2)

1a) $\det(M) = k(k+5) - 6 = 0$

$k^2 + 5k - 6 = 0$

$(k+6)(k-1) = 0$

$k = 1, -6$

(b)

$$M^{-1} = \frac{1}{k^2+5k-6}\begin{pmatrix} k & 2 \\ 3 & k+5 \end{pmatrix}$$

Leave blank

4. The equation $2x^2 + 5x + 7 = 0$ has roots α and β

Without solving the equation

(a) determine the exact value of $\alpha^3 + \beta^3$

(3)

(b) form a quadratic equation, with integer coefficients, which has roots

$$\frac{\alpha^2}{\beta} \text{ and } \frac{\beta^2}{\alpha}$$

(5)

(a) $\alpha\beta = 7/2 \qquad \alpha+\beta = -5/2$

$\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= -\frac{125}{8} - 3(7/2)(-5/2)$

$= \frac{85}{8}$

(b) Sum $= \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{85/8}{7/2} = \frac{85}{28}$

prod $= \alpha\beta = 7/2$

$x^2 - 85/28\, x + 7/2 = 0$

$28x^2 - 85x + 98 = 0$

Leave blank

5. (a) Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that

$$\sum_{r=1}^{n}(r+1)(r+5) = \frac{n}{6}(n+7)(2n+7)$$

for all positive integers n.

(5)

(b) Hence show that

$$\sum_{r=n+1}^{2n}(r+1)(r+5) = \frac{7n}{6}(n+1)(an+b)$$

where a and b are integers to be determined.

(2)

(a) $\sum_{r=1}^{n} r^2+6r+5$

$= \frac{1}{6}n(n+1)(2n+1) + 6\cdot\frac{1}{2}n(1+n) + 5n$

$= \frac{n}{6}\left[2n^2+3n+1 + 18+18n + 30\right]$

$= \frac{n}{6}\left[2n^2+21n+49\right] = \boxed{\frac{n}{6}(n+7)(2n+7)}$

(b) $\frac{2n}{6}(2n+7)(4n+7) - \frac{n}{6}(n+7)(2n+7)$

$= \frac{n}{6}\left[2(8n^2+42n+49) - (2n^2+21n+49)\right]$

$= \frac{n}{6}\left[14n^2+63n+49\right]$

$= \frac{7n}{6}\left[2n^2+9n+7\right]$

$= \boxed{\frac{7n}{6}(n+1)(2n+7)}$

Leave blank

6. The complex number z is defined by

$$z = -\lambda + 3\text{i} \quad \text{where } \lambda \text{ is a positive real constant}$$

Given that the modulus of z is 5

(a) write down the value of λ $\quad$ 4

(1)

(b) determine the argument of z, giving your answer in radians to one decimal place.

(2)

In part (c) you must show detailed reasoning.

Solutions relying on calculator technology are not acceptable.

(c) Express in the form $a + \text{i}b$ where a and b are real,

(i) $\dfrac{z+3\text{i}}{2-4\text{i}} = \dfrac{z+3i}{2-4i} \times \dfrac{2+4i}{2+4i} = \dfrac{(-4+6i)(2+4i)}{20} = \dfrac{-32-4i}{20}$

$= -\frac{8}{5} - \frac{1}{5}i$

(ii) z^2

$= (-4-3i)(-4+3i) = 16 - 24i - 9$

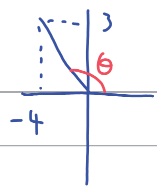
$= 7 - 24i$

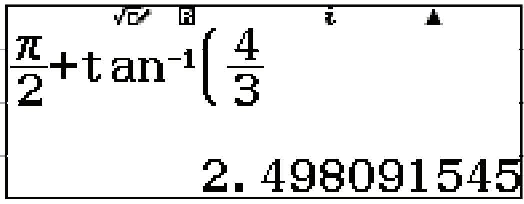
(5)

(d) Show on a single Argand diagram the points A, B, C and D that represent the complex numbers

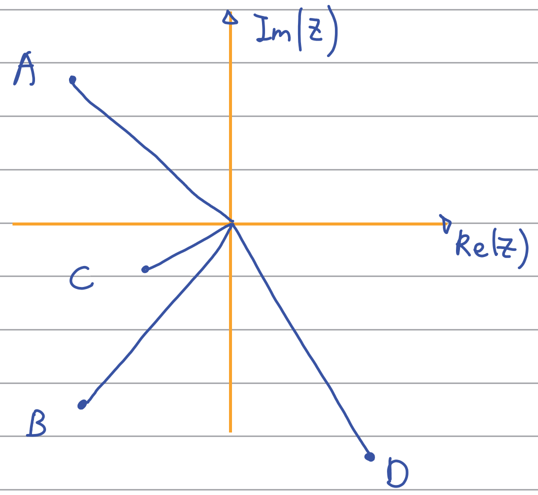
$$z,\ z^*,\ \frac{z+3\text{i}}{2-4\text{i}} \text{ and } z^2$$

(3)

(b) 



$\arg(z) = 2.5$

(d) 

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7. The matrix **A** is defined by

$$\mathbf{A} = \begin{pmatrix} 4 & -5 \\ -3 & 2 \end{pmatrix}$$

The transformation represented by **A** maps triangle T onto triangle T'

Given that the area of triangle T is 23 cm²

(a) determine the area of triangle T' $\quad S_{T'} = 23 \times |8 - 15| = 161$

(2)

The point P has coordinates $(3p + 2, 2p - 1)$ where p is a constant. The transformation represented by **A** maps P onto the point P' with coordinates $(17, -18)$

(b) Determine the value of p.

(2)

Given that

$$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(c) describe fully the single geometrical transformation represented by matrix **B**

(2)

The transformation represented by matrix **A** followed by the transformation represented by matrix **C** is equivalent to the transformation represented by matrix **B**

(d) Determine **C**

(3)

(b) $\begin{pmatrix} 4 & -5 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 3p+2 \\ 2p-1 \end{pmatrix} = \begin{pmatrix} 12p+8-10p+5 \\ -9p-6+4p-2 \end{pmatrix} = \begin{pmatrix} 2p+13 \\ -5p-8 \end{pmatrix}$

$2p + 13 = 17 \qquad -5p - 8 = -18$

$2p = 4 \qquad -5p = -10$

$p = 2 \qquad p = 2$

$p = 2$

(c) Rotation 90° clockwise about origin

Leave blank

Question 7 continued

(d) $B = CA$

$BA^{-1} = C$

$A^{-1} = -\frac{1}{7}\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$

$B \cdot A^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} -\frac{1}{7}\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix} = -\frac{1}{7}\begin{pmatrix} 3 & 4 \\ -2 & -5 \end{pmatrix}$

$C = -\frac{1}{7}\begin{pmatrix} 3 & 4 \\ -2 & -5 \end{pmatrix}$

Leave blank

8. The hyperbola H has Cartesian equation $xy = 25$

The parabola P has parametric equations $x = 10t^2$, $y = 20t$

The hyperbola H intersects the parabola P at the point A

(a) Use algebra to determine the coordinates of A

(3)

The point B with coordinates $(10, 20)$ lies on P

(b) Find an equation for the normal to P at B

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers to be determined.

(5)

(c) Use algebra to determine, in simplest form, the exact coordinates of the points where this normal intersects the hyperbola H

(6)

(a) $xy = 200t^3 = 25$

$t^3 = 1/8$ $\quad$ $A\left(5/2,\ 10\right)$

$t = 1/2$

(b) $t=1$ $\quad$ $\frac{dx}{dt} = 20t = 20$ $\quad$ $\boxed{\frac{dy}{dx} = 1}$

$\frac{dy}{dt} = 20$

NORMAL : $y - 20 = -(x-10)$

$x + y - 30 = 0$

(c) $x = \frac{25}{y}$: $\quad \frac{25}{y} + y - 30 = 0$

$y^2 - 30y + 25 = 0$

$y = \frac{30 \pm 20\sqrt{2}}{2} = 15 \pm 10\sqrt{2}$

$x = 15 \mp 10\sqrt{2}$

$(15 - 10\sqrt{2},\ 15 + 10\sqrt{2})$ $\quad$ $(15 + 10\sqrt{2},\ 15 - 10\sqrt{2})$

Leave blank

9. (i) A sequence of numbers $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{3}(2u_n - 1) \qquad u_1 = 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3\left(\frac{2}{3}\right)^n - 1$$

(6)

(ii) $f(n) = 2^{n+2} + 3^{2n+1}$

Prove by induction that, for $n \in \mathbb{Z}^+$, $f(n)$ is a multiple of 7

(6)

(i) $P(1)$ $\quad u_1 = 3(2/3) - 1 = 2 - 1 = 1$ $\quad P(1)$ TRUE.

ASSUME $P(n)$ TRUE $\quad u_n = 3(2/3)^n - 1$

CONSIDER: $\quad u_{n+1} = 1/3(2u_n - 1)$

$$= \frac{1}{3}\left(6(2/3)^n - 2 - 1\right)$$

$$= 2(2/3)^n - 1$$

$$= 3(2/3)^{n+1} - 1 \Rightarrow P(n+1)$$

$P(n) \rightarrow P(n+1)$ by induction.

$P(1)$ TRUE, so $P(n)$ TRUE FOR $n \in \mathbb{Z}^*$

(b) $P(1)$ $\quad f(1) = 2^3 + 3^3 = 8 + 27 = 35 = 7 \times 5$ $\quad$ TRUE

ASSUME $P(n)$ TRUE: $\quad f(n) = 2^{n+2} + 3^{2n+1} = 7k \quad k \in \mathbb{Z}$.

CONSIDER $\quad f(n+1) - f(n)$

$$= 2^{n+3} + 3^{2n+3} - 2^{n+2} - 3^{2n+1}$$

$$= 2 \cdot 2^{n+2} + 9 \cdot 3^{2n+1} - 2^{n+2} - 3^{2n+1}$$

$$= 2^{n+2} + 8 \cdot 3^{2n+1}$$

$$= 2^{n+2} + 3^{2n+1} + 7 \cdot 3^{2n+1}$$

$$= 7k + 7 \cdot 3^{2n+1}$$

$$= 7m, \quad m = k + 3^{2n+1} \in \mathbb{Z}$$

So $f(n+1)$ is a multiple of 7. $P(n) \rightarrow P(n+1)$ by induction.