Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Tuesday 14 January 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **WFM01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F1

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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1. $$\mathbf{A} = \begin{pmatrix} p & -5 \\ -2 & p+3 \end{pmatrix}$$

(a) Determine the values of the constant p for which $\mathbf{A}$ is singular. **(3)**

Given that $p = 3$

(b) determine $\mathbf{A}^{-1}$ **(3)**

(a) $\det(A) = p(p+3) - (-2)(-5) = 0$

$p^2 + 3p - 10 = 0$

$(p+5)(p-2) = 0$

$p = 2, -5$

(b) $A = \begin{pmatrix} 3 & -5 \\ -2 & 6 \end{pmatrix}$

$\det(A) = 18 - 10 = 8$

$A^{-1} = \frac{1}{8}\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$

Leave blank

2. Given that $x = -\frac{1}{3}$ is a root of the equation

$$3x^3 + kx^2 + 33x + 13 = 0 \qquad k \in \mathbb{R}$$

determine

(a) the value of k,

(2)

(b) the other 2 roots of the equation in the form $a + \mathrm{i}b$, where a and b are real numbers.

(4)

1a)
$$3\left(-\tfrac{1}{3}\right)^3 + k\left(-\tfrac{1}{3}\right)^2 + 33\left(-\tfrac{1}{3}\right) + 13 = 0$$

$$-\frac{1}{9} + k/9 - 11 + 13 = 0$$

$$-1 + k = -18$$

$$k = -17$$

(b)

$$\begin{array}{r|l} & x^2 \quad -6x \quad +13 \\ \hline 3x+1 & 3x^3 - 17x^2 + 33x + 13 \\ & \underline{3x^3 + x^2} \\ & -18x^2 + 33x \\ & \underline{-18x^2 - 6x} \\ & 39x + 13 \\ & \underline{39x + 13} \\ & 0 \end{array}$$

$$x^2 - 6x + 13 = 0$$

$$x = \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2}$$

$$= 3 \pm 2i$$

Leave blank

3. (a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that for all positive integers n

$$\sum_{r=1}^{n} r^2(2r+3) = \frac{n}{2}(n+1)(n^2+3n+1)$$

(4)

(b) Hence calculate the value of $\sum_{r=10}^{25} r^2(2r+3)$

(2)

(a) $$\sum_{r=1}^{n} r^2(2r+3) = \sum_{r=1}^{n} 2r^3 + 3r^2$$

$$= 2\left[\frac{1}{4}n^2(n+1)^2\right] + 3\left[\frac{1}{6}n(n+1)(2n+1)\right]$$

$$= \frac{1}{2}n^2(n+1)^2 + \frac{1}{2}n(n+1)(2n+1)$$

$$= \frac{n}{2}(n+1)\left[n^2+n+2n+1\right] = \frac{n}{2}(n+1)(n^2+3n+1)$$

(b) $$\sum_{r=10}^{25} r^2(2r+3) = \left(\sum_{r=1}^{25} - \sum_{r=1}^{9}\right) r^2(2r+3)$$

$$= \frac{25}{2}(26)(25^2+75+1) - \frac{9}{2}(10)(81+27+1)$$

$$= 222\,920$$

$$\sum_{x=10}^{25}(x^2(2x+3))$$

222920

Leave blank

4. $$z_1 = p + 5\text{i}, \quad z_2 = 9 + 8\text{i} \quad \text{and} \quad z_3 = \frac{z_1}{z_2}$$

where p is a real constant.

(a) Determine z_3 in the form $x + \text{i}y$, where x and y are in terms of p **(3)**

(b) Determine the exact value of the modulus of z_2 **(1)**

Given that the argument of z_1 is $\frac{\pi}{3}$

(c) (i) determine the exact value of p

(ii) determine the exact value of the modulus of z_3 **(3)**

1 a) $$z_3 = \frac{p+5i}{9+8i} \times \frac{9-8i}{9-8i}$$

$$= \frac{9p - 8pi + 45i + 40}{81+64}$$

$$= \frac{9p+40 + (45-8p)i}{145}$$

(b) $$|z_2| = \sqrt{9^2+8^2} = \sqrt{145}$$

(c) (i) $$\tan\left(\pi/3\right) = \frac{5}{p}$$

$$p = 5/\sqrt{3}$$

(ii) $$|z_1| = \sqrt{\frac{25}{3} + 25} = \frac{10}{\sqrt{3}}$$

$$|z_3| = \frac{10/\sqrt{3}}{\sqrt{145}} = \frac{2\sqrt{435}}{87}$$

Leave blank

5. $$f(x) = x^4 - 12x^{\frac{3}{2}} + 7 \qquad x \geqslant 0$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval [2, 3].

(2)

(b) Taking 2.5 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 2 decimal places.

(4)

(c) Show that your answer to (b) gives α correct to 2 decimal places.

(2)

(a)

2 → 2	$\text{Ans}^4-12\text{Ans}^{1.5}+7$ → -10.9411255
3 → 3	$\text{Ans}^4-12\text{Ans}^{1.5}+7$ → 25.64617093

$f(2) < 0$ f(x) is continuous and there is a sign change.

$f(3) > 0$ Hence there is a root between 2 and 3.

(b) $f'(x) = 4x^3 - 18x^{1/2}$

$$\alpha = 2.5 - \frac{f(2.5)}{f'(2.5)} =$$

$2.5-\frac{2.5^4-12(2.5)^{1.}}{4\times2.5^3-18\sqrt{2.}}$

2.540296269

$= 2.54$

(c)

$f($2.535$) < 0$

$f($2.545$) > 0$

2.535 → 2.535	$\text{Ans}^4-12\text{Ans}^{1.5}+7$ → -0.1373929334
2.545 → 2.545	$\text{Ans}^4-12\text{Ans}^{1.5}+7$ → 0.2312194188

Leave blank

6.

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$$

The transformation represented by **A** maps the point $R(3p - 13, p - 4)$, where p is a constant, onto the point $R'(7, -2)$

(a) Determine the value of p

(3)

The point S has coordinates $(0, 7)$

Given that O is the origin,

(b) determine the area of triangle ORS

(2)

The transformation represented by **A** maps the triangle ORS onto the triangle $OR'S'$

(c) Hence, using your answer to part (b), determine the area of triangle $OR'S'$

(2)

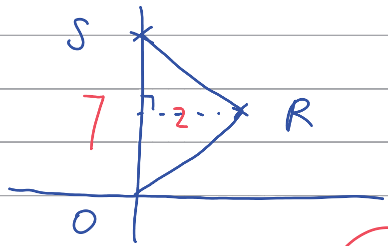
(a) $\begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} 3p-13 \\ p-4 \end{pmatrix} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}$

$6p - 26 + 3p - 12 = 9p - 38 = 7 \quad 9p = 45 \quad p = 5$

$3p - 13 - 4p + 16 = -p + 3 = -2$

(b) $R(2, 1)$

$S(0, 7)$



$S_{\triangle ORS} = \frac{1}{2} \times 7 \times 2 = 7$

(c) $\det(A) = -8 - 3 = -11$

$S_{\triangle OR'S'} = 11 \times 7 = 77$

Leave blank

7. The equation $3x^2 + px - 5 = 0$, where p is a constant, has roots α and β.

(a) Determine the value of

(i) $\alpha\beta$ $= -5/3$

(ii) $\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ $= \alpha\beta + 1 + 1 + \frac{1}{\alpha\beta} = -5/3 + 2 - 3/5 = -4/15$

(3)

(b) Obtain an expression, in terms of p, for

(i) $\alpha + \beta$ $= -p/3$

(ii) $\left(\alpha + \frac{1}{\beta}\right) + \left(\beta + \frac{1}{\alpha}\right)$ $= -p/3 + \frac{\alpha+\beta}{\alpha\beta} = -1/3\, p + \frac{-p/3}{-5/3}$

$= -1/3\, p + 1/5\, p = -2/15\, p$

(3)

Given that

$$\left(\alpha + \frac{1}{\beta}\right) + \left(\beta + \frac{1}{\alpha}\right) = 2\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$$

(c) determine the value of p.

(1)

(d) Using the value of p found in part (c), obtain a quadratic equation, with integer coefficients, that has roots $\left(\alpha + \frac{1}{\beta}\right)$ and $\left(\beta + \frac{1}{\alpha}\right)$

(2)

(c) $-\frac{2}{15} p = -8/15$

$p = 4$

(d)

Sum $= -2/15\,(4) = -8/15$

prod $= -4/15$

$x^2 + 8/15\, x - 4/15 = 0$

$15x^2 + 8x - 4 = 0$

Leave blank

8. A rectangular hyperbola, H, has Cartesian equation $xy = 16$

The point $P\left(4t, \dfrac{4}{t}\right)$, $t \neq 0$, lies on H.

(a) Use calculus to show that an equation of the normal to H at P is

$$ty - t^3x = 4 - 4t^4$$

(5)

The point A on H has parameter $t = 2$

The normal to H at A meets H again at the point B.

(b) Determine the exact value of the length of AB.

(6)

The tangent to H at A meets the y-axis at the point C.

(c) Determine the exact area of triangle ABC.

(3)

(a) $x \cdot y = 16$

$y + x\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = \dfrac{-y}{x} = \dfrac{-4/t}{4t} = -\dfrac{1}{t^2}$

NORMAL: $y - 4/t = t^2(x - 4t)$

$ty - 4 = t^3x - 4t^4$

$ty - t^3x = 4 - 4t^4$

(b) $A(8, 2)$ NORMAL: $2y - 8x = -60$

$\dfrac{32}{x} - 8x = -60$

$32 - 8x^2 = -60x$

$8x^2 - 60x - 32 = 0$

$2x^2 - 15x - 8 = 0$

$\begin{matrix} 2 & & +1 \\ 1 & x & -8 \end{matrix}$

$(2x+1)(x-8)$

Leave blank

Question 8 continued

$x = -\frac{1}{2}\ ,\ 8$

$B\left(-\frac{1}{2}\ ,\ -32\right)$

$A\left(8\ ,\ 2\right)$

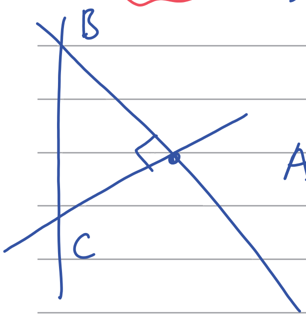
$AB =$ $\sqrt{8.5^2+34^2}$ $\quad \frac{17\sqrt{17}}{2}$

(c) TANGENT AT $A(8,2)$ $\quad t=2$

$y - 2 = -\frac{1}{4}(x-8)$

$C\ (x=0)$: $\quad y = 2+2 = 4$

$C(0,4)$



$AC = \sqrt{64+4} = \sqrt{68}$

$S_{\triangle ABC} = \frac{1}{2} \cdot \sqrt{68} \cdot \frac{17\sqrt{17}}{2}$

$= \frac{289}{2}$

Leave blank

9. (i) $$f(n) = 7^n(3n + 1) - 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$, f(n) is a multiple of 9

(6)

(ii) A sequence of numbers is defined by

$$u_1 = 2 \qquad u_2 = 6$$

$$u_{n+2} = 3u_{n+1} - 2u_n \qquad n \in \mathbb{Z}^+$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 2(2^n - 1)$$

(6)

(i) $f(1) = 7(3+1) - 1 = 28 - 1 = 27 = 3 \cdot 9$

$P(1)$ is true. Assume $P(n)$ is true.

So $f(n) = 7^n(3n+1) - 1 = 9k, \quad k \in \mathbb{Z}$

$P(n+1)$: $f(n+1) - f(n)$

$= \left[7^{n+1}(3n+4) - 1\right] - \left[7^n(3n+1) - 1\right]$

$= 7^{n+1}(3n+4) - 7^n(3n+1)$

$= 7 \cdot 7^n(3n+4) - 3n \cdot 7^n - 7^n$

$= 21n \cdot 7^n + 28 \cdot 7^n - 3n \cdot 7^n - 7^n$

$= 18n \cdot 7^n + 27 \cdot 7^n$

$= 9(2n \cdot 7^n + 3 \cdot 7^n)$

So $f(n+1)$ is also a multiple of 9.

$P(n) \Rightarrow P(n+1)$ by induction.

Leave blank

Question 9 continued

(ii) $n=1$ $\quad U_1 = 2(2^1-1) = 2$ TRUE.

$n=2$ $\quad U_2 = 2(2^2-1) = 6$ TRUE.

ASSUME :

$U_n = 2(2^n-1)$

CONSIDER $\quad U_{n+1} = 2(2^{n+1}-1)$

$U_{n+2} = 2(2^{n+2}-1)$

$U_{n+2} = 3U_{n+1} - 2U_n$

$2(2^{n+2}-1) = 3\cdot 2(2^{n+1}-1) - 2U_n$

$8\cdot 2^n - 2 = 6\cdot 2^{n+1} - 6 - 2U_n$

$8\cdot 2^n = 12\cdot 2^n - 4 - 2U_n$

$2U_n = 4\cdot 2^n - 4$

$U_n = 2\cdot 2^n - 2$

$U_n = 2(2^n-1)$

So $P(n) \rightarrow P(n+1)$ by induction.