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IAL FP1 MarkScheme

January 2020
WFM01/01 Further Pure Mathematics F1
Mark Scheme

Question Number	Scheme	Notes	Marks
1.	(a) $\mathbf{A} = \begin{pmatrix} p & -5 \\ -2 & p+3 \end{pmatrix}$ (b) $p=3$; $\mathbf{A} = \begin{pmatrix} a & -5 \\ -2 & d \end{pmatrix}$		
(a)	$\det(\mathbf{A}) = p(p+3)-(-5)(-2)$ $\{= p(p+3)-10\}$	Applies $p(p+3) \pm (-5)(-2)$	M1
	$p^2+3p-10=0 \Rightarrow (p+5)(p-2)=0 \Rightarrow p = ...$	Obtains a correct expression for $\det(\mathbf{A})$, sets their $\det(\mathbf{A}) = 0$ and solves their 3TQ $= 0$ by any valid method to give $p = ...$	M1
	$p=-5, 2$	$p=-5, 2$	A1
			(3)
(b)	$\left\{ p=3 \Rightarrow \mathbf{A} = \begin{pmatrix} 3 & -5 \\ -2 & 6 \end{pmatrix} \right\}$		
	For either $\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$ or $\det(\mathbf{A}) = 3(3+3)-10$ or 8	For **either** $\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$ **or** a correct numerical expression or value for $\det(\mathbf{A})$, which can be seen or implied	B1
	$\mathbf{A}^{-1} = \dfrac{1}{3(3+3)-(-5)(-2)}\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$	$\dfrac{1}{ad \pm (-5)(-2)}\text{Adj}(\mathbf{A})$, where a correct method has been employed for finding their $\text{Adj}(\mathbf{A})$	M1
	$\mathbf{A}^{-1} = \dfrac{1}{8}\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$ or $= \begin{pmatrix} \frac{3}{4} & \frac{5}{8} \\ \frac{1}{4} & \frac{3}{8} \end{pmatrix}$ or $= \begin{pmatrix} 0.75 & 0.625 \\ 0.25 & 0.375 \end{pmatrix}$ or $= \begin{pmatrix} \frac{6}{8} & \frac{5}{8} \\ \frac{2}{8} & \frac{3}{8} \end{pmatrix}$	Correct $\mathbf{A}^{-1}$	A1
			(3)
			6

	Question 1 Notes	
1. (b)	**Note**	$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \text{Adj}(\mathbf{A}) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ is a correct method for finding their $\text{Adj}(\mathbf{A})$
	Note	Allow B1 M1 A0 for just writing $\dfrac{1}{3(3+3)-(-5)(-2)}\begin{pmatrix} p+3 & 5 \\ 2 & p \end{pmatrix}$
	Note	Allow B0 M1 A0 for just writing $\dfrac{1}{3(3+3)+(-5)(-2)}\begin{pmatrix} p+3 & 5 \\ 2 & p \end{pmatrix}$
	Note	Allow B0 M1 A0 for just writing $\dfrac{1}{p(p+3)\pm(-5)(-2)}\begin{pmatrix} p+3 & 5 \\ 2 & p \end{pmatrix}$
	Note	Allow M1 for evidence of a correct numerical expression for $\det \mathbf{A} = ad \pm (-5)(-2)$ followed by $\dfrac{1}{\text{their } \det(A)}\text{Adj}(\mathbf{A})$ where a correct method has been employed for finding their $\text{Adj}(\mathbf{A})$
	Note	Give final A0 for $\dfrac{1}{18-10}\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$ without reference to $\dfrac{1}{8}\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$ or any other acceptable answer
	Note	Give B1 M1 A1 for writing down a correct final answer for $\mathbf{A}^{-1}$ from no working

Question Number	Scheme	Notes	Marks
2.	Let $\mathrm{f}(x)=3x^3+kx^2+33x+13$; $k\in\mathbb{R}$; $x=-\frac{1}{3}$ is a root of $\mathrm{f}(x)=0$		
	Note: Ignore labelling of parts when marking Q2		
(a) **Way 1**	$3\left(-\frac{1}{3}\right)^3+k\left(-\frac{1}{3}\right)^2+33\left(-\frac{1}{3}\right)+13=0\Rightarrow k=...$	Some evidence of substituting $x=-\frac{1}{3}$ into the given equation and solves to find $k=...$	M1
	$\left\{-\frac{1}{9}+\frac{1}{9}k-11+13=0\Rightarrow -1+k+18=0\Rightarrow\right\}$ $k=-17$	$k=-17$	A1
			(2)
(a) **Way 2**	$\mathrm{f}(x)=(3x+1)(x^2+Ax+13)$ x: $3(13)+A=33\Rightarrow A=-6$ x^2: $k=1+("-6")(3)$	Expresses $\mathrm{f}(x)=(3x\pm1)(x^2+Ax\pm13)$, equates x terms to find A and equates x^2 terms to find k	M1
	$k=-17$	$k=-17$	A1
			(2)
(b)	$\{\mathrm{f}(x)=\}$ $(3x+1)(x^2-6x+13)$ or $\{\mathrm{f}(x)=\}$ $\left(x+\frac{1}{3}\right)(3x^2-18x+39)$	Attempts to find the quadratic factor e.g. using long division to obtain $(3x\pm1)$ with $(x^2\pm qx+...)$ or $\left(x\pm\frac{1}{3}\right)$ with $(3x^2\pm qx+...)$; $q=$ value $\neq 0$ e.g. factorising/equating coefficients to obtain $\mathrm{f}(x)=(3x\pm1)(x^2\pm qx\pm r)$ or $\mathrm{f}(x)=\left(x\pm\frac{1}{3}\right)(3x^2\pm qx\pm r)$, $q=$ value $\neq 0$, r can be 0	M1
		$x^2-6x+13$ or $3x^2-18x+39$ seen in their working	A1
	$\{x^2-6x+13=0$ or $3x^2-18x+39=0\Rightarrow\}$		
	e.g. • $x=\dfrac{--6\pm\sqrt{(-6)^2-4(1)(13)}}{2(1)}$ • $(x-3)^2-9+13=0\Rightarrow x=...$	**dependent on the previous M mark** Correct method of applying the quadratic formula or completing the square **for solving their 3TQ** on their quadratic factor	dM1
	$\{x=\}$ $3\pm2\mathrm{i}$ (or $3\pm\mathrm{i}2$)	$3+2\mathrm{i}$ **and** $3-2\mathrm{i}$	A1
			(4)
			6

		Question 2 Notes
2. (b)	**Note**	You can assume $z\equiv x$ for solutions in this part
	Note	Give final dM1A1 for $x^2-6x+13=0$ or $3x^2-18x+39=0\Rightarrow x=3+2\mathrm{i},\,3-2\mathrm{i}$ with no intermediate working.
	Note	Give M1 A1 dM1 A1 for $3x^3-17x^2+33x+13=0\Rightarrow x=3+2\mathrm{i},\,3-2\mathrm{i}$ with no intermediate working.
	Note	They must be solving a 3TQ $"A"x^2+"B"x+"C"$ where A,B,C ***are all numerical values*** $\neq 0$ for the final dM1 mark.
	Note	**Special Case:** If ***their quadratic*** factor $x^2+"B"x+"C"$ **can** be factorised then allow dM1 for correct factorisation leading to $x=...$ Otherwise, give dM0 for applying a method of factorisation to solve their 3TQ = 0.

	Question 2 Notes Continued			
2. (b)	**Note**	**Reminder: Method mark for solving a 3TQ = 0** **Formula:** $Ax^2+Bx+C=0 \Rightarrow$ Attempt to use the correct formula (with values for A, B, C) **Completing the Square:** $x^2+Bx+C=0 \Rightarrow \left(x \pm \frac{B}{2}\right)^2 \pm q \pm C = 0,\ q \neq 0,$ leading to $x = \ldots$		
	Note:	**Comparing coefficients:** $\mathrm{f}(x)=(3x+1)(x^2+\alpha x+\beta) \equiv 3x^3-17x^2+33x+13$ $x^2: 3\alpha+1=-17 \Rightarrow \alpha=-6;\ \ z: 3\beta+\alpha=33 \Rightarrow 3\beta-6=33 \Rightarrow \beta=13;\ \ \text{constant}: \beta=13$ yielding quadratic factor = $x^2-6x+13$		
	Note	The solutions $3 \pm 2\mathrm{i}$ need to follow on from a correct $x^2-6x+13=0$ or $3x^2-18x+39=0$ in order to gain the final A mark.		
	Note	Give final A0 for writing $\frac{6 \pm 4\mathrm{i}}{2}$ followed by either $3 \pm 4\mathrm{i}$ or $6 \pm 2\mathrm{i}$		
2. (a) **ALT 1**	**Note**	**Long division:** $\begin{array}{r} 3x^2 \ -18x \ +39 \\ x+\tfrac{1}{3} \,\overline{)\, 3x^3 + kx^2 + 33x + 13} \\ \underline{3x^3 + x^2} \qquad\qquad \\ (k-1)x^2 + 33x \qquad \\ \underline{-18x^2 - 6x} \qquad \\ 39x + 13 \\ \underline{39x + 13} \\ 0 \end{array}$ or $\begin{array}{r} x^2 \ - \ 6x \ +13 \\ 3x+1 \,\overline{)\, 3x^3 + kx^2 + 33x + 13} \\ \underline{3x^3 + x^2} \qquad\qquad \\ (k-1)x^2 + 33x \qquad \\ \underline{-18x^2 - 6x} \qquad \\ 39x + 13 \\ \underline{39x + 13} \\ 0 \end{array}$		
		$(k-1)--18=0 \Rightarrow k=\ldots$	Full complete method of dividing by either $x+\frac{1}{3}$ or $(3x+1)$, applying remainder $=0$ and solving a relevant equation to find $k=\ldots$	M1
		$k=-17$	$k=-17$	A1
				(2)
	Note	Give M0 for dividing by either $x-\frac{1}{3}$ or $3x-1$		

Question 2 Notes Continued

2. (a) **ALT 2**

Note

Long division:

$$\begin{array}{r l} & x^2 + \left(\frac{k-1}{3}\right)x + \left(\frac{100-k}{9}\right) \\ 3x+1 \,|\, & 3x^3 + kx^2 + 33x + 13 \\ & \underline{3x^3 + x^2} \\ & (k-1)x^2 + 33x \\ & \underline{(k-1)x^2 + \left(\frac{k-1}{3}\right)x} \\ & \left(\frac{100-k}{3}\right)x + 13 \\ & \underline{\left(\frac{100-k}{3}\right)x + \left(\frac{100-k}{9}\right)} \\ & 13 - \left(\frac{100-k}{9}\right) \end{array}$$

or

$$\begin{array}{r l} & 3x^2 + (k-1)x + \left(\frac{100-k}{3}\right) \\ x+\tfrac{1}{3} \,|\, & 3x^3 + kx^2 + 33x + 13 \\ & \underline{3x^3 + x^2} \\ & (k-1)x^2 + 33x \\ & \underline{(k-1)x^2 + \left(\frac{k-1}{3}\right)x} \\ & \left(\frac{100-k}{3}\right)x + 13 \\ & \underline{\left(\frac{100-k}{3}\right)x + \left(\frac{100-k}{9}\right)} \\ & 13 - \left(\frac{100-k}{9}\right) \end{array}$$

$13-\left(\frac{100-k}{9}\right)=0 \Rightarrow k=...$ or $33-\left(\frac{k-1}{3}\right)=39 \Rightarrow k=...$	Full complete method of dividing by either $x+\frac{1}{3}$ or $(3x+1)$, applying remainder $=0$ and solving a relevant equation to find $k=...$	M1
$\left\{\frac{117-100+k}{9}=0 \Rightarrow\right\} k=-17$	$k=-17$	A1
		(2)

Note Give M0 for dividing by either $x-\frac{1}{3}$ or $3x-1$

Question Number	Scheme	Notes	Marks
3. (a)	$\sum_{r=1}^{n} r^2(2r+3) = 2\sum_{r=1}^{n} r^3 + 3\sum_{r=1}^{n} r^2$		
	$= 2\left(\frac{1}{4}n^2(n+1)^2\right) + 3\left(\frac{1}{6}n(n+1)(2n+1)\right)$	Attempts to expand $r^2(2r+3)$ and attempts to substitute at least one correct formula for either $\sum_{r=1}^{n} r^3$ or $\sum_{r=1}^{n} r^2$ into their resulting expression	M1
		Obtains an expression of the form $\alpha n^2(n+1)^2 + \beta n(n+1)(2n+1)$; $\alpha, \beta \neq 0$	M1
		$2\left(\frac{1}{4}n^2(n+1)^2\right) + 3\left(\frac{1}{6}n(n+1)(2n+1)\right)$ which can be simplified or un-simplified	A1
	$= \frac{1}{2}n(n+1)\big(n(n+1)+(2n+1)\big)$ $= \frac{1}{2}n(n+1)(n^2+3n+1)$ *	Achieves the given result via an appropriate intermediate step with no algebraic errors seen in their working	A1 * **cso**
			(4)
(b)	$\left\{\sum_{r=10}^{25} r^2(2r+3) = \right\}$	{**Note:** Let $f(n) = \frac{n}{2}(n+1)(n^2+3n+1)$ or their answer to part (a) or their un-simplified expression (for $f(n)$) of the form $\alpha n^2(n+1)^2 + \beta n(n+1)(2n+1)$; $\alpha, \beta \neq 0$}	
	$= \frac{25}{2}(25+1)((25)^2+3(25)+1) - \frac{9}{2}(9+1)((9)^2+3(9)+1)$	Applies $f(25) - f(9)$ **Note:** Give M0 for applying $f(25) - f(10)$	M1
	$\left\{= \frac{25}{2}(26)(701) - \frac{9}{2}(10)(109) = 227825 - 4905\right\}$		
	$= 222920$	222920 **cao**	A1
			(2)
			6

Question 3 Notes

3. (a)	**Note**	<u>**Final A mark:**</u> $\text{LHS} = \frac{1}{2}n^2(n+1)^2 + \frac{1}{2}n(n+1)(2n+1) = \frac{1}{2}n^2(n^2+2n+1) + \frac{1}{2}n(2n^2+3n+1)$ $= \frac{1}{2}n^4 + n^3 + \frac{1}{2}n^2 + n^3 + \frac{3}{2}n^2 + \frac{1}{2}n \;\; = \frac{1}{2}n^4 + 2n^3 + 2n^2 + \frac{1}{2}n$ $\text{RHS} = \frac{n}{2}(n+1)(n^2+3n+1) = \frac{n}{2}(n^3+3n^2+n+n^2+3n+1) = \frac{n}{2}(n^3+4n^2+4n+1)$ $= \frac{1}{2}n^4 + 2n^3 + 2n^2 + \frac{1}{2}n$ Give final A1 cso for using algebra to show that the LHS and RHS are the same with some acknowledgment (e.g. 'proved', LHS = RHS, QED or □) that their proof is complete.

	Question 3 Notes Continued	
3. (a)	**Note**	Give final A0 for • jumping from $\frac{1}{2}n^4 + 2n^3 + 2n^2 + \frac{1}{2}n$ to $\frac{n}{2}(n+1)(n^2+3n+1)$ with no intermediate working
	Note	Condone final A1 for • jumping from $\frac{n}{2}(n^3 + 4n^2 + 4n + 1)$ to $\frac{n}{2}(n+1)(n^2+3n+1)$ with no intermediate working
	Note	Achieving the given result via an appropriate intermediate step with no algebraic errors seen in their working includes e.g. • $2\left(\frac{1}{4}n^2(n+1)^2\right) + 3\left(\frac{1}{6}n(n+1)(2n+1)\right) = \frac{1}{2}n^2(n+1)^2 + \frac{1}{2}n(n+1)(2n+1)$ $= \frac{1}{2}n(n+1)(n^2+3n+1)$ • $2\left(\frac{1}{4}n^2(n+1)^2\right) + 3\left(\frac{1}{6}n(n+1)(2n+1)\right) = \frac{1}{2}n(n+1)(n^2+n) + \frac{1}{2}n(n+1)(2n+1)$ $= \frac{1}{2}n(n+1)(n^2+3n+1)$ • $2\left(\frac{1}{4}n^2(n+1)^2\right) + 3\left(\frac{1}{6}n(n+1)(2n+1)\right) = \frac{1}{2}n(n+1)[n(n+1)] + \frac{1}{2}n(n+1)(2n+1)$ $= \frac{1}{2}n(n+1)(n^2+3n+1)$
3. (b)	**Note**	Allow M1 for $227825 - 4905$ and A1 for obtaining 222920
	Note	Allow M1 for $\left(\frac{1}{2}(25)^2(26)^2 + \frac{1}{2}(25)(26)(51)\right) - \left(\frac{1}{2}(9)^2(10)^2 + \frac{1}{2}(9)(10)(19)\right)$ $\{= (211250 + 16575) - (4050 + 855) = 227825 - 4905\}$ and A1 for obtaining 222920
	Note	Give M0 A0 for writing 222920 by itself with no supporting working
	Note	Allow M1 A1 for writing $\sum_{r=1}^{25} r^2(2r+3) - \sum_{r=1}^{9} r^2(2r+3) = 222920$
	Note	Give M0 A0 for listing individual terms i.e. $\sum_{r=10}^{25} r^2(2r+3) = (10)^2(23) + (11)^2(25) + (12)^2(27) + ... + (25)^2(53)$ $= 2300 + 3025 + 3888 + ... + 33125 = 222920$ by itself is M0 A0
	Note	Give M0 A0 for applying $\text{f}(25) - \text{f}(10) = \frac{25}{2}(25+1)((25)^2 + 3(25)+1) - \frac{10}{2}(10+1)((10)^2 + 3(10)+1)$ $= \frac{25}{2}(26)(701) - 5(11)(131) = 227825 - 7205 = 220620$
	Note	For M1 allow only one slip when substituting in $n = 25$ and $n = 9$
	Note	Give M0 for • $\frac{25}{2}(25+1)((25)^2 + 3(25)+1) - \frac{9}{2}(9+1)((10)^2 + 3(10)+1)$ $\{= 227825 - 5895 = 221930\}$

Question Number	Scheme	Notes	Marks
4.	$z_1 = p + 5\mathrm{i},\ z_2 = 9 + 8\mathrm{i},\ z_3 = \dfrac{z_1}{z_2}$; $\arg(z_1) = \dfrac{\pi}{3}$		
(a) **Way 1**	$z_3 = \dfrac{(p+5\mathrm{i})}{(9+8\mathrm{i})} \times \dfrac{(9-8\mathrm{i})}{(9-8\mathrm{i})}$	Multiplies numerator **and** denominator of z_3 by $9-8\mathrm{i}$	M1
	$= \dfrac{9p - 8p\mathrm{i} + 45\mathrm{i} + 40}{81+64}$	Applies $\mathrm{i}^2 = -1$ to give either • a correct expression in terms of p for the numerator **or** • a correct numerical expression or value for the denominator	A1
	$= \dfrac{9p+40}{145} + \left(\dfrac{-8p+45}{145}\right)\mathrm{i}$	Correct answer written in the form $x + \mathrm{i}y$ o.e. **or** writes a correct $x = \dfrac{9p+40}{145},\ y = \dfrac{-8p+45}{145}$	A1
			(3)
(a) **Way 2**	$z_3 = \dfrac{(p+5\mathrm{i})}{(9+8\mathrm{i})} \times \dfrac{(-9+8\mathrm{i})}{(-9+8\mathrm{i})}$	Multiplies numerator **and** denominator of z_3 by $-9+8\mathrm{i}$	M1
	$= \dfrac{-9p + 8p\mathrm{i} - 45\mathrm{i} - 40}{-81-64}$	Applies $\mathrm{i}^2 = -1$ to give either • a correct expression in terms of p for the numerator **or** • a correct numerical expression or value for the denominator	A1
	$= \dfrac{-9p-40}{-145} + \left(\dfrac{8p-45}{-145}\right)\mathrm{i}$	Correct answer written in the form $x + \mathrm{i}y$ o.e. **or** writes a correct $x = \dfrac{-9p-40}{-145}$ and $y = \dfrac{8p-45}{-145}$	A1
			(3)
(b)	$\left\{\lvert z_2\rvert = \sqrt{9^2+8^2} \Rightarrow\right\}\ \lvert z_2\rvert = \sqrt{145}$	$\sqrt{145}$	B1
			(1)
(c)(i) **Way 1**	$\left\{\arg(z_1) = \dfrac{\pi}{3} \Rightarrow\right\}$		
	e.g. $\arctan\left(\dfrac{5}{p}\right) = \dfrac{\pi}{3}$ or $\tan\left(\dfrac{\pi}{3}\right) = \dfrac{5}{p}$ or $\sqrt{3} = \dfrac{5}{p}$	Uses trigonometry to form a correct equation in p	M1
	$p = \dfrac{5}{\sqrt{3}}$ or $\dfrac{5}{3}\sqrt{3}$ or $\sqrt{\dfrac{25}{3}}$	Correct exact value for p **Note:** You can apply isw	A1
(c)(i) **Way 2**	$\left\{z_1 = \sqrt{p^2+25}\left(\cos\dfrac{\pi}{3} + \mathrm{i}\sin\dfrac{\pi}{3}\right) = p + 5\mathrm{i} \Rightarrow\right\}$		
	e.g. $\sqrt{p^2+25}\left(\cos\dfrac{\pi}{3}\right) = p$ or $\sqrt{p^2+25}\left(\sin\dfrac{\pi}{3}\right) = 5$	Uses trigonometry to form a correct equation in p	M1
	$p = \dfrac{5}{\sqrt{3}}$ or $\dfrac{5}{3}\sqrt{3}$ or $\sqrt{\dfrac{25}{3}}$	Correct exact value for p **Note:** You can apply isw	A1
(ii)	• $\lvert z_3\rvert = \dfrac{\lvert z_1\rvert}{\lvert z_2\rvert} = \dfrac{\sqrt{\left(\dfrac{5}{\sqrt{3}}\right)^2 + (5)^2}}{\sqrt{145}} = \dfrac{\sqrt{\dfrac{100}{3}}}{\sqrt{145}}$ • $z_3 = \dfrac{8+3\sqrt{3}}{29} + \dfrac{27-8\sqrt{3}}{87} \Rightarrow \lvert z_3\rvert = \sqrt{\left(\dfrac{8+3\sqrt{3}}{29}\right)^2 + \left(\dfrac{27-8\sqrt{3}}{87}\right)^2}$		
	$\lvert z_3\rvert = \dfrac{10}{\sqrt{435}}$ or $\dfrac{10}{435}\sqrt{435}$ or $\dfrac{2}{87}\sqrt{435}$ or $\dfrac{2\sqrt{435}}{87}$	Correct exact answer written in the form $\dfrac{a}{\sqrt{b}}$ or $c\sqrt{b}$; $a, b \in \mathbb{Z},\ c \in \mathbb{Q}$	B1
	Note: Give B1 for $\lvert z_3\rvert = \sqrt{\dfrac{20}{87}}$		**(3)**
			7

	Question 4 Notes	
4. (a)	**Note**	Give 2nd A0 for $z_3 = \frac{9p+40}{81+64} + \left(\frac{-8p+45}{81+64}\right)\text{i}$ without reference to $z_3 = \frac{9p+40}{145} + \left(\frac{-8p+45}{145}\right)\text{i}$
	Note	$\frac{9p+40+(45-8p)\text{i}}{145}$ is not considered to be in the form $x+\text{i}y$
	Note	Allow final A1 for $z_3 = \frac{9p}{145} + \frac{8}{29} + \left(\frac{9}{29} - \frac{8p}{145}\right)\text{i}$
	Note	Allow final A1 for $z_3 = \frac{9p+40}{145} - \left(\frac{8p-45}{145}\right)\text{i}$
	Note	y written as $y = \left(\frac{-8p+45}{145}\right)\text{i}$ is incorrect
	Note	M1 A1 can be implied for writing $z_3 = \frac{(p+5\text{i})}{(9+8\text{i})} = \frac{9p-8p\text{i}}{145} + \frac{8+9\text{i}}{29}$ and final A1 is then given for $z_3 = \frac{9p}{145} + \frac{8}{29} + \left(\frac{9}{29} - \frac{8p}{145}\right)\text{i}$
(b)	**Note**	You can apply isw after seeing $\sqrt{145}$
	Note	Give B0 for writing 12, 12.0 or awrt 12.0 without reference to $\sqrt{145}$
(c)(i)	**Note**	Give M1 for any of $\arctan\left(\frac{5}{p}\right) = 60$, $\tan 60 = \frac{5}{p}$, $\arctan\left(\frac{p}{5}\right) = \frac{\pi}{6}$, $\tan 30 = \frac{p}{5}$
	Note	Give M1 A0 for $p = 2.88$ (truncated) or $p =$ awrt 2.89 without reference to a correct exact value
	Note	Give A0 for $p = \pm\frac{5}{\sqrt{3}}$ with no evidence of rejecting the negative value of p
(c)(ii)	**Note**	Allow B1 for $\lvert z_3 \rvert = \frac{\sqrt{1740}}{87}$

Question Number	Scheme	Notes	Marks
5.	$\mathrm{f}(x) = x^4 - 12x^{\frac{3}{2}} + 7\,;\quad x \geqslant 0$		
(a) **Way 1**	$\mathrm{f}(2) = -10.9411255...$ $\mathrm{f}(3) = \ \ 25.64617093...$	Attempts to evaluate both $\mathrm{f}(2)$ and $\mathrm{f}(3)$ **and** either $\mathrm{f}(2) = -10$ (truncated) or awrt -11 **or** $\mathrm{f}(3) = 25$ (truncated) or awrt 26	M1
	Sign change {negative, positive} {and $\mathrm{f}(x)$ is continuous} therefore a root $\{\alpha\}$ exists in the interval {[2, 3]}	Both values correct awrt (or truncated) to 2 sf, reason and a valid conclusion	A1 **cso**
			(2)
(b)	$\mathrm{f}'(x) = 4x^3 - 18x^{\frac{1}{2}}$	At least one of either $x^4 \to \pm Ax^3$ or $-12x^{\frac{3}{2}} \to \pm Bx^{\frac{1}{2}}$; $A, B \neq 0$	M1
		Correct differentiation, which can be un-simplified or simplified	A1
	$\left\{\alpha \simeq 2.5 - \dfrac{\mathrm{f}(2.5)}{\mathrm{f}'(2.5)} \Rightarrow\right\}\ \alpha \simeq 2.5 - \dfrac{(2.5)^4 - 12(2.5)^{\frac{3}{2}} + 7}{4(2.5)^3 - 18(2.5)^{\frac{1}{2}}}$	**dependent on the previous M mark** Valid attempt at Newton-Raphson using the applied $\mathrm{f}(2.5)$ and their applied $\mathrm{f}'(2.5)$	dM1
	$\left\{\alpha \simeq 2.5 - \dfrac{-1.3716649...}{34.0395011...} = 2.5 + 0.0402962...\right\}$		
	$\alpha = 2.54$ (2 dp)	**dependent on all 3 previous marks** 2.54 on first iteration (Ignore any subsequent iterations)	A1 **cao** **cso**
	Correct differentiation followed by 2.54 (with no working seen) scores full marks in part (b)		**(4)**
(c) **Way 1**	$\mathrm{f}(2.535) = -0.137392933...$ $\mathrm{f}(2.545) = \ \ 0.231219419...$	Chooses a suitable interval $[x_L, x_U]$ for x, which is within ± 0.005 and either side of their answer to (b) and attempts to find either $\mathrm{f}(x_L)$ or $\mathrm{f}(x_U)$	M1
	Sign change {negative, positive} {and $\mathrm{f}(x)$ is continuous} therefore (a root) $\alpha = 2.54$ {2 dp}	Both values correct awrt 1 sf, reason and a valid conclusion	A1
			(2)
(c) **Way 2**	**Condoned Method: Applying Newton-Raphson again.** E.g. Using $\alpha = 2.54,\ 2.5402962...$		
	• $\alpha \simeq 2.54 - \dfrac{0.046101609...}{36.8609766...} = 2.538751631...$ • $\alpha \simeq 2.5402962... - \dfrac{0.05693746...}{36.88822382...} = 2.538752436...$	Evidence of applying Newton-Raphson for a second time on their answer to part (b)	M1
	So $\alpha = 2.54$ (2 dp)	Obtains either a truncated 2.538 or awrt 2.539 and a valid conclusion	A1
	Note: Work for Way 2 can be recovered in part (b)		**(2)**
			8

		Question 5 Notes
5. (a)	**Note**	**Way 1: A1, correct solution only** Required to state **both** values for $\mathrm{f}(2)$ and $\mathrm{f}(3)$ correct awrt (or truncated) to 2 sf along with **a reason and a conclusion.** Reference to change of sign **or** e.g. $\mathrm{f}(2) \times \mathrm{f}(3) < 0$ **or** $\mathrm{f}(2) < 0 < \mathrm{f}(3)$ **or** a diagram **or** < 0 and > 0 **or** one negative, one positive are sufficient reasons. There must be a conclusion, e.g. $\{x \text{ or}\}\ \alpha \in [2, 3]$ or $\{x \text{ or}\}\ \alpha \in (2, 3)$ or root lies between 2 and 3. Ignore the presence or absence of any reference to continuity.
	Note	A minimal acceptable reason and conclusion is "change of sign, so $\alpha \in [2, 3]$" **or** "change of sign, so root is between 2 and 3" **or** "change of sign, so root" **or** "$\mathrm{f}(2) = -10.9 < 0,\ \mathrm{f}(3) = 25.6 > 0$, so root" **or** "change of sign, so in the interval"

		Question 5 Notes Continued
5. (a)	**Note**	Give final A0 for writing as their conclusion "root lies between $\mathrm{f}(2)$ and $\mathrm{f}(3)$"
5. (a)	**Note**	**ALT** The root of $\mathrm{f}(x)=0$ is $2.5388...$, so they can choose x_1 which is less than $2.5388...$, and choose x_2 which is greater than $2.5388...$ with both x_1 and x_2 lying in the interval $[2, 3]$. **M1:** Finds $\mathrm{f}(x_1)$ and $\mathrm{f}(x_2)$ with one of these values correct awrt (or truncated) to 2 sf **A1:** Both values correct awrt (or truncated) to 2 sf, reason (e.g. sign change) and conclusion
	Note	**Helpful Table** (see below)
(b)	**dM1**	This mark can be implied by applying at least one correct *value* of either $\mathrm{f}(2.5)$ or their $\mathrm{f}'(2.5)$ (where $\mathrm{f}'(2.5)$ is found using their $\mathrm{f}'(x)$) to awrt 2 significant figures in $2.5-\dfrac{\mathrm{f}(2.5)}{\mathrm{f}'(2.5)}$. So ***just writing*** $2.5-\dfrac{\mathrm{f}(2.5)}{\mathrm{f}'(2.5)}$ with an incorrect ft answer on their $\mathrm{f}'(2.5)$ scores dM0 A0.
	Note	Allow M1 A1 dM1 A1 for $2.5-\dfrac{\mathrm{f}(2.5)}{\mathrm{f}'(2.5)}=2.54$ with no algebraic differentiation
	Note	Allow M1 A1 dM1 A1 for correct answer 2.54 given with no other working
	Note	You can imply the M1 A1 marks for the absence of algebraic differentiation by either • $\mathrm{f}'(2.5)=4(2.5)^3-18(2.5)^{\frac{1}{2}}$ • $\mathrm{f}'(2.5)$ applied correctly in $\alpha \simeq 2.5-\dfrac{(2.5)^4-12(2.5)^{\frac{3}{2}}+7}{4(2.5)^3-18(2.5)^{\frac{1}{2}}}$ • $\mathrm{f}'(2.5)=$ awrt 34
	Note	**Differentiating INCORRECTLY to give** $\mathrm{f}'(x)=4x^3+18x^{\frac{1}{2}}$ leads to $\alpha \simeq 2.5-\dfrac{-1.3716649...}{90.9604989...}=2.51507978...=2.52$ (2 dp) **This response should be given M1 A0 dM1 A0**
	Note	**Differentiating INCORRECTLY to give** $\mathrm{f}'(x)=4x^3+18x^{\frac{1}{2}}$ and $\alpha \simeq 2.5-\dfrac{\mathrm{f}(2.5)}{\mathrm{f}'(2.5)}=2.52$ is M1 A0 dM1 A0

Helpful Table

x	$\mathrm{f}(x)$
2	$-10.9411255...$
2.1	$-10.0701694...$
2.2	$-8.731928012...$
2.3	$-6.873372451...$
2.4	$-4.439168148...$
2.5	$-1.371664903...$
2.6	$2.389111651...$
2.7	$6.90546741...$
2.8	$12.24204622...$
2.9	$18.46583545...$
3	$25.64617093...$

	Question 5 Notes Continued	
5. (c)	**Note**	If they obtain a correct answer 2.54 by an incorrect method in part (b) then M1 A1 is allowed in part (c).
	Note	**Way 1: A1, correct solution only** Required to state **both** values for $\mathrm{f}(x_L)$ and $\mathrm{f}(x_U)$ correct awrt (or truncated) to 1 sf along with **a reason and a conclusion.** Reference to change of sign **or** e.g. $\mathrm{f}(2.535)\times\mathrm{f}(2.545)<0$ **or** $\mathrm{f}(2.535)<0<\mathrm{f}(2.545)$ **or** a diagram **or** <0 and >0 **or** one negative, one positive are sufficient reasons. There must be a (minimal, not incorrect) conclusion e.g. $\alpha=2.54$, root (or α to part (b)) is correct, QED or □ are all acceptable. Ignore the presence or absence of any reference to continuity.
	Note	A minimal acceptable reason and conclusion is any of • "change of sign, hence root" • "change of sign, so $\alpha=2.54$" • "change of sign, so $x=2.54$" • "change of sign, so α is correct {to 2 decimal places}" • "$\mathrm{f}(2.535)=-0.1<0,\ \mathrm{f}(2.545)=0.2>0,$ so root" • "$\mathrm{f}(2.535)=-0.1<0,\ \mathrm{f}(2.545)=0.2>0,$ so $\alpha=2.54$"
	Note	No explicit reference to 2 decimal places is necessary for the conclusion
	Note	Give A0 for stating "root is in between 2.535 and 2.545" or "root lies in the given interval" without reference to either $\alpha=2.54$, root (or α to part (b)) is correct, QED or □
(c)	**Note**	**Way 1: ALT** The root of $\mathrm{f}(x)=0$ is $2.5388...$, so they can choose x_L which is less than $2.5388...$, and choose x_U which is greater than $2.5388...$ with both x_L and x_U lying in the interval $[2.535, 2.545]$ and evaluate $\mathrm{f}(x_L)$ and $\mathrm{f}(x_U)$ **M1:** Chooses a suitable interval $[x_L, x_U]$ and attempts to find either $\mathrm{f}(x_L)$ or $\mathrm{f}(x_U)$ **A1:** Both values correct awrt (or truncated) to 1 sf, reason (e.g. sign change) and conclusion
	Note	**Helpful Table** (see below)
(c) **Way 2**	**Note**	If $\alpha=2.54$ in part (b), then give M1 A1 in part (c) for any of • "$\alpha_2=2.538\Rightarrow\alpha_2=2.54$" • "$\alpha_2=2.539\Rightarrow\alpha_2=2.54$" • "$\alpha_2=2.539$, so answer to part (b) is correct"
	Note	If $\alpha=2.54$ in part (b), then give M1 A0 in part (c) for writing "$\alpha\simeq 2.54-\dfrac{\mathrm{f}(2.54)}{\mathrm{f}'(2.54)}=2.54$"

Helpful Table

x	$\mathrm{f}(x)$
2.535	$-0.137392933...$
2.536	$-0.100854301...$
2.537	$-0.064244144...$
2.538	$-0.027562401...$
2.539	$0.00919099...$
2.54	$0.046016091...$
2.541	$0.082912964...$
2.542	$0.119881671...$
2.543	$0.156922274...$
2.544	$0.194034836...$
2.545	$0.231219419...$

Question Number	Scheme	Notes	Marks
6.	$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$; $A: R(3p-13,\, p-4) \mapsto R'(7,-2)$		
(a) **Way 1**	$\left\{\begin{pmatrix} x_{R'} \\ y_{R'} \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} 3p-13 \\ p-4 \end{pmatrix} = \right\}$ $= \begin{pmatrix} 2(3p-13)+3(p-4) \\ 1(3p-13)-4(p-4) \end{pmatrix}$	Correct method of multiplying out either $\begin{pmatrix} 2 & 3 \end{pmatrix}\begin{pmatrix} 3p-13 \\ p-4 \end{pmatrix}$ or $\begin{pmatrix} 1 & -4 \end{pmatrix}\begin{pmatrix} 3p-13 \\ p-4 \end{pmatrix}$ to give a linear expression in terms of p for either $x_{R'}$ or $y_{R'}$ **Note:** Allow one slip in their multiplication	M1
	• $2(3p-13)+3(p-4)=7 \Rightarrow p=\ldots$ • $1(3p-13)-4(p-4)=-2 \Rightarrow p=\ldots$	**dependent on the previous M mark** Solves either their $x_{R'}=7$ or their $y_{R'}=-2$ to give $p=\ldots$	dM1
	$\{9p-38=7$ or $-p+3=-2 \Rightarrow\}$ $p=5$	$p=5$	A1
			(3)
(a) **Way 2**	$\{\mathbf{AR}=\mathbf{R}' \Rightarrow \mathbf{R}=\mathbf{A}^{-1}\mathbf{R}' \Rightarrow\}$		
	$\mathbf{R} = \dfrac{1}{-8-3}\begin{pmatrix} -4 & -3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 7 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$	Applies $\mathbf{A}^{-1}\begin{pmatrix} 7 \\ -2 \end{pmatrix}$ to find the value for either x_R or y_R **Note:** Allow one slip in finding $\mathbf{A}^{-1}$	M1
	• $3p-13=2 \Rightarrow p=\ldots$ • $p-4=1 \Rightarrow p=\ldots$	**dependent on the previous M mark** Solves either $3p-13=$ their x_R or $p-4=$ their y_R to give $p=\ldots$	dM1
	$p=5$	$p=5$	A1
			(3)
(a) **Way 3**	$\{\mathbf{AR}=\mathbf{R}' \Rightarrow\}$ $\begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}$ $\begin{matrix} 2a+3b=7 \\ a-4b=-2 \end{matrix} \Rightarrow a=2$ or $b=1$	Correct method of applying $\begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}$ to form a pair of simultaneous equations and attempts to find either $a=\ldots$ or $b=\ldots$ **Note:** Allow one slip in their multiplication	M1
	• $3p-13=2 \Rightarrow p=\ldots$ • $p-4=1 \Rightarrow p=\ldots$	**dependent on the previous M mark** Solves either $3p-13=$ their a or $p-4=$ their b to give $p=\ldots$	dM1
	$p=5$	$p=5$	A1
			(3)
(b) **Way 1**	$\{R(3(5)-13,\, 5-4) = R(2,1)\}$ $\{\text{Area}(ORS)=\}$ $\dfrac{1}{2}(7)(\text{"2"})$	A correct method for finding their x_R **and** applies $\dfrac{1}{2}(7)(\text{their } x_R)$	M1
	$= 7 \text{ (units)}^2$	7	A1 cao
			(2)
(c)	$\{\text{Area}(OR'S')=\}$ $\lvert 2(-4)-3(1)\rvert \times (7)$	$\pm\,(2(-4)-3(1)) \times (\text{their area}(ORS))$	M1
	$= 77$	Correct answer of 77, which must be positive. Only allow follow through of the value for $11\times$ their positive answer to (b)	A1 ft
			(2)
			7

Question Number	Scheme	Notes	Marks
6. (b) **Way 2**	{Area (ORS)} $=\frac{1}{2}\begin{vmatrix}0 & 2 & 0 & 0\\ 0 & 1 & 7 & 0\end{vmatrix}=\frac{1}{2}\lvert(0+14+0)-(0+0+0)\rvert$	A correct method for finding their $R(2,\ 1)$ with a complete applied method for finding area(ORS) using $S(0,\ 7)$ and their $R(2,\ 1)$	M1
	$= 7\ \text{(units)}^2$	7	A1 **cao**
			(2)

		Question 6 Notes
6.	**Note**	$ORS \mapsto OR'S' \Rightarrow \begin{pmatrix}0 & 2 & 0\\ 0 & 1 & 7\end{pmatrix} \mapsto \begin{pmatrix}0 & 7 & 21\\ 0 & -2 & -28\end{pmatrix}$
(b) **Way 1**	**Note**	A correct method for finding their x_R includes any of • $x_R = 3(\text{"5"}) - 13 = 2$, where $p = \text{"5"}$ is found using part (a), Way 1 • their x_R found by applying $\mathbf{A}^{-1}\mathbf{R}'$ using part (a), Way 2 • $x_R =$ their a found using part (a), Way 3
(b) **Way 2**	**Note**	Give M1 A1 for $\frac{1}{2}\begin{vmatrix}2 & 0\\ 1 & 7\end{vmatrix}=\frac{1}{2}\lvert 14-0\rvert = 7$
	Note	Give M0 A0 for $\begin{vmatrix}0 & 2 & 0 & 0\\ 0 & 1 & 7 & 0\end{vmatrix}=\lVert(0+14+0)-(0+0+0)\rVert = 14$
	Note	There are other ways to find Area(ORS). All ways require a complete correct method for the M mark and a correct area of 7 for the A mark.
	Note	Give M1 for $\frac{1}{2}(1)(\text{"2"}) + \frac{1}{2}(6)(\text{"2"})$ as this method is equivalent to writing $\frac{1}{2}(7)(\text{"2"})$
	Note	Give M0 for the calculation $\frac{1}{2}(7)(7)\ \left\{=\frac{49}{2}\right\}$
(c)	**Note**	Give M1 A0 for applying $(2(-4)-3(1))\times(7)$ to give -77 with no reference to 77
	Note	Part (c) requires the use of the answer to part (b). So give M0 A0 for • Area $(OR'S')=\frac{1}{2}\begin{vmatrix}0 & 7 & 21 & 0\\ 0 & -2 & -28 & 0\end{vmatrix}=\frac{1}{2}\lvert(0-196+0)-(0-42+0)\rvert=\frac{1}{2}(154)=77$ • Area $(OR'S')=\frac{1}{2}\begin{vmatrix}7 & 21\\ -2 & -28\end{vmatrix}=\frac{1}{2}\lvert(-196)-(-42)\rvert=\frac{1}{2}(154)=77$ • Area $(OR'S')=(28)(21)-\frac{1}{2}(21)(28)-\frac{1}{2}(7)(2)-\frac{1}{2}(2+28)(14)$ $= 588-294-7-210 = 77$
	Note	Allow M1 A1 for • $\dfrac{\begin{vmatrix}7 & 21\\ -2 & -28\end{vmatrix}}{\begin{vmatrix}2 & 0\\ 1 & 7\end{vmatrix}}\times 7 = \dfrac{\lvert(-196)-(-42)\rvert}{\lvert 14-0\rvert}\times 7 = \dfrac{154}{14}\times 7 = 11\times 7 = 77$

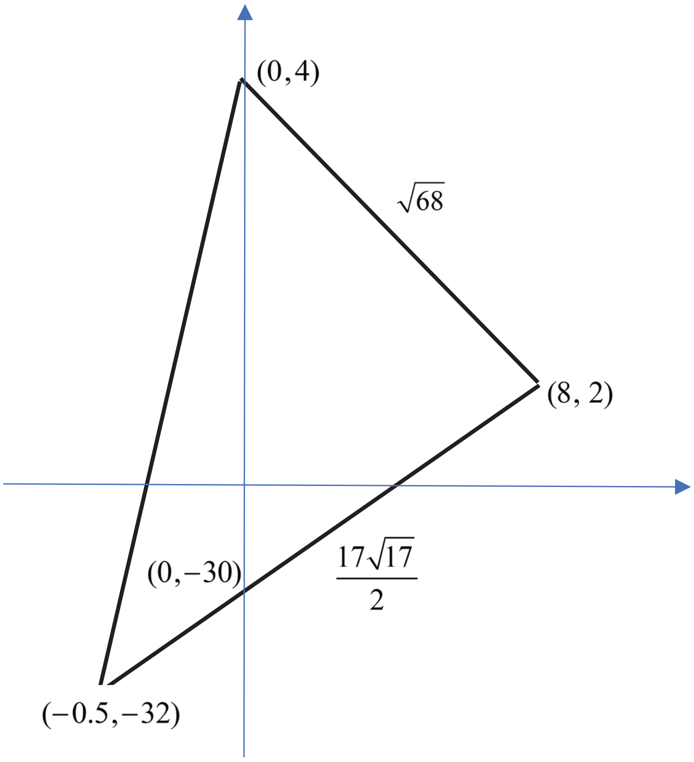
Question Number	Scheme	Notes	Marks
7.	$3x^2 + px - 5 = 0$ has roots α, β; p is a constant (c) $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)=2\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$		
(a) (i)	$\alpha\beta = -\frac{5}{3}$	$\alpha\beta = -\frac{5}{3}$	B1
(ii)	$\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$ $= \alpha\beta + 2 + \frac{1}{\alpha\beta} = -\frac{5}{3} + 2 + \frac{1}{\left(-\frac{5}{3}\right)}$	Expands to give $\frac{1}{\alpha\beta} + 1 + 1 + \alpha\beta$; and uses their value of $\alpha\beta$ at least once in a resulting expression	M1
	$= -\frac{4}{15}$	$-\frac{4}{15}$	A1
			(3)
(b)(i)	$\alpha + \beta = -\frac{p}{3}$	$\alpha + \beta = -\frac{p}{3}$ (may be recovered from (a))	B1
(ii)	$\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right) = \alpha + \beta + \frac{\alpha+\beta}{\alpha\beta}$	Evidence of $\frac{1}{\beta}+\frac{1}{\alpha}$ rewritten as $\frac{\alpha+\beta}{\alpha\beta}$ Can be implied	M1
	$= -\frac{p}{3} + \frac{-\frac{p}{3}}{-\frac{5}{3}}$ or $-\frac{p}{3}+\frac{p}{5}$ or $-\frac{2p}{15}$	$-\frac{p}{3} + \frac{-\frac{p}{3}}{-\frac{5}{3}}$ or $-\frac{p}{3}+\frac{p}{5}$ or $-\frac{2p}{15}$ or an equivalent fraction in terms of p **Note:** You can apply isw	A1
			(3)
(c)	$-\frac{2p}{15} = 2\left(-\frac{4}{15}\right) \Rightarrow p = 4$	Correctly obtains $p = 4$	B1
			(1)
(d)	$\sum = 2\left(-\frac{4}{15}\right) = -\frac{8}{15}$; $\prod = -\frac{4}{15}$		
	$x^2 - -\frac{8}{15}x - \frac{4}{15} = 0$	Valid method for finding (their sum) **and applies** $x^2 - (\text{their sum})x + \text{their product}$ (can be implied), for their numerical values of the sum and product. **Note:** " $=0$ " is not required for this mark **Note:** E.g. Using $(\text{their sum}) = \alpha + \beta = -\frac{p}{3} = -\frac{4}{3}$ is not considered a valid method for finding (their sum)	M1
	$15x^2 + 8x - 4 = 0$	***Any integer multiple*** of $15x^2 + 8x - 4 = 0$, including the " $=0$ "	A1 **cso**
			(2)
			9

Question Number	Scheme	Notes	Marks
(a)(ii) **Way 2**	$\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$ $=\frac{(\alpha\beta+1)(\alpha\beta+1)}{\alpha\beta}=\frac{\left(-\frac{5}{3}+1\right)\left(-\frac{5}{3}+1\right)}{\left(-\frac{5}{3}\right)}=\frac{\frac{4}{9}}{-\frac{5}{3}}$	Expands to give $\frac{(\alpha\beta+1)(\alpha\beta+1)}{\alpha\beta}$ and uses their value of $\alpha\beta$ at least once in a resulting expression	M1
	$=-\frac{4}{15}$	$-\frac{4}{15}$	A1
(b)(ii) **Way 2**	$\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)$ $=\frac{(\alpha\beta+1)}{\beta}+\frac{(\alpha\beta+1)}{\alpha}=\frac{\alpha^2\beta+\alpha+\alpha\beta^2+\beta}{\alpha\beta}$	Embedded evidence of $\frac{1}{\beta}+\frac{1}{\alpha}$ rewritten as $\frac{\alpha+\beta}{\alpha\beta}$ Can be implied	M1
	$=\frac{\alpha\beta(\alpha+\beta)+\alpha+\beta}{\alpha\beta}$		
	$=\frac{(-\frac{5}{3})(-\frac{p}{3})+(-\frac{p}{3})}{(-\frac{5}{3})}$ or $\frac{\frac{5p}{9}-\frac{p}{3}}{-\frac{5}{3}}$ or $\frac{\frac{2p}{9}}{-\frac{5}{3}}$ or $-\frac{2p}{15}$	Correct expression in terms of p **Note:** You can apply isw	A1

	Question 7 Notes	
7. (d)	**Note**	Valid method for finding (their sum) includes • applying their $p=\ldots$ in (c) to $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)=$ their $-\frac{2p}{15}$ found in (b)(ii) • applying $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)=2\left(\text{their }-\frac{4}{15}\text{ from (a)(ii)}\right)$
	Note	Defining a quadratic equation $px^2+qx+r=0$ and a correct method leading to $p=15,\ q=8,\ r=-4$ without writing a final answer of $15x^2+8x-4=0$ is final M1 A0
	Note	Give M0 for $\sum=-\frac{8}{15}$, $\Pi=-\frac{4}{15}$ leading to $x^2+\frac{8}{15}-\frac{4}{15}=0$ (without recovery)
	Note	Allow M1 for $\sum=-\frac{8}{15}$, $\Pi=-\frac{4}{15}$ with $x^2-(\text{sum})x+(\text{product})$ leading to $x^2+\frac{8}{15}-\frac{4}{15}=0$
	Note	Give A1 for $15y^2+8y-4=0$ (i.e. writing their answer completely in another variable)
	Note	$\alpha,\beta=\frac{-2\pm\sqrt{19}}{3}$ and $\alpha+\frac{1}{\beta},\ \beta+\frac{1}{\alpha}=\frac{-4\pm2\sqrt{19}}{15}$ may be used in (d) to find the sum and product of $\alpha+\frac{1}{\beta}$ and $\beta+\frac{1}{\alpha}$

	Question 7 Notes Continued	
7.	**ALT**	For finding $\alpha, \beta = \frac{-p+\sqrt{p^2+60}}{6}, \frac{-p-\sqrt{p^2+60}}{6}$
(a) (i)	**Note**	Give B1 for $\alpha, \beta = \frac{-p+\sqrt{p^2+60}}{6}, \frac{-p-\sqrt{p^2+60}}{6}$ and then finding $\alpha\beta = -\frac{5}{3}$ or $-\frac{60}{36}$
(b) (i)	**Note**	Give B1 for $\alpha, \beta = \frac{-p+\sqrt{p^2+60}}{6}, \frac{-p-\sqrt{p^2+60}}{6}$ and then finding $\alpha+\beta = -\frac{p}{3}$
	Note	Allow B1 for writing $\alpha+\beta = \frac{-p+\sqrt{p^2+60}}{6} + \frac{-p-\sqrt{p^2+60}}{6}$
(b)(ii)	**Note**	Allow M1 A1 for writing $\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)$ as $\frac{-p+\sqrt{p^2+60}}{6} + \frac{-p-\sqrt{p^2+60}}{6} + \frac{6}{-p+\sqrt{p^2+60}} + \frac{6}{-p-\sqrt{p^2+60}}$

Question Number	Scheme	Notes	Marks
8.	$H: xy = 16;\ P\left(4t, \dfrac{4}{t}\right), t \neq 0$, and $A: t = 2$ lies on H. $A(8, 2)$		
(a)	$y = \dfrac{16}{x} = 16x^{-1} \Rightarrow \dfrac{dy}{dx} = -16x^{-2}$ or $-\dfrac{16}{x^2}$	$\dfrac{dy}{dx} = \pm k x^{-2};\ k \neq 0$	M1
	$xy = 16 \Rightarrow x\dfrac{dy}{dx} + y = 0$	Uses implicit differentiation to give $\pm x\dfrac{dy}{dx} \pm y$	
	$x = 4t,\ y = \dfrac{4}{t} \Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{dt}.\dfrac{dt}{dx} = -\left(\dfrac{4}{t^2}\right)\left(\dfrac{1}{4}\right)$	their $\dfrac{dy}{dt} \times \dfrac{1}{\text{their } \dfrac{dx}{dt}}$; **Condone** $p \equiv t$	
	So at P, $m_T = -\dfrac{1}{t^2}$	Correct calculus work leading to $m_T = -\dfrac{1}{t^2}$	A1
	So, $m_N = t^2$	Applies $m_N = \dfrac{-1}{m_T}$, where m_T is found using calculus	M1
	• $y - \dfrac{4}{t} = "t^2"(x - 4t)$ • $\dfrac{4}{t} = "t^2"(4t) + c \Rightarrow y = "t^2"x +$ their c	Correct straight line method for an equation of a normal where $m_N\ (\neq m_T)$ is found by using calculus	M1
	Correct algebra leading to $ty - t^3x = 4 - 4t^4$ *	Correct solution only	A1 cso
			(5)
(b)	$\{t = 2 \Rightarrow\}$ **N**$: 2y - 8x = 4 - 64\ \{\Rightarrow y = 4x - 30\}$	Uses $t = 2$ to find the equation of the normal to H at A	M1
	• $x(4x - 30) = 16\ \{\Rightarrow 2x^2 - 15x - 8 = 0\}$ • $\left(\dfrac{y+30}{4}\right)y = 16\ \{\Rightarrow y^2 + 30y - 64 = 0\}$ • $\dfrac{4}{t} = 4(4t) - 30\ \{\Rightarrow 8t^2 - 15t - 2 = 0\}$	Substitutes the equation of the normal into the equation of the curve H to obtain an equation in x only or y only or t only	M1
	• $(x - 8)(2x + 1) = 0 \Rightarrow x_B = -\dfrac{1}{2}$ • $(y - 2)(y + 32) = 0 \Rightarrow y_B = -32$ • $(t - 2)(8t + 1) = 0 \Rightarrow t_B = -\dfrac{1}{8}$	**dependent on the first two M marks** Solves their 3 TQ $= 0$ to obtain a value for the x (or y) coordinate of B or a value of t at B	ddM1
	$B(-0.5, -32)$	Correct coordinates for B	A1
	$AB = \sqrt{(8 - -0.5)^2 + (2 - -32)^2}$	**dependent on the second M mark** Correct Pythagoras method to find the length of AB	dM1
	$= \dfrac{17\sqrt{17}}{2}$ or $\dfrac{\sqrt{4913}}{2}$ or $\sqrt{\dfrac{4913}{4}}$ or $\sqrt{1228.25}$	Correct exact length	A1
			(6)
(c)	$y - 2 = -\dfrac{1}{4}(x - 8)$ and $x = 0 \Rightarrow y_C = 2 + 2 = 4$	Finds the equation of the tangent at $(8, 2)$ to H, and sets $x = 0$ to find $y_C = \ldots$	M1
	$AC = \sqrt{(8-0)^2 + (2-4)^2}\ \{= \sqrt{68}\}$ Area $ABC = \dfrac{1}{2}\left(\dfrac{17\sqrt{17}}{2}\right)\left(\sqrt{68}\right)$	Uses the points $(8, 2)$, $(-0.5, -32)$ and $(0, 4)$ in a complete method to find the area of triangle ABC	M1
	$= 144.5$ or $\dfrac{289}{2}$	Correct answer	A1
			(3)
			14

	Question 8 Notes	
8. (b)	**Note**	The correct coordinates of B can be implied. e.g. embedded in the distance expression for AB
	Note	An incorrect **N**: $y = 4x + 30$ leads to the correct length AB for $A(-8, -2)$ and $B(0.5, 32)$
	Note	Condone final dM1 for $x_B = -\frac{1}{2}$ leading to $B(-2, -8)$ and $AB = \sqrt{(8 - -2)^2 + (2 - -8)^2}$
(c)	**Note**	Give 1st M0 for setting $x = 0$ in the equation of the normal to find $y_C = ...$
	Note	The 2nd M mark can only be gained by using all 3 correct points $(8, 2)$, $(-0.5, -32)$ and $(0, 4)$. Complete area methods include • Area $ABC = \frac{1}{2}\left(\frac{17\sqrt{17}}{2}\right)\left(\sqrt{68}\right)\{= 144.5\}$ • AB crosses y-axis at $(0, -30)$ and so Area $ABC = \frac{1}{2}(34)\left(\frac{1}{2}\right) + \frac{1}{2}(34)(8)\{= 8.5 + 136 = 144.5\}$ • Area $ABC = \frac{1}{2}\begin{vmatrix} 8 & -0.5 & 0 & 8 \\ 2 & -32 & 4 & 2 \end{vmatrix} = \frac{1}{2}\left\|(-256 - 2 + 0) - (-1 + 0 + 32)\right\| \left\{= \frac{1}{2}\left\|(-289)\right\| = 144.5\right\}$ • Area $ABC = (32+4)\left(\frac{1}{2}+8\right) - \frac{1}{2}(32+2)\left(\frac{1}{2}+8\right) - \frac{1}{2}(32+4)\left(\frac{1}{2}\right) - \frac{1}{2}(2)(8)$ $\{= 306 - 144.5 - 9 - 8 = 144.5\}$ • Area $ABC = \frac{1}{2}(8+8.5)(36) - \frac{1}{2}(32+2)\left(\frac{1}{2}+8\right) - \frac{1}{2}(2)(8)$ $\{= 297 - 144.5 - 8 = 144.5\}$
	Note	<u>**Helpful Sketch**</u> 

Question Number	Scheme	Notes	Marks
9.	$f(n)=7^n(3n+1)-1$ is a multiple of 9	$u_1=2,\ u_2=6,\ u_{n+2}=3u_{n+1}-2u_n \Rightarrow u_n=2(2^n-1)$	
(i) **Way 1**	$f(1)=7(4)-1=27$ {is a multiple of 9}	$f(1)=27$ is the minimum	B1
	$f(k+1)-f(k)=\underline{7^{k+1}(3(k+1)+1)-1}-(7^k(3k+1)-1)$	Attempts $f(k+1)-f(k)$	M1
		A correct expression for $\underline{f(k+1)}$	A1
	$=7^{k+1}(3k+4)-1-7^k(3k+1)+1=7^k(21k+28)-7^k(3k+1)$		
	$=18k(7^k)+27(7^k)$ or $7^k(18k+27)$	**dependent on the previous M mark** Uses correct algebra to achieve an expression where each term is an obvious multiple of 9	dM1
	$f(k+1)=9(7^k)(2k+3)+7^k(3k+1)-1$ or $f(k+1)=18k(7^k)+27(7^k)+f(k)$	Correct algebra leading to either e.g. $f(k+1)=9(7^k)(2k+3)+7^k(3k+1)-1$ or $f(k+1)=18k(7^k)+27(7^k)+f(k)$	A1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n $(\in\mathbb{Z}^+)$		A1 cso
			(6)
(i) **Way 2**	$f(1)=7(4)-1=27$ {is a multiple of 9}	$f(1)=27$ is the minimum	B1
	$f(k+1)=7^{k+1}(3(k+1)+1)-1$	Attempts $f(k+1)$	M1
		A correct expression for $f(k+1)$	A1
	$=7^{k+1}(3k+4)-1=7^k(21k+28)-1$		
	$=18k(7^k)+27(7^k)+7^k(3k+1)-1$ **or** $=(7^k)(18k+27)+7^k(3k+1)-1$ **or** $=9(7^k)(2k+3)+7^k(3k+1)-1$	**dependent on the previous M mark** Uses correct algebra to express $f(k+1)=g(k)+7^k(3k+1)-1$ or $f(k+1)=g(k)+f(k)$ where each term in $g(k)$ is an obvious multiple of 9	dM1
		Correct algebra leading to either e.g. $f(k+1)=9(7^k)(2k+3)+7^k(3k+1)-1$ or $f(k+1)=18k(7^k)+27(7^k)+f(k)$	A1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n $(\in\mathbb{Z}^+)$		A1 cso
			(6)
(ii)	$\{n=1,\}\ u_1=2(2^1-1)=2$;	Checks that the general formula works for either u_1 or u_2	M1
	$\{n=2,\}\ u_2=2(2^2-1)=6$	Checks that the general formula works for both u_1 and u_2	A1
	$\{u_{k+2}=3u_{k+1}-2u_k \Rightarrow\}$ $u_{k+2}=3(2(2^{k+1}-1))-2(2(2^k-1))$	Finds u_{k+2} by attempting to substitute $u_{k+1}=2(2^{k+1}-1)$ and $u_k=2(2^k-1)$ into $u_{k+2}=3u_{k+1}-2u_k$ Condone one slip	M1
	$\{u_{k+2}\}=6(2^{k+1})-6-4(2^k)+4$		
	$\{u_{k+2}\}=3(2^{k+2})-2^{k+2}-2$	Valid evidence of working in the same power of 2	M1
	$=2(2^{k+2})-2=2(2^{k+2}-1)$	***Uses algebra*** in a complete method to achieve this result with no errors	A1
	If the result is true for $n=k$ and for $n=k+1$, then it is true for $n=k+2$. As the result has been shown to be true for $n=1$ and $n=2$, then the result is true for all n $(\in\mathbb{Z}^+)$		A1 **cso**
			(6)
			12

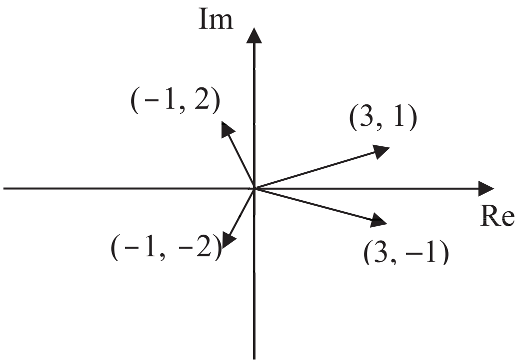
	Question 9 Notes	
9. (i)	**Note**	**Final A1** is dependent on all previous marks being scored. It is gained by candidates conveying the ideas of **all** four underlined points **in part (i)** **either** at the end of their solution **or** as a narrative in their solution.
	Note	Shows $\text{f}(k+1)-\text{f}(k)=7^k(18k+27)$ **or** $\text{f}(k+1)-\text{f}(k)=9(7^k)(2k+3)$ and writing if $\text{f}(k+1)-\text{f}(k)=9(7^k)(2k+3)$ o.e. is a multiple of 9 then $\text{f}(k+1)$ is a multiple of 9 is acceptable for the penultimate A mark in part (i). This means that the final A mark can potentially be available.
	Note	Only showing $\text{f}(k+1)=\ 7\text{f}(k)\ +6+21(7^k)$ (see Way 4) does not get the final dM mark because $6+21(7^k)$ is not an obvious multiple of 9
	Note	Allow dM1 for obtaining e.g. $\text{f}(k+1)-\text{f}(k)=18k(7^k)-27(7^k)$ or $\text{f}(k+1)-\text{f}(k)=7^k(18k-27)$
	Note	Allow dM1 for obtaining $\text{f}(k+1)=18k(7^k)-27(7^k)+7^k(3k+1)-1$ or $\text{f}(k+1)=9(7^k)(2k-3)+\text{f}(k)$
(ii)	**Note**	**1st M1:** At least one check is correct. **1st A1:** Both checks are correct • Check 1: Shows $u_1=2$ by writing an intermediate step of e.g. $2(2^1-1)$ or 2×1 • Check 2: Shows $u_2=6$ by writing an intermediate step of e.g. $2(2^2-1)$ or 2×3
	Note	Ignore $u_3=3u_2-2u_1=3(6)-2(2)=14$ as part of their solution to (ii)
	Note	Ignore $\{n=3,\}\ u_2=2(2^3-1)=14$ as part of their solution to (ii)
	Note	Valid evidence of working in the same power of 2 includes: • $6(2^{k+1})-4(2^k)\rightarrow 6(2^{k+1})-2(2^{k+1})$ or $2(3(2^{k+1})-2^{k+1})$ • $3(2(2^{k+1}))-2(2(2^k))\ \rightarrow 3(2^{k+2})-(2^{k+2})$ • $3(2(2^{k+1}))-2(2(2^k))\ \rightarrow 12(2^k)-4(2^k)$ • $6(2^{k+1})-4(2^k)\rightarrow 8(2^k)$ (by implication) • $6(2^{k+1})-4(2^k)\rightarrow 4(2^{k+1})$ (by implication)
	Note	Writing $u_{k+2}=3(2(2^{k+1}-1))-2(2(2^k-1))=2(2^{k+2}-1)$ is 2nd M1, 3rd M0, 2nd A0
	Note	Showing $\{\text{RHS}=\}\ u_{k+2}=2(2^{k+2}-1)=8(2^k)-2$ and writing $\{\text{LHS}=\}\ u_{k+2}=3(2(2^{k+1}-1))-2(2(2^k-1))$ and using valid algebra to show that $u_{k+2}=8(2^k)-2\ \{=\text{RHS}\}$ is fine for the 2nd M, 3rd M and 2nd A marks
	Note	**Final A1** is dependent on all previous marks being scored. It is gained by candidates conveying the ideas of **all** four underlined points **in part (ii)** **either** at the end of their solution **or** as a narrative in their solution.
	Note	"Assume for $n=k,\ u_k=2(2^k-1)$ and for $n=k+1,\ u_{k+1}=2(2^{k+1}-1)$" satisfies the requirement "true for $\boldsymbol{n=k}$ and $n=k+1$"
	Note	"For $n\in\mathbb{Z}^+,\ u_n=2(2^n-1)$ " satisfies the requirement "true for all n"
	Note	Full marks in (ii) can be obtained for an equivalent proof where e.g. • $n=k,\ n=k+1,\ \rightarrow n=k-2,\ n=k-1$; i.e. $k\equiv k-2$ • $n=k,\ n=k+1,\ \rightarrow n=k-1,\ n=k$; i.e. $k\equiv k-1$
(i), (ii)	**Note**	Allow as part of their conclusion "true for all positive values of n"
	Note	Allow as part of their conclusion "true for all values of n"
	Note	Allow as part of their conclusion "true for all $n\in\mathbb{N}$"
	Note	Condone referring to n as any integer in their conclusion for the final A1
	Note	Condone $n\in\mathbb{Z}^*$ as part of their conclusion for the final A1
	Note	Referring to n as a real number their conclusion is final A0

Question Number	Scheme	Notes	Marks
9.	$\mathrm{f}(n) = 7^n(3n+1) - 1$ is a multiple of 9; $P \in \mathbb{Z}^+$		
(i) **Way 3**	$\mathrm{f}(1) = 7(4) - 1 = 27$ {is a multiple of 9}	$\mathrm{f}(1) = 27$ is the minimum	B1
	$\mathrm{f}(k+1) - (9P+1)\mathrm{f}(k)$	Attempts $\mathrm{f}(k+1) - (9P+1)\mathrm{f}(k)$	M1
	$= \underline{7^{k+1}(3(k+1)+1) - 1} - (9P+1)(7^k(3k+1) - 1)$	A correct expression for $\underline{\mathrm{f}(k+1)}$	A1
	$= 7^k(21k + 28 - (9P+1)(3k+1)) - 1 + 9P + 1$		
	$= 7^k(21k + 28 - (27Pk + 9P + 3k + 1)) - 1 + 9P + 1$		
	$= 7^k(21k + 28 - 27Pk - 9P - 3k - 1) + 9P$		
	$= 7^k(18k - 27Pk - 9P + 27) + 9P$	**dependent on the previous M mark** Uses correct algebra to achieve an expression where each term is an obvious multiple of 9	dM1
	$\mathrm{f}(k+1) = 7^k(18k - 27PK - 9P + 27) + 9P + (9P+1)\mathrm{f}(k)$	Achieves a correct result for $\mathrm{f}(k+1) = \ldots$	A1
	If the result is <u>true for $n = k$</u>, then it is <u>true for $n = k+1$.</u> As the result has been shown to be <u>true for $n = 1$,</u> then the result <u>is true for all n</u> $(\in \mathbb{Z}^+)$		A1 cso
			(6)
	Note: $P = 0 \Rightarrow \mathrm{f}(k+1) - \mathrm{f}(k) = 7^k(18k + 27)$ $P = 1 \Rightarrow \mathrm{f}(k+1) - 10\mathrm{f}(k) = 7^k(18 - 9k) + 9$ $P = 2 \Rightarrow \mathrm{f}(k+1) - 19\mathrm{f}(k) = 7^k(9 - 36k) + 18$ $P = 3 \Rightarrow \mathrm{f}(k+1) - 28\mathrm{f}(k) = 7^k(-63k) + 27 = 27 - 9k(7^{k+1})$		

Question Number	Scheme	Notes	Marks
9.	$f(n) = 7^n(3n+1) - 1$ is a multiple of 9		
(i) **Way 4**	$f(1) = 7(4) - 1 = 27$ {is a multiple of 9}	$f(1) = 27$ is the minimum	B1
	$f(k+1) = 7^{k+1}(3(k+1)+1) - 1$	Attempts $f(k+1)$	M1
		A correct expression for $f(k+1)$	A1
	$= 7(7^k)(3k+3+1) - 1$		
	$= 7(7^k)(3k+1) + 3(7)(7^k) - 1$		
	$= 7[(7^k)(3k+1) - 1] + 7 + 21(7^k) - 1$ $= 7f(k) + 6 + 21(7^k)$ Let $g(n) = 6 + 21(7^n)$ $g(1) = 6 + 21(7^1) = 153$ {is a multiple of 9} {Assume the result is true for $\boldsymbol{n = k}$}	**dependent on the previous M mark** Uses correct algebra to express $f(k+1) = \alpha(7^k(3k+1) - 1) + g(k)$ or $f(k+1) = \alpha f(k) + g(k);\ \alpha \neq 0$ and uses correct algebra to achieve an expression for $g(k+1)$ where each term is an obvious multiple of 9	M1
	$g(k+1) = 6 + 21(7^{k+1})$ $= 6 + 147(7^k)$ $= 6 + 21(7^k) + 126(7^k)$ or $= g(k) + 9(14)(7^k)$	Correct algebra leading to $f(k+1) = 7f(k) + 6 + 21(7^k)$ o.e. and $g(k+1) = 6 + 21(7^k) + 126(7^k)$ where $g(n) = 6 + 21(7^n)$	A1
	Proves that $g(n) = 6 + 21(7^n)$ is a multiple of 9 and proves that for $f(n)$ if the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n $(\in \mathbb{Z}^+)$		A1 cso
			(6)
	Note: An alternative Way 4 method shows • $f(k+1) = 7f(k) + 6 + 21(7^k) = 7f(k) + 9(7^k + 1) + 3(7^k) - 3$ • Defines $g(n) = 3(7^n) - 3$ and proceeds to show that $g(n)$ is also a multiple of 9		

Question Number	Scheme	Notes	Marks
1(a)	$\mathrm{f}(x)=x^3-\dfrac{10\sqrt{x}-4x}{x^2} \qquad x>0$		
	$\mathrm{f}(1.4)=-0.435673...$ $\mathrm{f}(1.5)=0.598356...$	Attempts both f(1.4) and f(1.5)	M1
	Sign change (positive, negative) (and $\mathrm{f}(x)$ is continuous) therefore (a **root**) α is between $x=1.4$ and $x=1.5$	Both f(1.4) = awrt -0.4 and f(1.5) = awrt 0.6, sign change and conclusion. For 'sign change' indication that f(1.4) < 0 and f(1.5) > 0 is sufficient. Also f(1.4) f(1.5) < 0 is sufficient. 'Therefore root' (without mention of the interval) is a sufficient conclusion. Mention of 'continuous' is not required.	A1
			(2)
(b)	$\left(\mathrm{f}(x)=x^3-\dfrac{10\sqrt{x}-4x}{x^2}=x^3-10x^{-\frac{3}{2}}+4x^{-1}\right)$		
	$\mathrm{f}'(x)=3x^2+15x^{-\frac{5}{2}}-4x^{-2}$ (or equivalent, see below)	$x^n\to x^{n-1}$ for one term	M1
		2 correct terms simplified or unsimplified	A1
		All correct simplified or unsimplified	A1
			(3)
(c)	$(x_1)=1.4-\dfrac{\mathrm{f}(1.4)}{\mathrm{f}'(1.4)}$ $\left(=1.4-\dfrac{-0.43567...}{10.30720...}\right)=...$	Correct application of N-R leading to an answer. <u>Values</u> of f(1.4) and f '(1.4) need not be seen before their final answer.	M1
	$=1.442$	cao (must be corrected to 3 d.p.) isw if x_2, etc. have been found, but the answer for 'one use of N-R' must be seen as 1.442 to score this mark.	A1
			(2)
			Total 7
(b)	Equivalent unsimplified versions are acceptable, e.g. (using quotient rule); $3x^2-\dfrac{\left(5x^{\frac{3}{2}}-4x^2\right)-20x^{\frac{3}{2}}+8x^2}{x^4}$	The 'two correct terms' still applies for the first A1. Here a 'term' would be, for example, the $x^{-\frac{5}{2}}$ terms in unsimplified form. Isw after a correct unsimplified form.	
(b)(c)	A common error in (b) is to have $+4x^{-2}$ instead of $-4x^{-2}$, giving 1.430 in (c). This, if otherwise correct, would score (b) 110 and (c) 10		

Question Number	Scheme	Notes	Marks
2	$5x^2 - 2x + 3 = 0$		
(a)	$\alpha+\beta = \frac{2}{5},\quad \alpha\beta = \frac{3}{5}$	Both correct	B1
			(1)
(b)(i)	$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$	Uses a correct identity	M1
	$= \left(\frac{2}{5}\right)^2 - 2\left(\frac{3}{5}\right) = -\frac{26}{25}$	Correct value (allow -1.04), even after $\alpha+\beta = -\frac{2}{5}$ in (a)	A1
(ii)	$\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$ or $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$	Uses a correct identity	M1
	$= \left(\frac{2}{5}\right)^3 - 3\left(\frac{3}{5}\right)\left(\frac{2}{5}\right) = -\frac{82}{125}$	Correct value (allow -0.656)	A1
			(4)
(c)	$\text{Sum} = \alpha+\beta+\alpha^2+\beta^2 = \frac{2}{5} - \frac{26}{25}\left(= -\frac{16}{25}\right)$	Attempts value of sum	M1
	$\text{Product} = \alpha\beta + \alpha^3 + \beta^3 + (\alpha\beta)^2 = \frac{3}{5} - \frac{82}{125} + \left(\frac{3}{5}\right)^2 \left(= \frac{38}{125}\right)$	Attempts value of product, using the <u>correct</u> expansion of $(\alpha+\beta^2)(\beta+\alpha^2)$	M1
	$x^2 + \frac{16}{25}x + \frac{38}{125}(=0)$	Applies $x^2 -$ (their sum)x + their product Accept unsimplified versions. The '= 0' is not required	M1
	$125x^2 + 80x + 38 = 0$	Allow any integer multiple. Must be a fully correct equation, including the '= 0' Not just $p = 125$, $q = 80$, $r = 38$	A1
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
3	$\mathrm{f}(z) = z^4 + az^3 + bz^2 + cz + d$		
(a)	$(z =)3-\mathrm{i}$ or $(z =)-1+2\mathrm{i}$		B1
	$(z =)3-\mathrm{i}$ and $(z =)-1+2\mathrm{i}$		B1
			(2)
(b)		$3\pm\mathrm{i}$ correctly plotted with vectors or dots or crosses etc. or $-1\pm2\mathrm{i}$ correctly plotted with vectors or dots or crosses etc.	B1
		All 4 correct roots correctly plotted with scaling approximately correct (e.g. (-1, 2) higher than (3, 1), etc.) There should be approximate symmetry about the real axis, but be generous	B1
			(2)
(c)	$z = 3\pm\mathrm{i} \Rightarrow (z-(3+\mathrm{i}))(z-(3-\mathrm{i})) = ...$ or $z = -1\pm2\mathrm{i} \Rightarrow (z-(-1+2\mathrm{i}))(z-(-1-2\mathrm{i})) = ...$	Correct strategy to find at least one quadratic factor. Throughout this part ignore the use of x (or other variable) instead of z	M1
	$z^2-6z+10$ or z^2+2z+5	One correct quadratic	A1
	$z^2-6z+10$ and z^2+2z+5	Both correct	A1
	$(z^2-6z+10)(z^2+2z+5) = ...$	Attempts product of their two <u>3-term</u> quadratic factors... no 'missing terms' in the expansion	M1
	$a=-4, b=3, c=-10, d=50$ or $\mathrm{f}(z) = z^4-4z^3+3z^2-10z+50$	All correct values or correct quartic	A1
			(5)
			Total 9

(c) Way 2	$(z-(3+\mathrm{i}))(z-(-1\pm 2i))=\cdots$ or $(z-(3-\mathrm{i}))(z-(-1\pm 2\mathrm{i}))=\cdots$	Correct strategy to find at least one quadratic factor. Throughout this part ignore the use of x (or other variable) instead of z	M1
	$z^2+z(-2+\mathrm{i})+(-1-7\mathrm{i})$ (i) or $z^2+z(-2-\mathrm{i})+(-1+7\mathrm{i})$ (ii) or $z^2+z(-2+3\mathrm{i})+(-5-5\mathrm{i})$ (iii) or $z^2+z(-2-3\mathrm{i})+(-5+5\mathrm{i})$ (iv)	One correct quadratic	A1
	(i) and (ii) correct or (iii) and (iv) correct	A correct pair	A1
	e,g $[z^2+z(-2+i)+(-1-7i)]\times[z^2+z(-2-\mathrm{i})+(-1+7\mathrm{i})]=\ldots$	Attempts product of their two 3-term quadratic factors... no 'missing terms' in the expansion	M1
	$a=-4, b=3, c=-10, d=50$ or $\mathrm{f}(z)=z^4-4z^3+3z^2-10z+50$	All correct values or correct quartic	A1

(c) Way 3	$\sum\propto=(3+\mathrm{i})+(3-\mathrm{i})+(-1-2\mathrm{i})+(-1+2\mathrm{i})=..$ or $\alpha\beta\gamma\delta=(3+\mathrm{i})(3-\mathrm{i})(-1-2\mathrm{i})(-1+2\mathrm{i})=..$	Attempts one of these	M1
	$a=-4$ or $d=50$		A1
	Both $a=-4$ and $d=50$		A1
	$\sum\alpha\beta=\ldots$ and $\sum\alpha\beta\gamma=\ldots$	Attempts $\sum\alpha\beta$ (all 6 terms) and $\sum\alpha\beta\gamma$ (all 4 terms)	M1
	$a=-4, b=3, c=-10, d=50$ or $\mathrm{f}(z)=z^4-4z^3+3z^2-10z+50$	All correct values or correct quartic	A1

(c) Way 4	$(3+\mathrm{i})^4+a(3+\mathrm{i})^3+b(3+\mathrm{i})^2+c(3+\mathrm{i})+d=0$ $(\ldots\ldots)+(\ldots\ldots)\,\mathrm{i}=0$	Substitutes one of the roots into the given quartic and fully multiplies out	M1
	$(28+18a+8b+3c+d)+\mathrm{i}(96+26a+6b+c)$ or $(-7+11a-3b-c+d)+\mathrm{i}(-24+2a+4b-2c)$	One correct expansion	A1
	$(28+18a+8b+3c+d)+\mathrm{i}(96+26a+6b+c)$ and $(-7+11a-3b-c+d)+\mathrm{i}(-24+2a+4b-2c)$	Obtains a second correct expansion using another root.	A1
	$(28+18a+8b+3c+d)=0$, etc leading to $a=$, $b=$, $c=$, $d=$	Solves 4 simultaneous equation to find values of a, b, c and d	M1
	$a=-4, b=3, c=-10, d=50$ or $\mathrm{f}(z)=z^4-4z^3+3z^2-10z+50$	All correct values or correct quartic	A1

Question Number	Scheme	Notes	Marks
4(a)	$(2r-1)^2 = 4r^2-4r+1$	Correct expansion	B1
	$\sum_{r=1}^{n}(4r^2-4r+1) = 4\times\frac{1}{6}n(n+1)(2n+1) - 4\times\frac{1}{2}n(n+1)+n$ M1: Attempt to use at least one of the standard results correctly A1: Correct expression		M1A1
	$=\frac{1}{3}n\left[2(n+1)(2n+1)-6(n+1)+3\right]$	Attempt to factorise $\frac{1}{3}n(\dots.)$ Condone one slip but there must have been $+n$, not $+1$ in their expression for the sum	M1
	$=\frac{1}{3}n\left[4n^2-1\right]$*	Correct proof with no errors. There should be an intermediate step showing the expansion of $(n+1)(2n+1)$, or equivalent	A1*
	Condone poor or incorrect use of notation, e.g. Σ used at every step of the proof		
			(5)
(b)	$2r-1=499 \Rightarrow r=250$	Identifies the correct upper limit (may be implied)	B1
	$2r-1=201 \Rightarrow r=101$	Identifies the correct lower limit (may be implied)	B1
	$\sum_{r=101}^{250}(2r-1)^2 = \frac{1}{3}\times 250\left(4\times250^2-1\right)-\frac{1}{3}\times100\left(4\times100^2-1\right)$ Uses the result from part (a) together with their upper limit and their lower limit – 1. A common mistake is to assume 500 and 200 are the limits, and in this case the mark is scored if 199 is used		M1
	= 19 499 950	Cao	A1
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
5(a)	$xy=64 \Rightarrow y=64x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-64x^{-2}$ or $xy=64 \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{y}{x}$ or $x=8p, y=\frac{8}{p} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-8p^{-2}}{8}$	Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ This can be in any form, simplified or unsimplified. The parameter could be a different variable, e.g. t	B1
	$m_T=-\frac{64}{64p^2} \Rightarrow m_N=p^2$ or $m_T=-\frac{8/p}{8p} \Rightarrow m_N=p^2$ or $m_T=-p^{-2} \Rightarrow m_N=p^2$	Correct use of the perpendicular gradient rule and the point P to obtain the normal gradient	M1
		Correct normal gradient of p^2	A1
	$y-\frac{8}{p}=p^2(x-8p)$ or $y=p^2x+c, \frac{8}{p}=p^2\times 8p+c \Rightarrow c=\ldots$	Correct straight line method for normal	M1
	$p^3x-py=8(p^4-1)$*	cso. (No errors, but possibly direct from the version in line 1 above)	A1*
			(5)
(b)	$p^3x-py=8(p^4-1), xy=64 \Rightarrow$ $p^3x-p\frac{64}{x}=8(p^4-1)$ or $p^3\frac{64}{y}-py=8(p^4-1)$	Uses both equations to obtain an equation in one variable	M1
	$p^3x^2+8(1-p^4)x-64p=0$ or $py^2+8(p^4-1)y-64p^3=0$	Correct quadratic. Must have the x^2 or y^2 term, but the x or y terms need not be combined. The terms do not need to be 'all on one side', and the coefficients could involve fractions, e.g. $p^2x^2+\frac{8x}{p}-8p^3x=64$	A1
	$(x-8p)(p^3x+8)=0 \Rightarrow x=\ldots$ or $(py-8)(y+8p^3)=0 \Rightarrow y=\ldots$	Solves their 3TQ (usual rules) to obtain the <u>other</u> value of x or y. The <u>other</u> value must be picked out as a solution. This could be done by algebraic division... (see below)	dM1
	$x=-\frac{8}{p^3}$ $y=-8p^3$ or $\left(-\frac{8}{p^3},-8p^3\right)$	Correct coordinates (ignore coordinates of P if they are also given as an answer). $-8p^{-3}$ may be seen rather than $-\frac{8}{p^3}$	A1
			(4)
			Total 9

5(b)	Rather than solving the 3TQ for the dM1, algebraic division can be used. To score the mark the division should follow the usual rules for solution by factorisation, so in the first case, e.g. if the quadratic is correct, the quotient should be $\pm p^3 x \pm 8$, then this must lead to the other value $x_2 = \cdots$

5(b)	Note that another way to find the other value for the dM1 is to use the 'sum of roots' $= -\frac{b}{a}$, e.g. $8p + x_2 = \dfrac{-8(1-p^4)}{p^3} \qquad x_2 = \cdots$

(b) **Way 2**	$p^3 x - py = 8\left(p^4 - 1\right), \left(8q,\ \dfrac{8}{q}\right) \Rightarrow$ $p^3 8q - p\dfrac{8}{q} = 8\left(p^4 - 1\right)$	Uses the given normal equation and the parametric form for Q to form an equation in p and q	M1
	$p^3 q^2 - p = qp^4 - q$	Correct quadratic. Must have the q^2 term, but the q terms need not be combined. The terms do not need to be 'all on one side', and the coefficients could involve fractions.	A1
	$(p-q)\left(p^3 q + 1\right) = 0 \Rightarrow q = \ldots$	Solves their 3TQ (usual rules) to obtain the value of q. This could be done by algebraic division (condition as for main scheme)	dM1
	$q = -\dfrac{1}{p^3} \Rightarrow x = -\dfrac{8}{p^3} \quad y = -8p^3$ or $\left(-\frac{8}{p^3}, -8p^3\right)$	Correct coordinates (ignore coordinates of P if they are also given as an answer). $-8p^{-3}$ may be seen rather than $-\frac{8}{p^3}$	A1
			(4)

Question Number	Scheme	Notes	Marks
6(i)(a)	Stretch scale factor 3 parallel to the y-axis	Stretch (<u>not</u> enlargement)	B1
		Scale factor 3 parallel to the y-axis. Allow, e.g. '3 times y values', 'y increased by 3 factor', or similar. Allow, e.g. 'direction of y', 'along y', 'vertical', or similar. Ignore any mention of the origin. If additional transformations are included, send to Review	B1
			(2)
(b)	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$	Correct matrix. $\frac{1}{\sqrt{2}}$ may be seen rather than $\frac{\sqrt{2}}{2}$	B1
			(1)
(c)	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$	Attempt to multiply the right way round, i.e. **BA**, not **AB** At least two correct terms (for their matrix **B**) are needed to indicate a correct multiplication attempt	M1
	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} \end{pmatrix}$ or equiv. e.g. $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 3 \\ -1 & 3 \end{pmatrix}$	Correct matrix	A1
			(2)
(ii)	Trapezium area$=\frac{1}{2}(5+2)(k+8)$	Correct method for the area of the trapezium	M1
	$\begin{vmatrix} 5 & 1 \\ -2 & 3 \end{vmatrix} = 5\times 3-(-2)\times 1=17$	Correct method for the determinant	M1
		17 (Allow $\pm$ 17)	A1
	$\frac{1}{2}(5+2)(k+8)\times 17=510 \Rightarrow k=\ldots$	Multiplies their trapezium area by their determinant, sets equal to 510 and solves for k. Or equivalently: Equates their trapezium area to (510 $\div$ determinant) and solves for k	M1
	$k=\frac{4}{7}$	$\frac{4}{7}$ or exact equivalent. If additional answers such as $-\frac{4}{7}$ are given and not rejected, this is A0	A1
			(5)

(ii) **Way 2**	$\begin{pmatrix} 5 & 1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} -2 & -2 & 5 & 5 \\ 0 & k & 8 & 0 \end{pmatrix}$ $= \begin{pmatrix} -10 & -10+k & 33 & 25 \\ 4 & 4+3k & 14 & -10 \end{pmatrix}$	Multiplies correct matrices to find the coordinates for T'	2nd M
		Correct coordinates (can be left in matrix form)	A1
	$\frac{1}{2}[-10(4+3k)+14(-10+k)-330+100-4(-10+k)-33(4+3k)-350-100]$	Correct method for area of T' ('shoelace rule' with or without a modulus), using their coordinates for T'	1st M
	$\pm\frac{1}{2}(952+119k)=510$, $k = \ldots$	Sets area of T' equal to 510 and solves for k	M1
	$k = \frac{4}{7}$	$\frac{4}{7}$ or exact equivalent. If additional answers such as $-\frac{4}{7}$ are given and not rejected, this is A0	A1
			Total 10

Question Number	Scheme	Notes	Marks
7(a) Way 1	$3x-4y+48=0 \Rightarrow x=\dfrac{4y-48}{3}$ $y^2=4ax \Rightarrow y^2=4a\left(\dfrac{4y-48}{3}\right)$ or $3x-4y+48=0 \Rightarrow y=\dfrac{3x+48}{4}$ $y^2=4ax \Rightarrow \left(\dfrac{3x+48}{4}\right)^2=4ax$ or $x=\dfrac{y^2}{4a} \Rightarrow \dfrac{3y^2}{4a}-4y+48=0$	Uses both equations to obtain an equation in one variable.	M1
	$3y^2-16ay+192a=0$ or $9x^2+(288-64a)x+2304=0$ or $3x-8\sqrt{a}\sqrt{x}+48=0$	Correct 3TQ (Coefficients could be 'fractional') (This could be a quadratic in $\sqrt{x}$)	A1
	Equal roots: $(16a)^2=4\times3\times192a$ or $(288-64a)^2=4\times9\times2304$ $\Rightarrow a=\ldots$	Uses "$b^2=4ac$" to find a value for a	M1
	$a=9$ *	cso	A1*
	Beware the use of the given result $a=9$, but there may be cases where 'working backwards' deserves merit (if in doubt, send to Review).		
			(4)

(a) Way 2	$y^2=4ax \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x}=4a$ $3x-4y+48=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{4}$ $\Rightarrow 2y\times\frac{3}{4}=4a$	Uses differentiation to obtain the gradient of C and substitutes the gradient of l to obtain an equation connecting y and a, or connecting x and a, or an equation in t	M1
	$y=2a^{\frac{1}{2}}x^{\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=a^{\frac{1}{2}}x^{-\frac{1}{2}} \Rightarrow a^{\frac{1}{2}}x^{-\frac{1}{2}}=\frac{3}{4}$		
	$x=at^2,\ y=2at \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{t}$ $\frac{1}{t}=\frac{3}{4}$		
	$y=\frac{8a}{3}$ or $x=\frac{16a}{9}$	Correct y value, or correct x value (possibly implied in subsequent work, particularly if using the parametric equations)	A1
	$y^2=4ax \Rightarrow \frac{64a^2}{9}=4ax \Rightarrow x=\frac{16a}{9}$ $3\times\frac{16a}{9}-4\times\frac{8a}{3}+48=0 \Rightarrow a=\ldots$	Uses $y^2=4ax$ or l to find a value for x (or y) and substitutes their x and y into the other equation to find a value for a If using parameter t, substitutes their value for t into $3(at^2)-4(2at)+48=0$ and solves to find a value for a	M1
	$x=\frac{4y-48}{3}=\frac{32a}{9}-16$ $\frac{64a^2}{9}=4a\left(\frac{32a}{9}-16\right) \Rightarrow a=\cdots$		
	$y^2=4ax \Rightarrow y^2=\frac{64a^2}{9} \Rightarrow y=\frac{8a}{3}$ $3\times\frac{16a}{9}-4\times\frac{8a}{3}+48=0 \Rightarrow a=\ldots$		
	$y=\frac{3x+48}{4}=\frac{4a}{3}+12$ $\left(\frac{4a}{3}+12\right)^2=4a\left(\frac{16a}{9}\right) \Rightarrow a=\cdots$		
	$3(at^2)-4(2at)+48=0$ $3\left(\frac{16a}{9}\right)-4\left(\frac{8a}{3}\right)+48=0 \Rightarrow a=\cdots$		
	$a=9$ *	cso	A1*

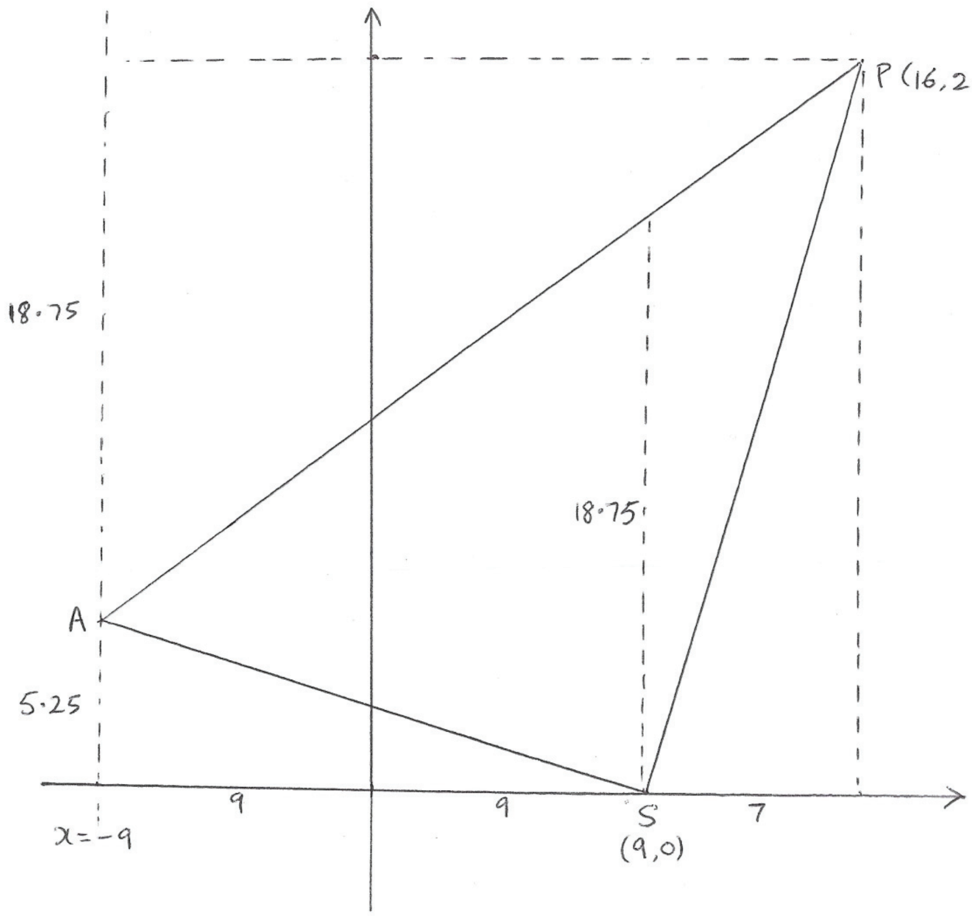
(b)	$a=9 \Rightarrow 3y^2-144y+1728=0 \Rightarrow y=24$ $9x^2-288x+2304=0 \Rightarrow x=16$	Uses $a=9$ to solve their 3TQ to obtain the repeated root for x or y.	M1
	$x=16$ and $y=24$	Correct values or coordinates.	A1
			(2)

(b) Way 2 follows (a)Way2	$a=9 \Rightarrow x=\cdots$ or $y=\cdots$	Substitutes $a=9$ into their expression for x or y, OR substitutes $a=9$ into at^2 to find x, or into $2at$ to find y.	M1
	$x=16$ and $y=24$	Correct values or coordinates.	A1

(c) Way 1	Focus is at (9, 0)	Correct focus (could be seen on a sketch or implied in working)	B1
	$x=-9 \Rightarrow 3(-9)-4y+48=0 \Rightarrow y=5.25$	Correct method with the correct directrix to find the y coordinate of A	M1
	E.g.Trapezium – 2 triangles $=\frac{1}{2}\left(\frac{21}{4}+24\right)\times 25-\frac{1}{2}\times 18\times\frac{21}{4}-\frac{1}{2}\times 7\times 24=\frac{1875}{8}$ Fully correct triangle area method (condone one slip if the intention seems clear)		dM1
	$=\frac{1875}{8}$ (234.375)	Correct area (exact)	A1
			(4)
			Total 10

(c) Way 2	Focus is at $(9,0)$	Correct focus (could be seen on a sketch or implied in working)	B1
	$x=9 \Rightarrow 3(9)-4y+48=0 \Rightarrow y=18.75$	Correct method to find the y coordinate when $x=9$, but also requires correct directrix at some stage of the solution	M1
	E.g. $A=\frac{1}{2}(18.75\times 18)+\frac{1}{2}(18.75\times(16-9))$	Fully correct triangle area method (condone one slip if the intention seems clear)	dM1
	$=\frac{1875}{8}$ (234.375)	Correct area (exact)	A1
(c) Way 3	Focus is at (9, 0)	Correct focus (could be seen on a sketch or implied in working)	B1
	$x=-9 \Rightarrow 3(-9)-4y+48=0 \Rightarrow y=5.25$	Correct method with the correct directrix to find the y coordinate of A	M1
	E.g. $\frac{1}{2}\begin{vmatrix} 9 & -9 & 16 & 9 \\ 0 & 5.25 & 24 & 0 \end{vmatrix}$ $=\frac{1}{2}\lvert 47.25-216-84-216\rvert$	Fully correct area method (condone one slip if the intention seems clear)	dM1
	$=\frac{1875}{8}$ (234.375)	Correct area (exact)	A1

(c) Way 4	Focus is at (9, 0)	Correct focus (could be seen on a sketch or implied in working)	B1
	$x=-9 \Rightarrow 3(-9)-4y+48=0 \Rightarrow y=5.25$	Correct method with the correct directrix to find the y coordinate of A	M1
	E.g.Rectangle – 3 triangles $(25\times 24)-\frac{1}{2}(18\times 5.25)-\frac{1}{2}(7\times 24)-\frac{1}{2}(25\times 18.75)$ Fully correct triangle area method (condone one slip if the intention seems clear)		dM1
	$=\dfrac{1875}{8}\ (234.375)$	Correct area (exact)	A1



Question Number	Scheme	Notes	Marks
8(i)	$$\sum_{r=1}^{n}\frac{2r^2-1}{r^2(r+1)^2}=\frac{n^2}{(n+1)^2}$$		
	$\frac{2(1)^2-1}{1^2(1+1)^2}=\frac{1}{4},\ \frac{1^2}{(1+1)^2}=\frac{1}{4}$	Getting $\frac{1}{2^2}$ or $\frac{1}{4}$ from each is the minimum	B1
	Assume $\sum_{r=1}^{k}\frac{2r^2-1}{r^2(r+1)^2}=\frac{k^2}{(k+1)^2}$		
	$\sum_{r=1}^{k+1}\frac{2r^2-1}{r^2(r+1)^2}=\frac{k^2}{(k+1)^2}+\frac{2(k+1)^2-1}{(k+1)^2(k+2)^2}$ Assumes the result is true for say $n=k$ and adds the next term		M1
	$\frac{k^2(k+2)^2+2(k+1)^2-1}{(k+1)^2(k+2)^2}$	Attempts common denominator	dM1
		Correct expression	A1
	$\frac{k^4+4k^3+4k^2+2k^2+4k+1}{(k+1)^2(k+2)^2}=\frac{k^4+4k^3+6k^2+4k+1}{(k+1)^2(k+2)^2}=\frac{(k+1)^4}{(k+1)^2(k+2)^2}$		
	$\frac{(k+1)^2}{(k+2)^2}$	Achieves this result with intermediate working and no errors	A1
	If the result is <u>true for $n=k$</u>, then it is <u>true for $n=k+1$</u>. As the result has been shown to be <u>true for $n=1$</u>, then the result is <u>true for all n</u>.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso
			(6)
(ii)	$f(n)=12^n+2\times5^{n-1}$		
	$f(1)=12+2\times1=14$	This is sufficient	B1
	$f(k+1)=12^{k+1}+2\times5^k$	Attempt $f(k+1)$	M1
	$f(k+1)-f(k)=12^{k+1}+2\times5^k-12^k-2\times5^{k-1}$ $f(k+1)-f(k)=11\times12^k+22\times5^{k-1}+10\times5^{k-1}-24\times5^{k-1}$	Working with $f(k+1)-f(k)$	
	$=11\times(12^k+2\times5^{k-1})-14\times5^{k-1}$	$11\times(12^k+2\times5^{k-1})$ or $11f(k)$	A1
		$-14\times5^{k-1}$	A1
	$f(k+1)=12f(k)-14\times5^{k-1}$	Makes $f(k+1)$ the subject **Dependent on at least one of the A marks**	dM1
	If the result is <u>true for $n=k$</u>, then it is <u>true for $n=k+1$</u>. As the result has been shown to be <u>true for $n=1$</u>, then the result is <u>true for all n</u>.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso
			(6)
			Total 12

ALT 1	$\mathrm{f}(1)=12+2\times1=14$	This is sufficient	B1
	$\mathrm{f}(k+1)=12^{k+1}+2\times5^{k}$	Attempt f(k+1)	M1
	$\mathrm{f}(k+1)=12\left(12^{k}+2\times5^{k-1}\right)+2\times5\times5^{k-1}-12\times2\times5^{k-1}$		
	$\mathrm{f}(k+1)=12\left(12^{k}+2\times5^{k-1}\right)-14\times5^{k-1}$	$12\left(12^{k}+2\times5^{k-1}\right)$ or $12\mathrm{f}(k)$	A1
		$-14\times5^{k-1}$	A1
	$\mathrm{f}(k+1)=12\mathrm{f}(k)-14\times5^{k-1}$	**Dependent on at least one of the A marks**	dM1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso
ALT 2	$\mathrm{f}(1)=12+2\times1=14$	This is sufficient	B1
	Let $12^{k}+2\times5^{k-1}=7M$		
	$\mathrm{f}(k+1)=12^{k+1}+2\times5^{k}$	Attempt f(k+1)	M1
	$\mathrm{f}(k+1)=12\left(7M-2\times5^{k-1}\right)+2\times5^{k}$	OR: $\mathrm{f}(k+1)=5(7M)+7\times12^{k}$	
	$\mathrm{f}(k+1)=84M-14\times5^{k-1}$ OR: $\mathrm{f}(k+1)=35M+7\times12^{k}$	$84M$	OR: $35M$	A1
		$-14\times5^{k-1}$	$+7\times12^{k}$	A1
	$\mathrm{f}(k+1)=12\mathrm{f}(k)-14\times5^{k-1}$ OR: $\mathrm{f}(k+1)=5\mathrm{f}(k)+7\times12^{k}$	**Dependent on at least one of the A marks**	dM1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso
ALT 3	$\mathrm{f}(1)=12+2\times1=14$	This is sufficient	B1
	$\mathrm{f}(k+1)=12^{k+1}+2\times5^{k}$	Attempts f(k + 1)	M1
	Working with f(k + 1) – mf(k) $\mathrm{f}(k+1)-m\mathrm{f}(k)=12^{k+1}+2\times5^{k}-m\left(12^{k}+2\times5^{k-1}\right)$ $\mathrm{f}(k+1)-\mathrm{f}(k)=(12-m)\times12^{k}+2\times(12-m)\times5^{k-1}+10\times5^{k-1}-24\times5^{k-1}$		
	$=(12-m)\times\left(12^{k}+2\times5^{k-1}\right)-14\times5^{k-1}$	$(12-m)\times\left(12^{k}+2\times5^{k-1}\right)$ or $(12-m)\mathrm{f}(k)$	A1
		$-14\times5^{k-1}$	A1
	$\mathrm{f}(k+1)=12\mathrm{f}(k)-14\times5^{k-1}$	Makes f(k + 1) the subject **Dependent on at least one of the A marks**	dM1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso

ALT 4	$\mathrm{f}(1)=12+2\times1=14$	This is sufficient	B1
	$\mathrm{f}(k+1)=12^{k+1}+2\times5^{k}$	Attempts f(k + 1)	M1
	$\mathrm{f}(k+1)-5\mathrm{f}(k)=12^{k+1}+2\times5^{k}-5\left(12^{k}+2\times5^{k-1}\right)$	Working with f(k + 1) – 5f(k)	
	$=7\times12^{k}+2\times5^{k}-2\times5^{k}$	7×12^{k}	A1
		$2\times5^{k}-2\times5^{k}$ (or zero)	A1
	$\mathrm{f}\left(k+1\right)=5\mathrm{f}\left(k\right)+7\times12^{k}$	Makes f(k + 1) the subject **Dependent on at least one of the A marks**	dM1
	If the result is <u>true for $n=k$</u>, then it is <u>true for $n=k+1$</u>. As the result has been shown to be <u>true for $n=1$</u>, then the result is <u>true for all n</u>.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso

<u>NOTES</u>:

<u>Part (i)</u>

This approach may be seen:
Assume result is true for $n = k$ <u>and</u> $n = k + 1$
Subtract: (sum to (k + 1) terms) minus (sum to k terms)
Show that this is equal to the (k + 1)th term
Please send any such response to Review.

<u>Part (ii)</u>

Apart from the given alternatives, other versions will work and can be marked equivalently.
If in any doubt, send to Review.

Question Number	Scheme	Marks
1.(a)	$f(0.2)=...$ and $f(0.6)=...$	M1
	$f(0.2)=-0.5973...$ and $f(0.6)=0.2707...$	
	Continuous function with change of sign so root (in given interval)	A1 (2)
(b)	$f(0.4)=-0.1788$	B1
	$f(0.5)=...$	M1
	$f(0.5)=0.04114... \Rightarrow 0.4\leqslant\alpha\leqslant 0.5$	A1 (3)
		[5]

	Notes
	Must see correct values for the accuracy marks. But allow signs only for attempts at values for the method marks.
(a)	
M1	Attempts both values, accept f(0.2)=... and f(0.6) =… with any values. (NB f(0.2) = −0.2069…, f(0.6) = 1.379 are the values with calculator in degrees mode.)
A1	Both values correct (rounded or truncated to 1d.p.) and a correct conclusion (continuous and sign change). Allusion to continuity must be mentioned somewhere in the solution. Allow other ways to show sign change e.g. <0, >0 etc.
(b)	
B1	Correct value of f (0.4); may be rounded or truncated to 1 dp
M1	Attempt value of f (0.5) or attempt value of f(0.3) if relevant for their sign of f(0.4).
A1	Correct value of f (0.5) which may be rounded to 2 dp and correct interval. Allow as open or closed interval. Accept any valid notation for the interval. Accept e.g. $0.4 < x < 0.5$

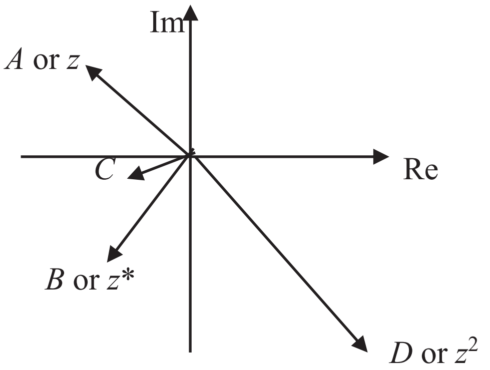
Question Number	Scheme	Marks
2		
(a)	$\frac{3}{8}-\frac{\sqrt{71}}{8}\text{i}$	B1 (1)
(b)	$\left(x-\frac{3}{8}-\frac{\sqrt{71}}{8}\text{i}\right)\left(x-\frac{3}{8}+\frac{\sqrt{71}}{8}\text{i}\right)\ \left((x-4)=0\right)$	
	$\left(x^2-\frac{3}{4}x+\frac{5}{4}\right)\ \left((x-4)=0\right)$	M1A1
	$x^3-\frac{19}{4}x^2+\frac{17}{4}x-5\ \ (=0)$	dM1
	$4x^3-19x^2+17x-20\ (=0)\quad p=17,\ q=-20$	A1 (4)
		[5]
(a) **B1**	Correct answer only	
(b) **M1**	Attempt the multiplication of the 2 brackets with the complex terms. Allow $(x\pm\text{root})$ for the brackets. Allow "invisible" brackets.	
A1	Correct quadratic obtained - may have multiplied by the 4 (or other constant factor) and this is fine. (Need not be fully simplified but must have real terms)	
dM1	Attempt to multiply their quadratic by $(x-4)$ or may divide their quadratic into the cubic or other full method leading to at least one of p or q.	
A1	Correct values. Values of p and q need not be shown explicitly but may been seen in a cubic, provided the cubic starts $4x^3-19x^2$ (isw after a correct cubic)	
	Note if a candidate uses a hybrid method, mark under main scheme unless an Alt scores more marks.	
Alt 1 (b)	$-\frac{q}{4}=4\times\left(\frac{3}{8}+\frac{\sqrt{71}}{8}\text{i}\right)\times\left(\frac{3}{8}-\frac{\sqrt{71}}{8}\text{i}\right)=\ldots\to q=..$ **or** $\frac{p}{4}=4\left(\frac{3}{8}+\frac{\sqrt{71}}{8}\text{i}\right)+4\left(\frac{3}{8}-\frac{\sqrt{71}}{8}\text{i}\right)+\left(\frac{3}{8}+\frac{\sqrt{71}}{8}\text{i}\right)\left(\frac{3}{8}-\frac{\sqrt{71}}{8}\text{i}\right)\to p=\ldots$	M1
	$\Rightarrow q=-16\times\left(\frac{9}{64}+\frac{71}{64}\right)=-20$ **or** $p=17$	A1
	E.g. $\text{f}(4)=0\Rightarrow 4(4)^3-19(4)^2+4p-\text{"}20\text{"}=0\Rightarrow p=\ldots$ or $\frac{p}{4}=4\left(\frac{3}{8}+\frac{\sqrt{71}}{8}\text{i}\right)+4\left(\frac{3}{8}-\frac{\sqrt{71}}{8}\text{i}\right)+\left(\frac{3}{8}+\frac{\sqrt{71}}{8}\text{i}\right)\left(\frac{3}{8}-\frac{\sqrt{71}}{8}\text{i}\right)\to p=\ldots$	dM1
	$p=17,\ q=-20$	A1 (4)

Question Number	Scheme	Marks
M1	A correct attempt to use product of roots is $-\frac{q}{4}$ to find a value for q **or** pair sum is $\frac{p}{4}$ to find a value of p.	
A1	Correct value for p or q	
dM1	Correct full method to find both p and q.	
A1	Correct values for both	
Alt 2	Attempts at using the factor theorem are possible but unlikely to succeed. Score as follows: M1: Uses the factor theorem to generate two equations in the two unknowns (note they will need to use a complex root to achieve this and equate real and imaginary parts.). A1: Correct equations. dM1: Solves their two equations to find values for p and q. A1: Correct values Send to review if unsure.	
Alt 3 (b)	$$\begin{array}{r} 4x^2-3x+p-12 \\ x-4\overline{)4x^3-19x^2+px+q} \\ 4x^3-16x^2 \\ -3x^2+px+q \\ -3x^2+12x \\ (p-12)x+q \\ (p-12)x-4(p-12) \end{array}$$	M1 A1
	$\Rightarrow q+4(p-12)=0 \;\; \& \;\; \frac{p-12}{4}=\left(\frac{3}{8}+\frac{\sqrt{71}}{8}\text{i}\right)\left(\frac{3}{8}-\frac{\sqrt{71}}{8}\text{i}\right)=\frac{5}{4}$	dM1
	$P=17,\ q=-20$	A1 (4)
(b) **M1** **A1** **dM1** **A1**	Divides $x-4$ into the cubic to achieve a 3TQ quotient and a remainder Correct quotient and remainder Correct full method to find p or q Correct values	

Question Number	Scheme	Marks
3(a)	$k(k+5)-6=0$	M1
	$k^2+5k-6=0$	
	$\left((k-1)(k+6)=0 \Rightarrow\right) \quad k=1, \; -6$	A1 (2)
(b)	$\dfrac{1}{"k^2+5k-6"}\begin{pmatrix} k & 2 \\ 3 & k+5 \end{pmatrix}$	M1A1 (2)
		[4]
(a)		
M1	Attempts determinant and sets equal to zero (or equivalent method) to obtain an unsimplified quadratic equation	
A1	Correct values for k (may solve the quadratic by any valid means)	
(b)		
M1	Forms the matrix of signed minors (must have at least three correct elements) divided or multiplied by an attempt at the determinant	
A1	Fully correct inverse	

Question Number	Scheme	Marks
4 (a)	$\alpha+\beta=-\frac{5}{2}\quad \alpha\beta=\frac{7}{2}$	B1
	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=\left(-\frac{5}{2}\right)^3-3\left(\frac{7}{2}\right)\left(-\frac{5}{2}\right)$	M1
	$=\frac{85}{8}$	A1 (3)
(b)	$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}=\ldots$	M1
	$=\left(\frac{85}{8}\right)\times\left(\frac{2}{7}\right)=\frac{85}{28}$	A1
	$\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}=\alpha\beta=\frac{7}{2}$	B1ft
	$x^2-"\frac{85}{28}"x+"\frac{7}{2}"\ (=0)$	M1
	$28x^2-85x+98=0$	A1 (5) **[8]**
	Note: if a candidate solves the equation and uses the roots to answer the question, then send to review.	
(a)		
B1	Both correct. (Seen anywhere in the working)	
M1	Uses their sum and product of roots in a correct expression for $\alpha^3+\beta^3$.	
A1	Correct value. Must be exact. Accept 10.625	
(b)		
M1	substitutes their values for $\alpha^3+\beta^3$ and $\alpha\beta$ into $\frac{\alpha^3+\beta^3}{\alpha\beta}=\ldots$ (allow slips in substitution).	
A1	Correct sum as a single fraction (may be seen or implied in their equation)	
B1ft	Correct product or follow through their product	
M1	Use $x^2-\text{sum of roots}\times x+\text{product of roots}$ with their values for sum and product. "= 0" may be missing.	
A1	A correct final equation as shown or any integer multiple of this. " = 0" must be included.	

Question Number	Scheme	Marks
5(a)	$\sum_{r=1}^{n}(r+1)(r+5)=\sum_{r=1}^{n}(r^2+6r+5)$	B1
	$=\sum_{r=1}^{n}r^2+6\sum_{r=1}^{n}r+5n$	
	$=\frac{n}{6}(n+1)(2n+1)+6\frac{n}{2}(n+1)+5n$	M1A1
	$=\frac{n}{6}(2n^2+3n+1+18n+18+30)$	dM1
	$=\frac{n}{6}(2n^2+21n+49)=\frac{n}{6}(n+7)(2n+7)$ *	A1 * (5)
(b)	$\sum_{r=n+1}^{2n}=\sum_{r=1}^{2n}-\sum_{r=1}^{n}=\frac{2n}{6}(2n+7)(4n+7)-\frac{n}{6}(n+7)(2n+7)$	M1
	$=\frac{n}{6}(2n+7)\{8n+14-(n+7)\}$	
	$=\frac{7n}{6}(2n+7)(n+1)$	A1 (2)
		[7]
(a)		
B1	Brackets multiplied out correctly. Summation signs not needed.	
M1	Use at least two correct formulae from $\sum_{r=1}^{n}r$, $\sum_{r=1}^{n}r^2$ and $\sum_{r=1}^{n}1=n$.	
A1	Fully correct expression.	
dM1	Attempt to remove factor $\frac{n}{6}$ from an expression with common factor n present. (if "5n" is just 5 then this mark will not be scored). Must be seen before the given answer is quoted. No need to simplify the remaining quadratic factor.	
A1*	Obtain the correct 3 term quadratic and factorise. This is a "show that" question, so the 3 TQ must be seen. No errors seen.	
(b)		
M1	Use $\sum_{r=n+1}^{2n}=\sum_{r=1}^{2n}-\sum_{r=1}^{n}$	
A1	Simplify to the correct answer.	

Question Number	Scheme	Marks
6(a)	$\lambda = 4$	B1 (1)
(b)	$\arctan\dfrac{3}{"4"}$ or $\arctan\dfrac{-3}{"4"}$	M1
	(Second quadrant so arg z = 2.498...) = 2.5 (rad)	A1 (2)
(c)(i)	$\dfrac{z+3\text{i}}{2-4\text{i}} = \dfrac{-4+6\text{i}}{2-4\text{i}} \times \dfrac{2+4\text{i}}{2+4\text{i}}$ or $\dfrac{z+3\text{i}}{2-4\text{i}} = a+\text{i}b \Rightarrow -4+6\text{i} = (a+\text{i}b)(2-4\text{i})$	M1
	$= \dfrac{-8+12\text{i}-16\text{i}+24\text{i}^2}{4+16} = -\dfrac{8}{5} - \dfrac{1}{5}\text{i}$ Accept e.g. $\dfrac{-32-4\text{i}}{20}$	dM1A1
	Or $2a+4b=-4, 2b-4a=6 \Rightarrow a=..,b=..$	
(ii)	$z^2 = (-4+3\text{i})^2 = 16-24\text{i}+9\text{i}^2 = 16-24\text{i}-9$	M1
	$= 7-24\text{i}$	A1ft (5)
(d)		B1 B1ft B1ft (3) **[11]**

(a) **B1**	Correct answer. No working needed.
(b) **M1**	For $\arctan\left(\pm\dfrac{3}{"4"}\right)$ with their "4". Can be awarded from $\tan\theta = \pm\dfrac{3}{"4"} \Rightarrow \theta = ...$ or by implication if correct value for either arctan or correct final answer (rounded or not rounded, may be degrees) is seen.
A1	Cao 2.5
(c)(i) **M1**	Multiplies numerator and denominator by complex conjugate of denominator. Award if denominator of 4+16 or 20 is seen instead of product. May still have z at this stage, or even allow with $\lambda + 3\text{i}$ as numerator. Alternatively, sets equal to $a + \text{i}b$ and cross multiplies.
dM1	Using their λ or 4 substitutes **correctly** for z, fully expands the numerator and uses $\text{i}^2 = -1$ Alt, uses $\text{i}^2 = -1$, equates real and imaginary terms and solves their equations for a and b
A1	Correct answer only, as shown or single fraction accepting equivalent fractions or with exact decimals $(-1.6 - 0.2\text{i})$.
(ii) M1	Squaring an expression of form $k + 3\text{i}$ (with a real value for k) to get 3 terms (may be implied) and uses $\text{i}^2 = -1$
A1ft	Correct answer, follow through their $\lambda > 0$ (ie for $"\lambda^2 - 9" - "6\lambda"\text{i}$ must be negative i term)
(d)	NB: Penalise once only (in the first mark due) for mislabelling or failing to label points as long as they look to be placed correctly. Award if lines/arrows not included. Points may be labelled by letter, name or their Cartesian coordinates (which may be given on the axes).
B1	Plots z in second quadrant and z* as mirror image in the Real axis. Both must be labelled.
B1ft	Plot and label C for their solution to (c)(ii) It must be the correct side of B (for their

	answers) and a correct relative scale (so noticeably closer to *O* than their *B* if correct values).
B1ft	Plot and label their *D* (- 24 need not be to scale, but should be further from *O* than their *B*).

Question Number	Scheme	Marks
7(a)	$\begin{vmatrix} 4 & -5 \\ -3 & 2 \end{vmatrix} = 8-15=-7 \Rightarrow \text{Area } T' = \text{"}\pm 7\text{"}\times 23 = ...$	M1
	$\text{Area } T' = 161$	A1 (2)
(b)	$\begin{pmatrix} 4 & -5 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 3p+2 \\ 2p-1 \end{pmatrix} = \begin{pmatrix} 17 \\ -18 \end{pmatrix}$ or $\begin{pmatrix} 3p+2 \\ 2p-1 \end{pmatrix} = \frac{1}{8-15}\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 17 \\ -18 \end{pmatrix}$	
	$4(3p+2)-5(2p-1)=17$ or $-3(3p+2)+2(2p-1)=-18$ or $3p+2=-\frac{1}{7}(34-90)$ or $2p-1=-\frac{1}{7}(51-72)$ $(\text{e.g. } 2p+13=17 \Rightarrow .....)$	M1
	$p=2$	A1 (2)
(c)	Rotation; through 90° clockwise (or 270° anticlockwise) about origin	B1;B1 (2)
(d)	**CA** = **B**	
	$\mathbf{A}^{-1} = -\frac{1}{7}\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$ or $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 4 & -5 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 4a-3b & -5a+2b \\ 4c-3d & -5c+2d \end{pmatrix}$	B1
	$\mathbf{C} = -\frac{1}{7}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix} = ...$ or $\begin{cases} 4a-3b=0 & -5a+2b=1 \\ 4c-3d=-1 & -5c+2d=0 \end{cases} \Rightarrow ...$	M1
	$\mathbf{C} = -\frac{1}{7}\begin{pmatrix} 3 & 4 \\ -2 & -5 \end{pmatrix}$ or $\begin{pmatrix} -\frac{3}{7} & -\frac{4}{7} \\ \frac{2}{7} & \frac{5}{7} \end{pmatrix}$ oe	A1 (3)
		[9]

(a)	
M1	Attempts to find the determinant of **M** and use as a scale factor. Accept if a slip in calculation is made and accept if negative is used for this mark. Dividing by the determinant is M0.
A1	Correct answer only. No working needed (correct answer implies the method).
(b)	
M1	Form a matrix equation using either **A** or an attempt at $\mathbf{A}^{-1}$, obtain a linear equation and solve for *p*.
A1	Correct value for *p*, obtained from a correct equation. (No need to check in other equation.)
(c)	
B1	Rotation, rotates, rotate or rotating (oe) Accept "turn"
B1	Correct angle (degrees or radians) with direction specified and about origin or (0, 0)
(d)	
B1	Correct matrix for $\mathbf{A}^{-1}$ May have been found in (b) but must be used in (d). Alternatively, correct **CA** with unknowns for entries of **C**.
M1	Multiply **B** by $\mathbf{A}^{-1}$ **on the right.** Alternatively, sets **CA** equal to **B** and solves equations.
A1	Correct matrix **C**(isw after a correct answer).

Question Number	Scheme	Marks
8(a)	$200t^3 = 25$ or $\left(\frac{25}{x}\right)^2 = 40x$ or $y^2 = 40\left(\frac{25}{y}\right)$	M1
	$t = \frac{1}{2}$ or $x = \frac{5}{2}$ or $y = 10$	A1
	$\left(\frac{5}{2}, 10\right)$	A1 (3)
(b)	$y^2 = 40x \Rightarrow x = \frac{y^2}{40} \Rightarrow \frac{dx}{dy} = \frac{2y}{40}$ or $2y\frac{dy}{dx} = 40 \Rightarrow \frac{dy}{dx} = \frac{40}{2y}$ or $\frac{dy}{dx} = \frac{\sqrt{10}}{\sqrt{x}}$ or by parametric differentiation: $\frac{dy}{dx} = \frac{1}{t}$	B1
	at $(10, 20)$: $\frac{dy}{dx} = 1$ or $\frac{dx}{dy} = 1$	M1
	Grad normal $= -1$	A1
	$y - 20 = -(x - 10)$	M1
	$x + y - 30 = 0$	A1 (5)
(c)	$xy = 25 \quad x + y - 30 = 0$	
	$x + \frac{25}{x} - 30 = 0$ or $\frac{25}{y} + y - 30 = 0$ or $\frac{5}{t} + 5t - 30 = 0$	M1
	$x^2 - 30x + 25 = 0$ or $y^2 - 30y + 25 = 0$ or $5t^2 - 30t + 5 = 0$	dM1
	$x = 15 \pm \sqrt{200}$ or $y = 15 \pm \sqrt{200}$ or $t = 3 \pm 2\sqrt{2}$ (or exact equivalents)	A1
	eg $x = 15 + \sqrt{200} \Rightarrow y = 15 - \sqrt{200}$	ddM1
	$\left(15 + 10\sqrt{2}, 15 - 10\sqrt{2}\right) \quad \left(15 - 10\sqrt{2}, 15 + 10\sqrt{2}\right)$	A1A1 (6)
		[14]
(a)		
M1	Attempt an equation in a single variable.	
A1	Correct value for x, y or t	
A1	Correct values for x and y. Need not be in coordinate brackets. No other points seen.	
(b)		
B1	Any correct expression involving the derivative, $\frac{dy}{dx}$ or $\frac{dx}{dy}$, for P	
M1	Attempt to obtain value of their derivative at (10, 20). May be from an incorrect curve.	
A1	Correct gradient of normal.	
M1	Equation of normal by any complete method. Must involve a sign change of their derivative and have numerical gradient. Cam be scored from an incorrect starting equation/point.	
A1	Correct equation in form demanded (though terms may be in different order).	
(c)		
M1	Use the equation of H and their equation of the normal from (b) to obtain an equation in a single variable	
dM1	Obtains a 3TQ and attempts to solve by any valid means.	
A1	Correct values for x or y or t. Must be exact but need not be fully simplified (but discriminant must be evaluated).	
ddM1	Use at least one of their values for x or y to obtain a value for the other coordinate or t to find at least one set of coordinates. (Can be scored with inexact values.)	
A1	Either pair of coordinates correct. Allow if unsimplified.	
A1	Second pair of coordinates correct and no extra solutions and both pairs in simplest form (as shown in scheme). Need not be coordinates as long as correctly paired. Award A1A0 if $x = 15 \pm 10\sqrt{2}, y = 15 \pm 10\sqrt{2}$ is given	

Question Number	Scheme	Marks
9(i)	$n=1 \quad u_1 = 3\times\frac{2}{3}-1=1$ (so true for $n = 1$ (†))	B1
	Assume true for $n=k$ ie $u_k = 3\left(\frac{2}{3}\right)^k - 1$ (†)	
	$u_{k+1} = \frac{1}{3}(2u_k - 1) = \frac{1}{3}\left(2\left(3\left(\frac{2}{3}\right)^k - 1\right) - 1\right) = \frac{1}{3}\left(6\left(\frac{2}{3}\right)^k - 2 - 1\right)$	M1A1
	$= \frac{1}{3}\left(2\times3\left(\frac{2}{3}\right)^{k+1}\times\left(\frac{3}{2}\right) - 2 - 1\right)$	dM1
	$= \frac{1}{3}\times2\times3\left(\frac{2}{3}\right)^{k+1}\times\left(\frac{3}{2}\right)+\frac{1}{3}(-2-1)$	
	$= 3\left(\frac{2}{3}\right)^{k+1} - 1$	A1
	$\therefore$ if true for $n=k$, also true for $n=k+1$ (†)	
	$(\text{True for } n=1)$ so $u_n = 3\left(\frac{2}{3}\right)^n - 1$ is true for all $n\in\mathbb{Z}^+$	A1cso (6)
(ii)	$\mathrm{f}(1) = 2^3 + 3^3 = 8 + 27 = 35$ (Multiple of 7) (so true for $n = 1$ (†))	B1
	Assume $\mathrm{f}(k)$ is a multiple of 7 $\mathrm{f}(k) = 2^{k+2} + 3^{2k+1}$ is a multiple of 7 (†)	
	$\mathrm{f}(k+1) - M\mathrm{f}(k) = 2^{k+3} + 3^{2k+3} - M\left(2^{k+2} + 3^{2k+1}\right)$	M1
	$= 2^{k+2}(2-M) + 3^{2k+1}(3^2 - M)$	A1
	$= (2-M)\left(2^{k+2}+3^{2k+1}\right) + 3^{2k+1}\times7$ or $(9-M)\left(2^{k+2}+3^{2k+1}\right) - 7\times2^{k+2}$ oe	dM1
	$\therefore \mathrm{f}(k+1) = 2\mathrm{f}(k) + 7\times3^{2k+1}$ oe e.g. $9\mathrm{f}(k) - 7\times2^{k+2}$	A1
	Or e.g. $7\times3^{2k+1}$ is a multiple of 7, so if f(k) is a multiple of 7 <u>then f(k+1) is also a multiple of 7</u>	
	If the result is true for $n = k$ it is also true for $n = k + 1$ (†)	
	As the result has been shown to be true for $n = 1$, it is true for all $n\in\mathbb{Z}^+$	A1 cso (6) **[12]**

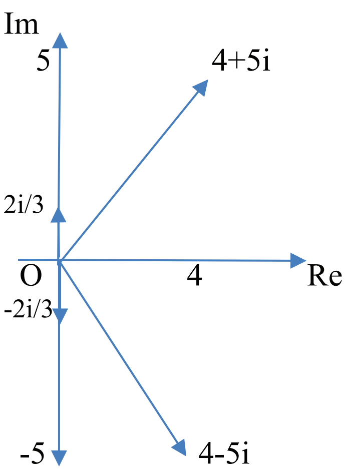
9(i)	
B1	Check that the formula gives 1 when $n = 1$ Working must be shown. (Need not state true for n =1 for this mark – but see final A)
M1	(Assume true for $n = k$ and) attempts to substitute the formula for u_k into $u_{k+1} = \frac{1}{3}(2u_k - 1)$ or equivalent with suffixes increased. Allow slips.
A1	Correct substitution.
dM1	Obtain an expression with $\left(\frac{2}{3}\right)^{k+1}$ and no other k. Alternatively, expands u_{k+1} to a matching expression (ie work from both directions).
A1	Correct expression when $n = k + 1$ At least one intermediate stage of working must be shown and no errors (though notational slips may be condoned). If working from both directions, it is for correct work to reach matching expressions.
A1cso	Correct concluding statements following correct solution which has included each of the points (†) at some stage during the working. Depends on all except the first B mark (e.g. if they think they have checked n =1 but have really checked n = 2). Note: Allow the M's and first two A's for students who go from k+1 to k+2 but treat it as k to k + 1.
(ii)	
B1	Checks the case $n = 1$. Minimum statement of $f(1) = 35$
M1	Attempts an expression for $f(k + 1) - Mf(k)$ with any value of M. Need not be simplified. Most likely with $M = 1$ but may be seen with other values of M. With $M = 0$, $f(k+1) = 2^{k+3} + 3^{2k+3}$ is all that is required.
A1	A correct expression with terms 2^{k+2} and 3^{2k+1} clearly identified.
dM1	Attempts to extract/identify f(k) within a correct expression to give terms divisible by 7. With $M = 0$ look for $f(k+1) = 2\times\left(2^{k+2} + 3^{3k+1}\right) + 7\times 3^{2k+1}$ or $9\times\left(2^{k+2} + 3^{3k+1}\right) - 7\times 2^{k+2}$ oe and similar for other value of M.
A1	One of the correct expressions for f(k+1) shown (or with powers of 2 and 3) or full reason why f(k+1) is divisible by 7, following a suitable expression.
A1cso	Correct concluding statements following correct solution which has included each of the points (†) at some stage during the working. Depends on all previous marks.

Question Number	Scheme	Notes	Marks
1	$f(x) = x^3 + 4x - 6$		
1(i)(a)	$f(1) = -1$ $f(1.5) = 3.375\left(= \frac{27}{8}\right)$	Attempts to evaluate at both end points. If substitution not seen accept $f(1) = -1$ **or** $f(1.5) = 3.375$ as evidence, with any value for the other end.	M1
	Sign change and $f(x)$ continuous therefore α is between $x = 1$ and $x = 1.5$	$f(1) = -1$ **and** $f(1.5) = 3.375$ <u>both correct</u> and mentions/indicates <u>sign change</u>, <u>continuous</u> and <u>conclusion</u>.	A1
			(2)
1(i)(b)	$f'(x) = 3x^2 + 4$	Correct derivative. Can be implied by a correct expression seen later such as $3(1.5)^2+4$	B1
	$x_2 = 1.5 - \frac{f(1.5)}{f'(1.5)} = 1.5 - \frac{3.375}{10.75}$	Attempt Newton-Raphson using the **correct** formula.	M1
	$x_2 = 1.186...$	Accurate first application either awrt 1.186, $\frac{51}{43}$ or a correct numerical expression.	A1
	$x_3 = \left(1.186... - \frac{0.412...}{8.220...}\right) = 1.1358...$		
	$\alpha \approx 1.136$	cao	A1
			(4)
1(ii)	$g(1.4) = 3.442116...$ **or** $g(1.5) = -3.601419...$	Evidence of at least one value correct to 3 d.p. or better. May be implied by correct answer if never seen.	B1
	$\frac{1.5-\beta}{\beta-1.4} = \frac{0-g(1.5)}{g(1.4)-0}$ or $\frac{\beta-1.4}{0-g(1.4)} = \frac{1.5-1.4}{g(1.5)-g(1.4)}$ oe	A correct linear interpolation statement as shown oe (with correct signs). May omit the zeroes or use evaluated values (e.g 0.1 instead of 1.5 – 1.4). Other forms are possible.	B1
	$\Rightarrow \beta \approx 1.448869...$	Evaluates β from an attempt at a linear interpolation statement, allow if signs are incorrect. $\beta \approx$ awrt 1.449	M1 A1
			(4)
			Total 10

Qn No	Scheme	Notes	Marks
2. (a)	$\frac{z_2 z_3}{z_1} = \frac{(p-\mathrm{i})(p+\mathrm{i})(2+\mathrm{i})}{(2-\mathrm{i})(2+\mathrm{i})}$	Multiply top and bottom by complex conjugate of their denominator. (The two M's may be scored if the given numbers are wrongly placed.)	M1
	$= \frac{(p^2+1)(2+\mathrm{i})}{5}$	Simplifies numerator with evidence that $\mathrm{i}^2 = -1$ and denominator real. Accept any equivalent form in the numerator as long as there are not i^2 terms if expanded.	M1
	$= \frac{2(p^2+1)}{5} + \frac{(p^2+1)}{5}\mathrm{i}$	Correct real +imaginary form with i factored out. Accept as single fraction with numerator in correct form. Accept $'a='$ and $'b='$.	A1
			(3)
ALT	$\frac{z_2 z_3}{z_1} = \frac{(p-\mathrm{i})(p+\mathrm{i})}{(2-\mathrm{i})} = a+b\mathrm{i}$ $p^2+1 = (a+b\mathrm{i})(2-\mathrm{i})$ $\left.\begin{array}{l}2a+b=p^2+1\\2b-a=0\end{array}\right\}$	Cross multiplies by $2-\mathrm{i}$ (or their denominator), expands and equates real and imaginary parts. (The two M's may be scored if the given numbers are wrongly placed.)	M1
	$\left.\begin{array}{l}2a+b=p^2+1\\2b-a=0\end{array}\right\} \Rightarrow a=...,b=...$	Attempt to solve their equations.	M1
	$a+b\mathrm{i} = \frac{2(p^2+1)}{5} + \frac{(p^2+1)}{5}\mathrm{i}$	Correct real +imaginary form with i factored out. Accept as single fraction with numerator in correct form. Accept $'a='$ and $'b='$.	A1
			(3)
2(b)	$\left\lvert\frac{z_2 z_3}{z_1}\right\rvert^2 = \frac{4(p^2+1)^2}{25} + \frac{(p^2+1)^2}{25}$	Correct attempt at the modulus or modulus squared. Accept with their answers to part (a). Any erroneous i or i^2 is M0.	M1
	$\frac{4(p^2+1)^2}{25} + \frac{(p^2+1)^2}{25} = \left(2\sqrt{5}\right)^2$	Their $\left\lvert\frac{z_2 z_3}{z_1}\right\rvert^2 = \left(2\sqrt{5}\right)^2$	dM1
	$(p^2+1)^2 = 100 \Rightarrow p = \pm 3$	Attempt to solve and achieves $p = ...$ (may be scored from use of $\lvert..\rvert^2 = 2\sqrt{5}$) $p = \pm 3$	M1 A1
			(4)
ALT 1	$\left\lvert\frac{z_2 z_3}{z_1}\right\rvert = 2\sqrt{5} \Rightarrow \lvert z_2 z_3\rvert = \sqrt{4+1} \times 2\sqrt{5}$	Cross multiplies and attempts $\lvert z_1 \rvert$	M1
	$\Rightarrow \lvert z_2\rvert^2 = \sqrt{4+1} \times 2\sqrt{5} \Rightarrow p^2+1 = ...$	Attempts $\lvert z_2 z_3 \rvert$ either directly or using $\lvert z_2 z_2^*\rvert = \lvert z_2\rvert^2$ to get an equation in p.	dM1
	$(p^2+1) = 10 \Rightarrow p = \pm 3$	Attempt to solve and achieves $p = ...$ $p = \pm 3$	M1 A1
			(4)
			Total 7

Question Number	Scheme	Notes	Marks
3(a)	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 2 & 0 \\ 1 & 3 & 1 \end{pmatrix}=\begin{pmatrix} 1 & 3 & 1 \\ -2 & -2 & 0 \end{pmatrix}$	Attempt to multiply in the correct order with at least four correct elements. May be done as three separate calculations, so look for at least 4 correct values.	M1
	(1,−2), (3,−2) and (1,0)	Accept as individual column vectors but not as a single 2x3 matrix.	A1
			(2)
3(b)	Rotation	Accept rotate or turn oe.	B1
	270° (anticlockwise) **about the origin**	Accept −90° (anticlockwise) or 90° clockwise (must be stated) and (0,0) or O. *Assume anticlockwise unless otherwise stated.*	B1
			(2)
3(c)	$\mathbf{Q}=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \mathbf{R}=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	One correct, both correct	B1,B1
			(2)
3(d)	$\mathbf{RQ}=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Multiplication in correct order for their matrices and at least 1 row or 1 column correct.	M1
	$=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Correct matrix	A1
			(2)
3(e)	Reflection	Correct type identified.	B1
	in (the line) $y=x$	Correct line of reflection specified, accepting equivalent forms (e.g. line at angle 45° (anticlockwise) to the (positive) x-axis).	B1
			(2)
			Total 10

Question Number	Scheme	Notes	Marks
4(a)	$y=25x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-25x^{-2}$ $xy=25 \Rightarrow y+x\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}x}=-\frac{5}{t^2}.\frac{1}{5}$	Attempts a derivative expression, such as $\frac{\mathrm{d}y}{\mathrm{d}x}=kx^{-2}$ or $x\frac{\mathrm{d}y}{\mathrm{d}x}=ky$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\text{their}\frac{\mathrm{d}y}{\mathrm{d}t}}{\text{their}\frac{\mathrm{d}x}{\mathrm{d}t}}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-25x^{-2}$ or $\frac{-y}{x}$ or $\frac{-1}{t^2}$	Correct derivative.	A1
	$y-\frac{5}{t}=-\frac{1}{t^2}(x-5t)$	Uses $y-\frac{5}{t}=$ (their gradient) x $(x-5t)$ or $y=$ (their gradient)$x+c$ using $x=5t, y=\frac{5}{t}$ in an attempt to find c. Their gradient must be a function of t for marks to be awarded.	M1
	$t^2y+x=10t$ *	cso	A1
			(4)
Alt 4(a)	Intersect when $(10t-t^2y)y=25$	Substitutes given line equation into the curve equation to find intersections	M1
	$\Rightarrow t^2y^2-10ty+25=0$	Correct equation	A1
	$\Rightarrow (ty-5)^2=0 \Rightarrow y=\frac{5}{t}$ is single root.	Shows the equation has a single root.	M1
	Single root when $y=\frac{5}{t}$ means the line is the tangent to H at P.	All work correct, with correct conclusion.	A1
			(4)
4(b)	$t^2(-5)+15=10t$	Substitute $(15,-5)$ into equation of tangent	M1
	$(t+3)(t-1)=0 \Rightarrow t=..$	Solves via any valid means	M1
	$\Rightarrow t=-3, t=1$ (or one correct point)	Both values of t **or** one correct coordinate pair.	A1
	$\left(-15,-\frac{5}{3}\right)$ and $(5,5)$	Both correct sets of coordinates.	A1
			(4)
ALT	$t_1{}^2y+x=10t_1$ $t_2{}^2y+x=10t_2$	Form equations with their t_1,t_2 and attempt to solve for x and y	M1
	$(x=)\frac{10t_1t_2}{(t_1+t_2)}=15, (y=)\frac{10}{(t_1+t_2)}=-5$	Equates to coordinate and attempt to solve for t_1,t_2	M1
	$t_1=-3, t_2=1$ (or one correct point)	Both values of t **or** one correct coordinate pair.	A1
	$\left(-15,-\frac{5}{3}\right)$ and $(5,5)$	Both correct sets of coordinates.	A1
			(4)
			Total 8

Question Number	Scheme	Notes	Marks
	$\mathrm{f}(x)=\left(9x^2+d\right)\left(x^2-8x+(10d+1)\right)$		
5(a)	$9x^2+d=0\Rightarrow x=\pm\sqrt{-\dfrac{d}{9}}$ or $\pm\dfrac{\mathrm{i}\sqrt{d}}{3}$	or exact equivalents	B1
	or $x=\dfrac{8\pm\sqrt{64-4(10d+1)}}{2}$ or $(x-4)^2-16+10d+1=0\Rightarrow x=...$	Solve $x^2-8x+(10d+1)=0$ by formula or completing the square. Must have complete constant term.	M1
	$x=4+\sqrt{15-10d}$ and $x=4-\sqrt{15-10d}$	oe with discriminant simplified. Mark final answer, do not isw.	A1
			(3)
5(b)	$x=\pm\dfrac{2\mathrm{i}}{3}$ or f.t. their roots	Correct roots, or f.t.their answer for the $9x^2+d$	B1ft
	$x=4\pm5\mathrm{i}$ or f.t. their roots	Correct roots for the given quadratic, or f.t. their 3TQ	B1ft
	SC Award B1ftB0 if only one of **each** pair is given.		
			(2)
5(c)		Two roots on imaginary axis the same distance from *O*. Follow through their *imaginary* roots from (b). B0 if real roots found. Their two *complex* roots with real and imaginary parts, one the conjugate of the other, so reflected in the real axis. Must be correct relative scale compared with the imaginary roots **if the first B1 in (c) has been awarded** (ie clearly further from *O* if correct, or f.t. their answers). But if first B0 has been given, ignore scales. Accept points or vectors. Complex numbers must be labelled in some way e.g. via scales or coordinates or vectors.	B1ft B1ft
			(2)
			Total 7

Question Number	Scheme	Notes	Marks
6(a)	$y=\sqrt{8}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx}=\frac{1}{2}\sqrt{8}x^{-\frac{1}{2}}=\sqrt{2}x^{-\frac{1}{2}}$ $y^2=8x \Rightarrow 2y\frac{dy}{dx}=8$ or $\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}=4.\frac{1}{4t}$	Attempts a derivative expression, such as $\frac{dy}{dx}=kx^{-\frac{1}{2}}$ or $ky\frac{dy}{dx}=c$ or their $\frac{dy}{dt}\times\left(\frac{1}{\text{their }\frac{dx}{dt}}\right)$	M1
	$\frac{dy}{dx}=\frac{1}{2}\sqrt{8}x^{-\frac{1}{2}}\left(=\sqrt{2}x^{-\frac{1}{2}}\right)$ or $2y\frac{dy}{dx}=8$ or $\frac{dy}{dx}=4.\frac{1}{4t}\left(=\frac{1}{t}\right)$	Correct differentiation (need not have substituted for t etc)	A1
	At P, gradient of normal is $m_N=-p$	Correct gradient for the normal.	A1
	$y-4p=-p(x-2p^2)$	$y-4p=(\text{their } m_N)\times(x-2p^2)$ or $y=(\text{their } m_N)x+c$ using $x=2p^2, y=4p$ in an attempt to find c. Their gradient must be a function of p for marks to be awarded. Must use a changed gradient, not tangent gradient.	M1
	$y+px=2p^3+4p$ *	cso	A1
			(5)
6(b)	$y+qx=2q^3+4q$	oe	B1
			(1)
6(c)			
	$y+px=2p^3+4p$ $\underline{y+qx=2q^3+4q}$ $px-qx=2p^3+4p-2q^3-4q$	Attempt to solve simultaneous equations. A correct equation in only one variable	M1 A1
	$(p-q)x=2((p-q)(p^2+pq+q^2)+2(p-q))$	Attempts to simplify the expression to required form. E.g. factorise difference of two cubes, $p^3-q^3=(p-q)(p^2+pq+q^2)$ or equivalent work to enable $p-q$ to cancel.	M1
	$x=2(p^2+pq+q^2+2)$*	cso for reaching the correct x coordinate.	A1
	$y+2p^3+2p^2q+2pq^2+4p=2p^3+4p$ $y=-2pq(p+q)$*	cso for reaching both coordinates correctly.	A1
			(5)
ALT	$qy+pqx=2p^3q+4pq$ $\underline{py+pqx=2pq^3+4pq}$ $py-qy=2pq^3-2p^3q$	Attempt to solve simultaneous equations. A correct equation in only one variable	M1 A1

	$(p-q)y=-2pq(p^2-q^2)$ $(p-q)y=-2pq(p-q)(p+q)$	Attempts to simplify the expression to required form. E.g. Attempt to factorise difference of two squares. $p^2-q^2=(p-q)(p+q)$ or equivalent work to enable $p-q$ to cancel.	M1
	$y=-2pq(p+q)$*	cso for reaching the correct y coordinate.	A1
	$-2p^2q^2-2pq^3+pqx=2p^3q+4pq$ $x=2(p^2+pq+q^2+2)$*	cso for reaching both coordinates correctly.	A1
			(5)
ALT 2	$-2pq(p+q)+2p(p^2+pq+q^2+2)=$ $-2p^2q-2pq^2+2p^3+2p^2q+2pq^2+4p$	Substitutes both coordinates into the equation for normal at P and expands brackets.	M1
	$=2p^3+4p$ so N is on normal at P	Simplifies correctly to $2p^3+4p$ and deduces N is on the normal at P	A1
	$-2pq(p+q)+2q(p^2+pq+q^2+2)=$ $-2p^2q-2pq^2+2p^2q+2pq^2+2q^3+4q$	Substitutes both coordinates into the equation for normal at Q and expands brackets.	M1
	$=2q^3+4q$ so N is on normal at Q	Simplifies correctly to $2q^3+4q$ and deduces N is on the normal at Q but penalise only the first instance for missing the deduction.	A1
	As N is on both normals it is therefore the intersection point of them.	Makes a suitable conclusion	A1
			(5)
6(d)	Grad $PQ=\dfrac{4q-4p}{2q^2-2p^2}\left(=\dfrac{2}{p+q}\right)$	Use of $\dfrac{y_2-y_1}{x_2-x_1}$ and substituting.	M1A1
	Grad $ON=\dfrac{-2pq(p+q)}{2(p^2+pq+q^2+2)}$	Can be unsimplified.	B1
	Grad $PQ\times$Grad ON $=\dfrac{2}{p+q}\times\dfrac{-2pq(p+q)}{2(p^2+pq+q^2+2)}=-1$	Using product of their gradients $=-1$	M1
	$(p+q)^2-3pq=-2$	-2	A1
			(5)
			Total 16

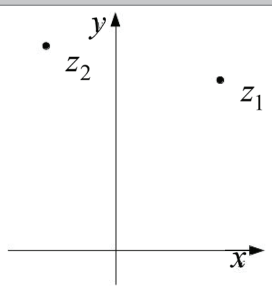
Question Number	Scheme	Notes	Marks
7	$\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$		
7(a)	$n=1, \text{LHS}=1^2=1,\ \text{RHS}=\frac{1}{6}.2.3=1$	Shows both LHS=1 and RHS=1 Accept LHS = 1 but must see at least $\frac{1}{6}.2.3=1$ for RHS.	B1
	Assume true for $n=k$		
	When $n=k+1$ $\sum_{r=1}^{k+1} r^2 = \frac{k}{6}(k+1)(2k+1)+(k+1)^2$	Adds $(k+1)^2$ to result for $n=k$	M1
	$=\frac{(k+1)}{6}(k(2k+1)+6(k+1))$	Attempt to factorise by $\frac{(k+1)}{6}$	dM1
	$=\frac{(k+1)}{6}(k+2)(2k+3)$ $=\frac{(k+1)}{6}((k+1)+1)((2(k+1)+1)$	Either factorised form. SC allow dM1A0 for fully factorising to a cubic expression and going direct to the fully factorised expression with no intermediate quadratic seen.	A1
	True for $n=1$. If true for $n=k$ then true for $n=k+1$ therefore true for all n.	Complete proof with no errors and these 4 statements seen anywhere. Depends on both M's and the A, but may be scored if the B is lost as long as some indication of true for $n = 1$ is given.	A1cso
			(5)
7(b)	$\sum_{r=1}^{n}(r^2+2)=\sum_{r=1}^{n} r^2+\sum_{r=1}^{n} 2$ $=\frac{n}{6}(n+1)(2n+1)+\ \ldots$	Split into the addition of 2 sums and applies the result of (a).	M1
	$=\frac{n}{6}(n+1)(2n+1)+2n$	Correct expression.	A1
	$=\frac{n}{6}(2n^2+3n+13)$	Factorises out the $\frac{n}{6}$ - must have a common factor n to achieve this mark; Simplifies to correct answer.	M1; A1
	$(a=2, b=3, c=13)$		
			(4)
7(c)	$\sum_{r=10}^{25}(r^2+2)=S_{25}-S_9$ $=\frac{25}{6}.(2\times25^2+3\times25+13)-\frac{9}{6}(2\times9^2+3\times9+13)$	Attempts $S_{25}-S_9$ or $S_{25}-S_{10}$ with some substitution.	M1
	$=\frac{25}{6}\times1338-\frac{9}{6}\times202=5575-303=5272$	For 5272	A1
	Note: Answer only (from calculator) is M0A0 as question requires use of part (b).		
			(2)
			Total 11

Question Number	Scheme	Notes	Marks
	$\mathrm{f}(n)=4^{n+2}+5^{2n+1}$ divisible by 21		
8	$n=1, 4^3+5^3=189=9\times 21$ (Or $n=0$, $4^2+5^1=21$)	$\mathrm{f}(1)=21\times 9$ Accept f(0) = 21 as an alternative starting point.	B1
	Assume that for $n=k$, $\mathrm{f}(k)=(4^{k+2}+5^{2k+1})$ is divisible by 21 for $k\in\mathbb{Z}^+$.		
	$\mathrm{f}(k+1)-\mathrm{f}(k)=4^{k+3}+5^{2k+3}-(4^{k+2}+5^{2k+1})$	Applies $\mathrm{f}(k+1)$ with at least 1 power correct. May be just as f(k + 1), or as part of an expression in f(k + 1) and f(k).	M1
	$=4.4^{k+2}+25.5^{2k+1}-4^{k+2}-5^{2k+1}$	For a correct expression in f(k + 1), and possibly f(k), with powers reduced to those of f(k).	A1
	$=3.4^{k+2}+24.5^{2k+1}$		
	$=3\mathrm{f}(k)+21.5^{2k+1}$ or $=24\mathrm{f}(k)-21.4^{k+2}$	For one of these expression or equivalent with obvious factor of 21 in each.	A1
	$\mathrm{f}(k+1)=4\mathrm{f}(k)+21.5^{2k+1}$	Makes $\mathrm{f}(k+1)$ the subject or gives clear reasoning of each term other than $\mathrm{f}(k+1)$ being divisible by 21. Dependent on at least one of the previous accuracy marks being awarded.	dM1
	{ $\mathrm{f}(k+1)$ is divisible by 21 as both $\mathrm{f}(k)$ and 21 are both divisible by 21}		
	If the result is **true for $n = k$**, then it is now **true for $n = k + 1$**. As the result has shown to be **true for n = 1 (or 0)**, then the result is **true for all n** ($\in\mathbb{Z}^+$).	Correct conclusion seen at the end. Condone true for n = 1 stated earlier. Depends on both M's andA's, but may be scored if the B is lost as long as at least f(1) = 189 was reached (so e.g. if the 21×9 was not shown)	A1 cso
			(6)
ALT for first 4 marks	$n=1, 4^3+5^3=189=9\times 21$ (Or $n=0$, $4^2+5^1=21$)	As main scheme.	B1
	$\mathrm{f}(k+1)-\alpha\mathrm{f}(k)=4^{k+3}+5^{2k+3}-\alpha(4^{k+2}+5^{2k+1})$	Attempts f(k + 1) in any equation (as main scheme).	M1
	$\mathrm{f}(k+1)-\alpha\mathrm{f}(k)=(4-\alpha)4^{k+2}+(25-\alpha)5^{2k+1}$	For a correct expression with any α, with powers reduced to match f(k).	A1
	$\mathrm{f}(k+1)-\alpha\mathrm{f}(k)=(4-\alpha)(4^{k+2}+5^{2k+1})+21.5^{2k+1}$ $\mathrm{f}(k+1)-\alpha\mathrm{f}(k)=(25-\alpha)(4^{k+2}+5^{2k+1})-21.4^{k+2}$	Any suitable equation with powers sorted appropriately to match f(k)	A1
	NB: $\alpha=0, \alpha=4, \alpha=25$ will make relevant terms disappear, but marks should be awarded accordingly.		
			Total 6

Question	Scheme	Marks
1.	$\det \mathbf{M} = 3x \times (2-x) - (4x+1) \times 7 = \ldots$	**M1**
	$= -3x^2 - 22x - 7$ or $3x^2 + 22x + 7$	**A1**
	$-3x^2 - 22x - 7 = 0 \Rightarrow (-3x-1)(x+7) = 0 \Rightarrow x = \ldots$	**M1**
	$-3x^2 - 22x - 7 > 0 \Rightarrow "-7" < x < "-\frac{1}{3}"$	**M1**
	So range is $-7 < x < -\frac{1}{3}$ or $(x \in)\left(-7, -\frac{1}{3}\right)$	**A1**
		(5)
		(5 marks)

Notes:

M1: Attempts to expand the determinant of **M.** Allow with + between the 2 products.
A1: Correct simplified quadratic with = or an inequality sign or neither
M1: Attempts to solve their three term quadratic, any valid means (usual rules – see front pages). Correct answers seen implies correct method. Can be awarded even if the roots are complex.
M1: Chooses the inside region for their roots, accept with strict or loose inequalities.
A1: Correct answer. Accept $x > -7 \cap x < -\frac{1}{3}$

Question	Scheme	Marks
2(a)		M1 A1
		(2)
(b) (i)	$\|z_1\| = \sqrt{3^2+5^2} = \sqrt{34}$	B1
		(1)
(ii)	$\frac{z_1}{z_2} = \frac{3+5\text{i}}{-2+6\text{i}} \times \frac{-2-6\text{i}}{-2-6\text{i}} = \ldots$	M1
	$= \frac{-6-18\text{i}-10\text{i}+30}{40}$	A1
	$= \frac{3}{5} - \frac{7}{10}\text{i}$	A1
		(3)
(c)	$\arg\frac{z_1}{z_2} = \arctan\frac{-7/10}{3/5} = \arctan\frac{-7}{6} = \ldots$ but allow $\arctan\frac{7}{6}$ for M1	M1
	$= -0.86$ or 5.42 (awrt)	A1
		(2)
		(8 marks)

Notes:

(a)
M1: Points in correct quadrants - z_1 in quadrant 1 and z_2 in quadrant 2. Must be clearly labelled either eg z_1 or 3 + 5i or correct numbers on the axes. (Accept with vector arrows.)
A1: Correct diagram, z_1 in first quadrant further away from real axis than imaginary and z_2 in second quadrant, closer to imaginary axis but above z_1 **OR** correct nos on their axes (imag axis may include i), **but not** dashes w/o any indication of scale.
Allow M1A0 for points unlabelled but diagram otherwise correct.
(b)(i)
B1: Correct modulus. Must be evaluated to $\sqrt{34}$ Question says "without using your calculator" so decimal answers can be ignored (isw) but exact answer must be seen somewhere.

(ii)
M1: Multiplies numerator and denominator by the conjugate of their denominator.
A1: Correct unsimplified (or simplified) numerator, with the i^2 correctly dealt with, and correct denominator.
A1: Correct answer. Allow as shown, $\frac{6}{10} - \frac{7}{10}\text{i}$, or 0.6 - 0.7i.

(c)

M1: For $\arctan\left(\pm"\frac{7}{6}"\right)$ (not necessarily simplified to this) or $\tan\alpha = \pm\frac{7}{6} \quad \alpha = ...$ This mark is available if answer is given in degrees. Can use $\arctan\left(\pm"\frac{6}{7}"\right)$ provided a complete method to reach the correct arg is seen.
A1: For awrt −0.86 or awrt 5.42 Must be radians.

ALT for (c):
M1: Use argz_1 – arg z_2 correctly
A1: Correct answer

Question	Scheme	Marks
3(a)	$\left(\frac{9}{2},0\right)$	B1
		(1)
(b)	$PS=9$	B1
	$x_P=-\frac{9}{2}+9=\frac{9}{2}\Rightarrow OP=\sqrt{\left(\frac{9}{2}\right)^2+\left(18\times\frac{9}{2}\right)}=...$	M1
	So perimeter = $"\frac{9}{2}"+"9"+"\frac{9\sqrt{5}}{2}"$	dM1
	$=\frac{27+9\sqrt{5}}{2}$ oe	A1
		(4)
		(5 marks)

Notes:

(a)
B1: Correct coordinates**.**
(b)
B1: Deduces $PS=9$ from the focus directrix property (may be implied by seeing it embedded in an expression for the perimeter). May find coordinates of P first and then attempt Pythagoras theorem – must be correct. May be seen on the diagram. Allow even if incorrect value used later.
M1: Uses distance from directrix to find x coordinate of P and goes on to find OP by Pythagoras (with a plus sign).
dM1: Sums their **three** side lengths. Extras – including 0 – score M0. Depends on the previous M mark.
A1: Correct answer. Equivalents must be in simplified surd form.

Question	Scheme	Marks
4(a)	$4-3\text{i}$	B1
		(1)
(b)	$(x-(4+3\text{i}))(x-(4-3\text{i}))=\ldots$	M1
	$x^2-8x+25$	A1
		(2)
(c)	E.g. Product of roots is 225, so product of real roots is $\frac{225}{25}=9$ Or $x^4+Ax^3+Bx^2+Cx+225=(x^2-8x+25)(x^2+\ldots+9)$	M1
	Hence (as root is positive) repeated real root is 3	A1
		(2)
(d)	$(x^2-8x+25)(x^2-6x+9)$ $=x^4-6x^3+9x^2-8x^3+48x^2-72x+25x^2-150x+225$	M1
	$=x^4-14x^3+82x^2-222x+225$ Two correct middle term coefficients	A1
	So $A=-14$, $B=82$ and $C=-222$ (or accept in the quartic)	A1
		(3)
		(8 marks)

Notes:

(a)
B1: For $4-3\text{i}$
(b)
M1: Correct strategy to find a quadratic factor. May expand as shown in scheme, or may look for sum of roots and product of roots first and then write down the factor.
A1: Correct quadratic factor. Can be written down – give M1A1 if correct, M0A0 if incorrect.
Ignore "= 0" with their quadratic factor.
Alt for (b):
M1: Product of complex roots is 25, so product of real roots is $\frac{225}{25}=9$, so the (positive) real root is "3", hence quadratic factor is $(x-\text{"3"})^2$
A1: x^2-6x+9 **or** $(x-3)^2$
(c)
M1: A complete strategy to deduce the real root or its square. May consider product of roots, as in scheme, or may first attempt to factorise/long division to find the other quadratic factor – award at the point the quadratic factor with real roots is found. May have been seen in (b)
A1: Real root is 3. (No need to see rejection of the negative possibility.)
Not a "show that" so award M1A1 if correct root is written down with no working.

(d) **M1:** Attempts to expand the two quadratic factors – one of which must have a repeated root, so $\left(x^2 \pm 9\right)$ scores M0. (Alternative, may apply –(sum of roots) to find A, pair sum to find B etc – accept method for at least two constants.) **A1:** Two correct values of the three. Accept as embedded in a quartic equation. **A1:** All three correct. Accept as embedded in their quartic equation. If their answers are wrong a correct method would get M1A0A0 but w/o some working score M0

Question	Scheme	Marks
5(a)	Two of: Rotation; about O; through $60°$ $\left(\frac{\pi}{3}\right)$ (anticlockwise)	M1
	All of: Rotation about O through $60°$ $\left(\frac{\pi}{3}\right)$ (anticlockwise)	A1
		(2)
(b)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1
		(1)
(c)	$\mathbf{R} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	M1
	$= \begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ **QP** correctly found	A1
		(2)
(d)	$3\mathbf{R} = \begin{pmatrix} -\frac{3\sqrt{3}}{2} & -\frac{3}{2} \\ -\frac{3}{2} & \frac{3\sqrt{3}}{2} \end{pmatrix}$ or correctly deals with 3 as a multiple.	B1ft
	Required matrix is $(3\mathbf{R})^{-1} = \dfrac{1}{\left(-\frac{3\sqrt{3}}{2}\right)\left(\frac{3\sqrt{3}}{2}\right)-\left(-\frac{3}{2}\right)\left(-\frac{3}{2}\right)}\begin{pmatrix} \frac{3\sqrt{3}}{2} & \frac{3}{2} \\ \frac{3}{2} & -\frac{3\sqrt{3}}{2} \end{pmatrix} = \ldots$ Or $(\mathbf{R})^{-1} = \dfrac{1}{\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)}\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1\sqrt{3}}{2} \end{pmatrix} = \ldots$	M1
	$(3\mathbf{R})^{-1} = \frac{1}{-9}\begin{pmatrix} \frac{3\sqrt{3}}{2} & \frac{3}{2} \\ \frac{3}{2} & -\frac{3\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{6} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{\sqrt{3}}{6} \end{pmatrix}$	A1
		(3)
		(8 marks)

Notes:
(a) **M1:** Two aspects of the type, centre of rotation and angle correct. Accept equivalent angles or angle in radians. (E.g. 300° clockwise is fine). Assume anticlockwise unless otherwise stated. **A1:** Fully correct description. Accept just 60° for the angle, but 60° clockwise is incorrect **(b)** **B1:** Correct matrix. **(c)** **M1:** Attempts to multiply **Q** and **P** in the correct order. **A1: QP** correct **(d)** **B1ft:** Multiplies all elements of their matrix by 3, or multiplies all elements of their $\mathbf{R}^{-1}$ by $\frac{1}{3}$ **M1:** Attempts the inverse of their 3**R** or **R**. This must be a complete method – ie must transpose and evaluate the determinant and use it. Alternatively, they may attempt an inverse from first principles. Award this mark if a slip is made in solving their simultaneous equations. **A1:** Correct answer. Accept alternative forms

Question	Scheme	Marks
6(a)	(i) $\alpha+\beta=-\frac{5}{A}$	B1
	(ii) $\alpha\beta=-\frac{12}{A}$	B1
		(2)
(b)	$\left(\alpha-\frac{3}{\beta}\right)+\left(\beta-\frac{3}{\alpha}\right)=(\alpha+\beta)-3\left(\frac{\alpha+\beta}{\alpha\beta}\right)=-\frac{5}{A}-3\left(\frac{-5}{A}\right)\times\frac{-A}{12}$	M1
	$-\frac{5}{A}-\frac{15}{12}=\frac{5}{4}\Rightarrow A=\ldots$	dM1
	$A=-2$	A1
		(3)
(c)	$\left(\alpha-\frac{3}{\beta}\right)\left(\beta-\frac{3}{\alpha}\right)=\alpha\beta-6+\frac{9}{\alpha\beta}=-\frac{12}{A}-6+\frac{9}{-12/A}$	M1
	$-\frac{12}{"-2"}-6-\frac{9"-2"}{12}=\frac{B}{4}\Rightarrow B=\ldots$	dM1
	$B=6$	A1
		(3)
		(8 marks)

Notes:

(a)
(i) B1: Correct expression for $\alpha+\beta$
(ii) B1: Correct expression for $\alpha\beta$
(b)
M1: Attempts the sum of roots for second equation in terms of A using results from (a). Allow slips in signs.
dM1: Equates the sum of roots to $\frac{5}{4}$ and solves for A. Depends on the previous M mark.
A1: $A=-2$
(c)
M1: Attempts the product of roots for second equation in terms of A using results from (a). Allow slips in signs. May be using their value of A or A itself
dM1: Equates the product of roots to $\frac{B}{4}$ and solves for B using their value of A. Depends on first M mark of (c).
A1: $B=6$

Question	Scheme	Marks
7(a)	$\dfrac{dy}{dx}=-\dfrac{36}{x^2}$ oe	B1
	$m_t=-\dfrac{36}{4^2}\Rightarrow m_n=\dfrac{16}{36}=\dfrac{4}{9}$	M1
	Normal is $y-9=\dfrac{4}{9}(x-4)$	M1
	$\Rightarrow 9y-81=4x-16\Rightarrow 4x-9y+65=0$ *	A1*
		(4)
(b)	Normal meets H again when $4x-9\times\dfrac{36}{x}+65=0$ or $4\times\dfrac{36}{y}-9y+65=0$	M1
	$\Rightarrow 4x^2+65x-324=0\Rightarrow x=\ldots$ or $9y^2-65y-144=0\Rightarrow y=\ldots$	dM1
	$\Rightarrow Q=\left(-\dfrac{81}{4},-\dfrac{16}{9}\right)$	A1
	At $x=-\dfrac{81}{4}$, $\dfrac{dy}{dx}=-\dfrac{36}{\left(-\dfrac{81}{4}\right)^2}=\ldots$ so tangent is $y-\left(-\dfrac{16}{9}\right)=-\dfrac{64}{729}\left(x-\left(-\dfrac{81}{4}\right)\right)$	M1
	$y=-\dfrac{64}{729}x-\dfrac{32}{9}$	A1(5)
		(9 marks)

Notes:

(a)

B1: Correct expression for $\dfrac{dy}{dx}$, or any equivalent correct expression including it, such as $x\dfrac{dy}{dx}+y=0$ or $\dfrac{dy}{dx}=-\dfrac{1}{t^2}$

M1: Attempts negative reciprocal gradient at the point P

M1: Uses their normal (changed from tangent) gradient and $P(4,9)$ to find the equation of the normal. Look for $y-9=\text{"}m_n\text{"}(x-4)$ Working must be shown for their constant if $y=mx+c$ is used as this is a "show that" question.

A1*: Correct equation achieved from correct working with intermediate step.

(b)

M1: Substitutes hyperbola equation into the **given** normal to obtain an equation in one variable. Other valid means of obtaining an equation in a single variable are acceptable.

dM1: Gathers terms and solves the 3 term quadratic to find a value $\neq 4$ for x or $\neq 9$ for y. Solution by calculator allowed if correct roots (or values $\neq 4$ for x or $\neq 9$ for y) are shown

A1: Correct coordinates of intersection.

M1: Uses their x value to find the gradient at Q and then uses the intersection point with their gradient to form the equation of the line.

A1: Correct equation.

Question	Scheme	Marks
8(a)	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>f(x)</td><td>0.5</td><td>**−1.2401**</td><td>−0.2885</td><td>**0.1508**</td><td>0.5840</td></tr></table> One correct Both correct	B1 B1
		(2)
(b)	Identify an interval where the sig changes and mention the change of sign	M1
	f is continuous on [3,4], not on [1,2] hence the root is in [3,4]	A1
		(2)
(c)	$\mathrm{f}(3.5) = -0.064... < 0$ so root in [3.5,4]	M1
	$\mathrm{f}(3.75) = 0.0435... > 0$	M1
	Hence root is in the interval [3.5, 3.75]	A1
		(3)
(d)	E.g. $\dfrac{\mathrm{f}(-0.5)-\mathrm{f}(-1)}{-0.5-(-1)} = \dfrac{\mathrm{f}(-0.5)-0}{-0.5-\beta}$ or $\dfrac{\beta-(-1)}{0-\mathrm{f}(-1)} = \dfrac{\beta-(-0.5)}{0-\mathrm{f}(-0.5)}$ etc	M1
	$\Rightarrow \beta = -0.5 - \dfrac{0.5\times\mathrm{f}(-0.5)}{\mathrm{f}(-0.5)-\mathrm{f}(-1)} = ...$ or $\beta = \dfrac{-0.5\mathrm{f}(-1)+\mathrm{f}(-0.5)}{(\mathrm{f}(-1)-\mathrm{f}(-0.5))} = ...$ etc	dM1
	$= -0.5 - \dfrac{0.5\times 0.5786...}{0.5786...-(-0.875)} = -0.6990... = -0.699$ (awrt)	A1
		(3)
		(10 marks)

Notes:

Accept open or closed intervals throughout the question where relevant and intervals described by inequalities.

(a)

B1: One correct value of the two missing.

B1: Both values correct.

(b)

M1: Identifies at least one of the intervals on which a sign change occurs – must mention sign changing.

A1: Correct interval with reason given. Accept reasons such as f not defined at $\frac{5}{3}$ in [1,2] or $x = \frac{5}{3}$ is an asymptote as reason for dismissing this interval.

(c)

M1: Evaluates f at the midpoint of their chosen interval from (b) and selects interval of length 0.5 in which the root lies. This mark can be awarded if the interval was incorrect (even if no change of sign in that interval)

M1: Evaluates f at the midpoint of their interval of length 0.5, and considers the signs or chooses the "correct" interval of length 0.25. There must have been a change of sign in their initial interval for this mark to be awarded.

A1: Correct interval selected with all values correct to at least 1 s.f. rounded or truncated. No extra intervals included.

(d)
M1: Correct interpolation strategy. Accept any correct statement such as the one shown. Sign errors imply an incorrect formula unless they follow a correct general statement.
dM1: Rearranges to find β and evaluates.
A1: Accept awrt −0.699 following correct working.

Question	Scheme	Marks
9(a)	For $n = 1$, $\sum_{r=1}^{1} r^3 = 1$ and $\frac{1}{4}(1^2)(1+1)^2 = \frac{1}{4}\times 1\times 4 = 1$ So true for $n = 1$	B1
	(Assume the result is true for $n = k$, so $\sum_{r=1}^{k} r^3 = \frac{1}{4}k^2(k+1)^2$) Then $\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$	M1
	$= \frac{1}{4}(k+1)^2\left[k^2 + 4(k+1)\right] = \frac{1}{4}(k+1)^2\left[k^2 + 4k + 4\right]$	M1
	$= \frac{1}{4}(k+1)^2(k+2)^2$	A1
	$\left[= \frac{1}{4}(k+1)^2((k+1)+1)^2\right]$ Hence result is true for $n = k+1$. As **true for n = 1** and have shown **if true for n = k then it is true for n = k +1, so it is true for all** $n \in \mathbb{N}$ by induction.	A1
		(5)
(b)	$\sum_{r=1}^{n} r(r+1)(r-1) = \sum_{r=1}^{n} r^3 - r$	B1
	$= \frac{1}{4}n^2(n+1)^2 - \frac{1}{2}n(n+1)$	M1
	(Please note the mark above is incorrectly labelled as A1 on e-PEN)	
	$= \frac{1}{4}n(n+1)\left[n^2 + n - 2\right] = \frac{1}{4}n(n+1)(n+...)(n+...)$	M1
	$= \frac{1}{4}n(n+1)(n-1)(n+2)$	A1
		(4)
(c)	$\sum_{r=n}^{2n} r^2 = \frac{1}{6}(2n)(2n+1)(2(2n)+1) - \frac{1}{6}(n-1)(n)(2(n-1)+1)$	M1 A1
	$3\sum_{r=1}^{n} r(r+1)(r-1) = 17\sum_{r=n}^{2n} r^2$ $\Rightarrow \frac{3}{4}n(n+1)(n-1)(n+2) = \frac{17}{6}n\left(2(8n^2+6n+1) - (2n^2-3n+1)\right)$ $\Rightarrow 18(n+1)(n-1)(n+2) = 68\left(14n^2+15n+1\right) = 68(14n+1)(n+1)$	dM1

	$\Rightarrow 18(n-1)(n+2) = 68(14n+1)$ $\Rightarrow 18n^2 - 934n - 104 = 0 \Rightarrow n = ...$	**ddM1**
	$n = 52$	**A1**
		(5)
		(14 marks)

Notes:

(a)

B1: Checks the result for n = 1. Should see a clear substitution into both sides, accept minimum of seeing $\frac{1}{4}\times 1\times 4$, or $\frac{1}{4}\times 1\times 2^2$, or $\frac{1}{4}\times 1\times(1+1)^2 = 1$ for right hand side.

M1:(Makes or assumes the inductive assumption, and) adds $(k + 1)^3$ to the result for $n = k$

M1: Attempts to take at least $(k+1)^2$ as a factor out of the expression. Allow if an expansion to a quartic is followed by the factorised expression.

A1: Reaches the correct expression for $n = k + 1$from correct working with sufficient working seen, so expect at least seeing the quadratic before a factorised form.. Need not see the "k+1" explicitly for this mark.

A1: Completes the induction by demonstrating the result clearly, with suitable conclusion conveying "true for n = 1", "assumed true for $n = k$" and "shown true for $n = k + 1$", and "hence true for all n". All these statements (or equivalents) must be seen in their conclusion (not simply scattered through the work). Depends on all except the B mark, though a check for n = 1 must have been attempted.

(b)

B1: Correct expansion.

M1: Applies the standard formula for $\sum r$ and the result from (a) to their sum.

If the expansion is given as $\sum r^3 - r^2$, allow the use of $\sum r^2$ instead of $\sum r$

M1: Takes out the common factors n and $(n+1)$ and attempts to simplify to required form OR factorises their quartic.

A1: Correct answer. (Ignore A, B and C listed explicitly.) Correct answer can be obtained from a cubic or a quartic. Award M1A1 in either of these cases.

(c)

M1: Attempts to apply $\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n-1} r^2$ with the standard result for $\sum r^2$ Accept with n instead of $n-1$ in second expression.

A1: Correct expression for the RHS seen, no need to be simplified.

dM1: Applies the summations to the equation in the question and cancels/factorises out the factor n. Depends on the first M mark of (c)

ddM1: Simplifies the quadratic factor of the right hand side and cancels/factors out the n+1 and solves the resulting quadratic. Note M1M1 is implied by sights of the correct roots $-\frac{1}{9}, 52, 0, -1$ of the quartic. Depends on both previous M marks in (c)

A1: Correct answer. Must reject other roots. The correct answer obtained from a quartic or cubic equation solved by calculator gains all relevant marks.

Question	Scheme	Marks
1.(a)	$\begin{pmatrix} 2 & -1 & 3 \\ -2 & 3 & 0 \end{pmatrix}\begin{pmatrix} 1 & k \\ 0 & -3 \\ 2k & 2 \end{pmatrix}=\begin{pmatrix} 2+0+6k & 2k+3+6 \\ -2+0+0 & -2k-9+0 \end{pmatrix}$	M1
	$=\begin{pmatrix} 2+6k & 2k+9 \\ -2 & -2k-9 \end{pmatrix}$	A1cao
		(2)
(b)	det **AB** $=(2+6k)(-2k-9)-(-2)(2k+9)$	M1
	det **AB** $=0 \Rightarrow -12k^2-54k=0 \Rightarrow k=...$	dM1
	$k=-\frac{9}{2}$	A1
		(3)
		(5 marks)

Notes:

(a)

M1: Obtains a 2×2 matrix with at least two entries correct, unsimplified.

A1cao: Correct matrix with terms simplified.

(b)

M1: Attempts the determinant, be tolerant of minor slips, such as sign slips with the negatives, if the correct "$ad-bc$" form is apparent. They may give the $-(-2)(\ldots)$ as just $+2(\ldots)$. Accept if seen as part of the attempt at the inverse matrix.

dM1: Expands their determinant to a **quadratic**, sets equal to zero (may be implied) and achieves a value for k via correct method (allow if a factor k is cancelled, use of formula or calculator (a correct value for their quadratic)).

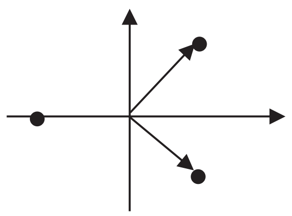
A1: cso for $-\frac{9}{2}$. Accept as decimal or equivalent fractions, such as $-\frac{54}{12}$. Ignore any reference to the 0 solution.

Question	Scheme	Marks
2.	$(7r-5)^2 = 49r^2 - 70r + 25$	**B1**
	$\sum_{r=1}^{n}(7r-5)^2 = 49\sum_{r=1}^{n}r^2 - 70\sum_{r=1}^{n}r + \sum_{r=1}^{n}25$ $= 49\times\underline{\frac{n}{6}(n+1)(2n+1)} - 70\times\underline{\frac{n}{2}(n+1)} + \underline{25\times n}$	**M1** **A1ft**
	$= \frac{n}{6}\left(49(2n^2+3n+1) - 210(n+1) + 150\right)$	**M1**
	$= \frac{n}{6}\left(98n^2 - 63n - 11\right)$	**A1**
	$= \frac{n(7n+1)(14n-11)}{6}$	**A1**
		(6)
		(6 marks)

Notes:

B1: Correct expansion.
M1: Attempts the summations with at least two of the underlined formulae correct.
A1ft: Fully correct application of all three summations. Follow through on their expansion as long as there are 3 terms.
M1: Attempts to factor out at least the factor of n from their **three term** expansion – must have a common factor of n throughout to be able to score this mark which must be extracted from each term. (If the last term is +25, it is M0.) Allow if there are minor slips but the process must be correct.

Alternatively allow this mark for an attempt to expand $\frac{n}{6}(7n+1)\left(An+B\right)$ and compare coefficients with their expanded equation.
A1: Gathers terms appropriately and achieves the correct quadratic. In the alternative approach allow for $A=14$ and $B=-11$ stated from their comparison.
A1cso: Correct answer from correct work. Any values found from the comparison approach must be substituted back in to achieve the result. Note from a correct unsimplified quadratic to correct answer, A0A1 can be awarded.

Question	Scheme	Marks
	$f(z) = 4z^3 + pz^2 - 24z + 108$, -3 a root.	
3(a)	$f(-3) = 0 \Rightarrow 4(-3)^3 + p(-3)^2 - 24(-3) + 108 = 0 \Rightarrow p = ...$	M1
	$p = -8$	A1
		(2)
(b)	$4z^3 - 8z^2 - 24z + 108 = (z+3)(4z^2 + ...z + 36)$	M1
	$= (z+3)(4z^2 - 20z + 36)$	A1
	$4z^2 - 20z + 36 = 0 \Rightarrow z = \dfrac{20 \pm \sqrt{400 - 4\times4\times36}}{8} = ...$	dM1
	Roots are $-3, \dfrac{5 \pm i\sqrt{11}}{2}$	A1
		(4)
(c)	e.g. Product of complex roots is $\dfrac{36}{4} = 9$, so modulus is $\sqrt{"9"}$ or Modulus is $\sqrt{\left(\dfrac{5}{2}\right)^2 + \left(\dfrac{\sqrt{11}}{2}\right)^2}$	M1
	Hence modulus is 3	A1
		(2)
(d)	 Complex conjugate pair in correct quadrant for their roots	M1
	All three roots correctly positioned.	A1
		(2)
		(10 marks)

Notes:

Mark the question as a whole - do not be concerned part labelling.

(a)

M1: A complete method to find the value of p. Use of the factor theorem is most direct, look for setting $f(-3) = 0$ and solving for p. May attempt to factor out $(z+3)$ and compare coefficients, e.g.

$\mathrm{f}(z)=4z^3+pz^2-24z+108=(z+3)\left(4z^2+bz+36\right)\Rightarrow 3b+36=-24, 12+b=p\Rightarrow b=.., p=...$ or may attempt long division and set remainder equal to zero to find p or variations on these.

A1: For $p=-8$

(b)

Note: Allow marks in (b) for work seen in (a) e.g. via attempts in (a) by long division.

M1: Correct strategy to find a quadratic factor. If factorising, look for correct first and last terms. May use long division, in which case look for the correct first term and attempt to use it - may have been seen in (a).

Question instructs use of algebra so an algebraic method must be seen.

A1: Correct quadratic factor - may have been seen in (a).

dM1: Uses the quadratic formula or completing the square or calculator to find the roots of their quadratic factor (allow for attempts at a quadratic factor via long division which had non-zero remainder). If a calculator is used (no method shown), there must be at least one correct **complex** root for their equation. Factorisation is M0.

A1: Correct roots in simplest form. All three should be included at some point in the solution in (b).

(c)

M1: Any correct method to find the modulus of the complex roots. Most likely to see Pythagoras, but some may deduce from product of roots. They must have complex roots to score the marks in (c).

A1: Modulus 3 only. If -3 is also given as a modulus then score A0.

(d)

Note: Allow the marks in (d) if the i's were missing in their roots in (b) but they clearly mean the correct complex roots on their diagram.

M1: Plots the complex roots as a conjugate pair in the correct quadrants for their roots.

A1: Fully correct diagram with one root on the negative real axis, and the other as a complex pair of roughly the same length in quadrants 1 and 4.

Question	Scheme	Marks
4(a)	(i) $\mathrm{f}'(x) = \underline{Ax^{-5}} + \underline{Bx^{-\frac{9}{2}}}$ oe for at least one power	M1
	$\mathrm{f}'(x) = -\dfrac{-4x^{-5}}{8} + \dfrac{2\times -\frac{7}{2}x^{-\frac{9}{2}}}{7} = \dfrac{1}{2x^5} - \dfrac{1}{x^{\frac{9}{2}}}$ oe	A1
	(ii) Since $\mathrm{f}'(0.25) = 512 - 512 = 0$ the process cannot be carried out as it would require division by zero.	B1
	(iii) $\alpha = 0.15 - \dfrac{\mathrm{f}(0.15)}{\mathrm{f}'(0.15)} = 0.15 - \dfrac{-27.332...}{1484.137...} = ...$	M1
	$= 0.168$ to 3 d.p.	A1cso
		(5)
(b)	e.g. $\dfrac{\mathrm{f}(0.25)-\mathrm{f}(0.15)}{0.25-0.15} = \dfrac{\mathrm{f}(0.15)-0}{0.15-\alpha}$ or $\dfrac{\alpha-0.25}{0-\mathrm{f}(0.25)} = \dfrac{\alpha-0.15}{0-\mathrm{f}(0.15)}$ etc	M1
	$\Rightarrow \alpha = 0.15 - \dfrac{0.1\times \mathrm{f}(0.15)}{\mathrm{f}(0.25)-\mathrm{f}(0.15)} = ...$ or $\alpha = \dfrac{0.25\,\mathrm{f}(0.15) - 0.15\,\mathrm{f}(0.25)}{(\mathrm{f}(0.15)-\mathrm{f}(0.25))} = ...$ etc	M1
	$= 0.15 - \dfrac{0.1\times -27.332...}{5.571... - (-27.332)} = 0.23306... = \text{awrt}\, 0.233$ (3 d.p.)	A1
		(3)
		(8 marks)

Notes:

(a)(i)
M1: Attempts to differentiate f(x), obtaining the correct power for at least one term.
A1: Correct differentiation, need not be simplified.
(ii)
B1: Correct reason given, accept e.g. "as f'(0.25) = 0" as a minimum and isw after a correct reason is given. Just stating f'(0.25) = 0 is not sufficient, there must be an indication this is the reason why the process cannot be used but accept any indication (such as "not valid") following this.
(iii)
M1: Correct Newton-Raphson process attempted using their derivative or implied by the correct answer from use of a calculator.
A1cso: Correct answer from correct work (derivative must have been correct). Must be 3dp.
(b)
M1: Correct interpolation strategy. Accept any correct statement such as the one shown. They may use e.g. x for $\alpha - 0.15$, in which case the the method will be gained once the correct overall strategy is clear.
M1: Proceeds from a recognisable attempt at interpolation to find a value for α. Not dependent, but they must have attempted to set up a suitable equation in α. If no working is shown accept any value for the root following the suitable statement.

A1: Accept awrt 0.233 following correct working.

Question	Scheme	Marks
5(a)	$\alpha+\beta=-\dfrac{3}{4}$	B1
	$\alpha\beta=\dfrac{k}{4}$	B1
		(2)
(b)	$\dfrac{\alpha}{\beta^2}+\dfrac{\beta}{\alpha^2}=\dfrac{\alpha^3+\beta^3}{\alpha^2\beta^2}$	B1
	$=\dfrac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^2};=\dfrac{\left(-\frac{3}{4}\right)^3-3\left(\frac{k}{4}\right)\left(-\frac{3}{4}\right)}{\left(\frac{k}{4}\right)^2}=\dots$	M1; M1
	$=\dfrac{36k-27}{4k^2}=\dfrac{9}{k}-\dfrac{27}{4k^2}$	A1
		(4)
(c)	Product of roots is $\dfrac{\alpha\beta}{\alpha^2\beta^2}=\dfrac{1}{\alpha\beta}=\dfrac{4}{k}$	B1ft
	Equation is $x^2-\left(\dfrac{36k-27}{4k^2}\right)x+\dfrac{4}{k}=0$	M1
	$4k^2x^2-(36k-27)x+16k=0$	A1
		(3)
		(9 marks)

Notes:

(a)

B1: Correct expression for $\alpha+\beta$

B1: Correct expression for $\alpha\beta$

(b)

B1: Combines the fractions correctly.

M1: For a correct identity for the sum of cubes.

M1: Substitutes their values for $\alpha+\beta$ and $\alpha\beta$ into their equation for sum of $\dfrac{\alpha}{\beta^2}+\dfrac{\beta}{\alpha^2}$ (not dependent, so there may be a slip in the identity used for $\alpha^3+\beta^3$).

A1: Correct expression in terms of k in a simplified form - e.g. either form as shown in scheme.

(c)

B1ft: Correct product of roots in terms of k, or follow through $\dfrac{1}{\text{their }\alpha\beta}$ from part (a).

M1: Applies $x^2 - (\text{their sum of roots})x + \text{their product of roots}\,(=0)$. Allow without the "=0" for this mark.
A1: Correct equation, as shown or an integer multiple thereof. Accept equivalents for the x term (e.g. $4k^2x^2 + (27-36k)x + 16k = 0$. Must include the "=0".

Question	Scheme	Marks
6(a)	$a=5$	B1
		(1)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{20}{x^2}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{2\sqrt{5}}{t^2}\div 2\sqrt{5}=-\frac{1}{t^2}$ or $x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0$ oe	B1
	Gradient of normal is $\frac{-1}{"-1/t^2"}=t^2$	M1
	Normal is $y-\frac{2\sqrt{5}}{t}=t^2\left(x-2t\sqrt{5}\right)$	M1
	$\Rightarrow ty-2\sqrt{5}=t^3x-2t^4\sqrt{5}\Rightarrow ty-t^3x-2\sqrt{5}\left(1-t^4\right)=0$ *	A1*
		(4)
(c)	$cy-c^3x-2\sqrt{5}\left(1-c^4\right)=0$ passes through $\left(-\frac{\sqrt{5}}{c},-4c\sqrt{5}\right)$ $\Rightarrow -4c^2\sqrt{5}+c^2\sqrt{5}-2\sqrt{5}\left(1-c^4\right)=0$	M1
	$\Rightarrow 2c^4-3c^2-2=0$ (oe)	A1
	$\Rightarrow c^2=\frac{3\pm\sqrt{9-4\times 2\times -2}}{4}=\ldots\left(2,-\frac{1}{2}\right)$	dM1
	$c^2>0\Rightarrow c^2=2\Rightarrow c=\pm\sqrt{2}$	A1
		(4)
		(9 marks)

Notes:

(a)

B1: Correct value stated.

(b)

B1: Correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$, or any equivalent correct expression including it, such as $x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{t^2}$

M1: Attempts negative reciprocal gradient at the point P. Allow with a instead of 5 for this mark, so score for e.g. $m_N=\frac{4at^2}{20}$.

M1: Uses their normal (changed from tangent) gradient and P to find the equation of the tangent. Look for $y-\frac{2\sqrt{5}}{t}="m_n"\left(x-t\sqrt{5}\right)$. If using $y=mx+c$ they must proceed as far as finding c.

A1*: Correct equation achieved from correct working with intermediate step.

(c) **M1:** Substitutes the parameter for A into the normal equation and attempts to substitute the coordinates of B to obtain an equation in one variable. Allow if there are slips during substitution. **A1:** Correct quadratic in c^2 need not be simplified. **dM1:** Solves their (at least two term) quadratic in c^2 to find a value for at least c^2 **A1:** Deduces correct values. Both required. Ignore reference to any complex roots.		
Alts		
(c)	$cy-c^3x-2\sqrt{5}\left(1-c^4\right)=0$ intersects H again $\Rightarrow \frac{20}{x}c-c^3x-2\sqrt{5}\left(1-c^4\right)=0$ or $cy-\frac{20}{y}c^3-2\sqrt{5}\left(1-c^4\right)=0$ $\Rightarrow c^3x^2+2\sqrt{5}\left(1-c^4\right)x-20c=0$ or $cy^2-2\sqrt{5}\left(1-c^4\right)y-20c^3=0$	**M1** **A1**
	$\Rightarrow\left(c^3x+2\sqrt{5}\right)\left(x-2c\sqrt{5}\right)=0$ or $\left(cy-2\sqrt{5}\right)\left(y+2\sqrt{5}c^3\right)=0$ $\left(x=2c\sqrt{5}\text{ is }A\text{ so}\right)$ for B $x=-\frac{2\sqrt{5}}{c^3}=-\frac{\sqrt{5}}{c}\Rightarrow c=\ldots$ $\left(\text{or }y=\frac{2\sqrt{5}}{c}\text{ is }A\text{ so}\right)$ for B $y=-2\sqrt{5}c^3=-4c\sqrt{5}\Rightarrow c=\ldots$	**M1**
	$\Rightarrow c^2=2\Rightarrow c=\pm\sqrt{2}$	**A1**
		(4)
Notes:		
(c) **M1:** Substitutes parameters for A and equation for H into normal to obtain a quadratic in x. **A1:** Correct quadratic in x or y **M1:** Solves the quadratic in x or y, identifies correct solution and equates to the relevant coordinate of B and solves for c **A1:** Deduces correct values. Both required.		
(c)	$m_{AB}=\dfrac{\frac{2\sqrt{5}}{c}-\left(-4c\sqrt{5}\right)}{2\sqrt{5}c-\left(-\frac{\sqrt{5}}{c}\right)}$	**M1**
	$m_{AB}=\frac{2+4c^2}{2c^2+1}=2$	**A1**
	From (b), normal at A has gradient $(t^2=)c^2\Rightarrow c^2=2$	**M1**
	$\Rightarrow c^2=2\Rightarrow c=\pm\sqrt{2}$	**A1**
		(4)

Notes:
(c) **M1:** Attempts the gradient of *AB*. **A1:** Correct gradient need not be simplified. **M1:** Finds/deduces the gradient of the normal at *A* and sets equal to their gradient of *AB*. **A1:** Deduces correct values. Both required.

Question	Scheme	Marks
7(i)(a)	Reflection or in the line $y = -x$	M1
	Reflection in the line $y = -x$	A1
		(2)
(b)	$\begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$ or $6\times\begin{pmatrix} \pm\cos 240° & \pm\sin 240° \\ \pm\sin 240° & \pm\cos 240° \end{pmatrix}$	M1
	$\begin{pmatrix} -3 & 3\sqrt{3} \\ -3\sqrt{3} & -3 \end{pmatrix}$	A1
		(2)
(c)	$\mathbf{R} = \mathbf{QP} = \begin{pmatrix} -3 & 3\sqrt{3} \\ -3\sqrt{3} & -3 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \ldots$	M1
	$= \begin{pmatrix} -3\sqrt{3} & 3 \\ 3 & 3\sqrt{3} \end{pmatrix}$ **QP** correctly found	A1
		(2)
(ii)	$\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix}\begin{pmatrix} \lambda \\ 1 \end{pmatrix} = \begin{pmatrix} 4\lambda \\ 4 \end{pmatrix} \Rightarrow \begin{pmatrix} -2\lambda + 2\sqrt{3} \\ 2\lambda\sqrt{3} + 2 \end{pmatrix} = \begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}$	M1
	$-2\lambda + 2\sqrt{3} = 4\lambda$ or $2\lambda\sqrt{3} + 2 = 4$	A1
	$\Rightarrow 6\lambda = 2\sqrt{3} \Rightarrow \lambda = \ldots$ or $2\sqrt{3}\lambda = 2 \Rightarrow \lambda = \ldots$	dM1
	$\lambda = \frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$ oe	A1
	Both $-2\lambda + 2\sqrt{3} = 4\lambda$ and $2\lambda\sqrt{3} + 2 = 4$ solved leading to $\lambda = \frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$	A1
		(5)
		(11 marks)

Notes:

(a)
M1: Identifies the transformation as a reflection or identifies the correct line of reflection.
A1: Fully correct description, with the equation of the line of reflection or suitable description (e.g. in the line through angle 135° with the positive x-axis). Ignore any references to a centre of reflection.

(b)
M1: Either the correct matrix for the rotation (with trig ratios evaluated) or an attempt at scaling a matrix of form shown by a factor 6 (need not evaluate ratio) – if no trig ratios seen this may be implied by the exact values in the right places. The correct answer implies the M.
A1: Correct matrix.
(c)
M1: Attempts to multiply **Q** and **P** in the correct order.
A1: QP correct
(ii)
M1: Attempts the product $\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix}\begin{pmatrix} \lambda \\ 1 \end{pmatrix}$ and sets equal to $\begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}$. Allow for poor notation as long as the intention is clear, and it may be implied by one correct equation or follow through equation.
A1: Extracts at least one correct equation (not part of the matrix equation). May be implied by correct value for λ following correct matrix equation.
dM1: Attempts to solve the equation. May be implied by the correct value following a correct matrix equation with no extraction of separate equations.
A1: Correct value for λ from at least one equation and isw if incorrectly simplified (allow if their second equation does not concur).
A1: Correct value for λ coming from both equations, solved explicitly, or checks the value of λ from the first equation works in the second equation.

Alt (ii)	$\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix}\begin{pmatrix} \lambda \\ 1 \end{pmatrix}=\begin{pmatrix} 4\lambda \\ 4 \end{pmatrix} \Rightarrow \begin{pmatrix} \lambda \\ 1 \end{pmatrix}=\frac{1}{-4-12}\begin{pmatrix} 2 & -2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix}\begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}$ $\Rightarrow \begin{pmatrix} \lambda \\ 1 \end{pmatrix}=-\frac{1}{16}\begin{pmatrix} 8\lambda-8\sqrt{3} \\ -8\lambda\sqrt{3}-8 \end{pmatrix}$	**M1**
	$2\lambda=\sqrt{3}-\lambda$ or $2=\lambda\sqrt{3}+1$	**A1**
	$\Rightarrow 3\lambda=\sqrt{3} \Rightarrow \lambda=...$ or $\sqrt{3}\lambda=1 \Rightarrow \lambda=...$	**dM1**
	$\lambda=\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$	**A1**
	Both $2\lambda=\sqrt{3}-\lambda$ and $2=\lambda\sqrt{3}+1$ solved leading to $\lambda=\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$	**A1**
		(5)

Notes:

M1: Correct attempt at inverse, attempts the product $\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix}^{-1}\begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}$ and sets equal to $\begin{pmatrix} \lambda \\ 1 \end{pmatrix}$.

Allow for poor notation as long as the intention is clear, and it may be implied by one correct equation or follow through equation.
A1: Extracts at least one correct equation (not part of the matrix equation). May be implied by correct value for λ following correct matrix equation.

dM1: Attempts to solve the equation. May be implied by the correct value following a correct matrix equation with no extraction of separate equations.
A1: Correct value for λ from at least one equation (allow if their second equation does not concur).
A1: Correct value for λ coming from both equations, solved explicitly, or checks the value of λ from the first equation works in the second equation.

Question	Scheme	Marks
8(a)	$y^2 = 4ax, y = k \Rightarrow P = \left(\dfrac{k^2}{4a}, k\right)$	B1
	Either $PS = \dfrac{k^2}{4a} + a = ...$ *or* $PS^2 = \left(\dfrac{k^2}{4a} - a\right)^2 + k^2 = ... \Rightarrow PS = ...$	M1
	$PS = \dfrac{k^2 + 4a^2}{4a}$ *	A1*
		(3)
(b)	Gradient of l_2 is $\dfrac{k}{\frac{k^2+4a^2}{4a}} = \dfrac{4ak}{k^2+4a^2}$ oe	B1
	$l_2 : y = \dfrac{4ak}{k^2+4a^2}(x+a) \Rightarrow y = \dfrac{4ak}{k^2+4a^2} \times (0+a) = ..$	M1
	$y\big\|_{x=0} = \dfrac{4a^2k}{k^2+4a^2}$ *	A1*
		(3)
(c)	Area $OSP = \dfrac{1}{2} \times a \times k$	B1
	Area $BPA = \dfrac{1}{2} \times \dfrac{k^2+4a^2}{4a} \times \left(k - \dfrac{4a^2k}{k^2+4a^2}\right)$ $\left(= \dfrac{k^3}{8a}\right)$	M1
	$\dfrac{\text{Area } BPA}{\text{Area } OSP} = \dfrac{\frac{k^2+4a^2}{4a} \times \left(k - \frac{4a^2k}{k^2+4a^2}\right)}{ak} = 4k^2$	M1
	$\Rightarrow k^3 + 4a^2k - 4a^2k = 16a^2k^3 \Rightarrow a = ...$	dM1
	$a = \dfrac{1}{4}$	A1
		(5)
		(11 marks)

Notes:

(a)
B1: Correct x coordinate at P found. May be seen on diagram.
M1: For a full method to find an expression for PS. Either use of focus-directrix property or may use Pythagoras with their coordinates.
A1*: Reaches the correct expression with a suitable intermediate step and no errors seen. If using Pythagoras the suitable step must be one with brackets expanded before factorising again.

(b)
B1: Correct expression for the gradient of l_2 given or implied by working. Need not be simplified. If using a similar triangles approach this may be scored for e.g. $\frac{k^2+4a^2}{4a} \div k = \frac{a}{y}$ or $k = \left(\frac{k^2+4a^2}{4a}\right)m$

M1: Full method to find the y intercept, e.g. by forming the equation of the line and substituting $x = 0$

May use $y - k = \frac{4ak}{k^2+4a^2}\left(x - \frac{k^2}{4a}\right) \Rightarrow y = k + \frac{-k^3}{k^2+4a^2}$

A1*: Reaches correct answer with no errors seen.
(c)
NB: If a value is chosen for k (or $k = 2a$) used, all marks are available, score for the relevant correct expressions/methods with their value.

B1: Correct area of OSP stated or implied. Note that if they go direct to ratios, the $\frac{1}{2}$ may not be seen (as it cancels with that in BPA)
M1: Correct method for the area of the triangle BPA. Allow sign slips if the method is clear (e.g. $\frac{k^2}{4a} - "-a" = \frac{k^2}{4a} - a$ if it is clear BP is meant). Allow if the negative of the area is found.
M1: Applies the ratio correctly to the problem.
dM1: Attempts to solve their equation.
A1: Correct answer. Allow if the negative of the area was found and later made positive as recovery.

Question	Scheme	Marks
9	For $n = 1$, $\sum_{r=1}^{1} \log(2r-1) = \log(2-1) = \log 1$ and $\log\left(\frac{(2\times1)!}{2^1 1!}\right) = \log 1$ So true for $n = 1$	B1
	(Assume the result is true for $n = k$, so $\sum_{r=1}^{k} \log(2r-1) = \log\left(\frac{(2k)!}{2^k k!}\right)$ Then) $\sum_{r=1}^{k+1} \log(2r-1) = \log\left(\frac{(2k)!}{2^k k!}\right) + \log(2(k+1)-1)$	M1
	$= \log\left(\frac{(2k)!}{2^k k!} \times (2k+1)\right)$	M1
	$= \log\left(\frac{(2k+1)!}{2^k k!} \times \frac{2k+2}{2k+2}\right) = \log\left(\frac{(2k+2)!}{2^k \times 2(k+1)!}\right)$	M1
	$= \log\left(\frac{(2k+2)!}{2^{k+1}(k+1)!}\right)$	A1
	Hence result is true for $n = k+1$. As true for $n = 1$ and have shown if true for $n = k$ then it is true for $n = k+1$, so it is true for all $n \in \mathbb{N}$ by induction.	A1
		(6)
		(6 marks)

Notes:

(a)

B1: Checks the result for $n = 1$. Must see **both** sides (possibly in one line) identified as log1 or 0 but may not see much more than this.

M1: Makes the inductive assumption (may be implied) and applies it to the question by adding the $(k+1)^{\text{th}}$ term to the expression for the sum to k terms. Allow if there are minor slips (e.g. a missing factorial) if the intent is clear.

M1: Attempts to combine or split log terms appropriately. Not dependent, so may be scored if the wrong term is added in the previous M as long as it is a log term.

M1: Introduces the relevant cancelling factors to achieve the $(2k + 2)!$ term. The introduction of the factors must shown or implied in an intermediate step. Alternatively, may decompose from the k+1 statement to achieve the same intermediate expression.

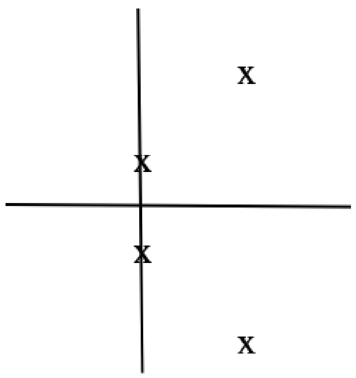
A1: Achieves correct expression from correct work (or correctly shows equivalence).

A1: Completes the induction by demonstrating the result clearly, with suitable conclusion conveying "true for $n = 1$", "assumed true for $n = k$" and "shown true for $n = k + 1$", and "hence true for all n". Depends on all except the B mark, though a check for $n = 1$ must have been attempted.

NB Allow the M's and first A if n is used throughout but the steps are correct, but must have used a different variable for the final A.

Alt steps if splitting logs:	$\sum_{r=1}^{k+1} \log(2r-1) = \log\left(\frac{(2k)!}{2^k k!}\right) + \log(2k+1)$	**M1**
	$= \log(2k)! - \log\left(2^k k!\right) + \log(2k+1)$	**M1**
	$= \log(2k+1)! - \log\left(\frac{2^{k+1}(k+1)!}{2\times(k+1)}\right) = \log\left(\frac{(2k+1)!(2k+2)}{2^k(k+1)!}\right)$	**M1**

Question Number	Scheme	Notes	Marks
1	$\sum_{r=1}^{n} r^2(r+2) = \sum_{r=1}^{n} r^3 + 2\sum_{r=1}^{n} r^2$ or $\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} 2r^2$	Correct split with 2 summations. Could be implied by correct work. Condone missing or incorrect summation limits.	B1
	$= \frac{1}{4}n^2(n+1)^2 + 2\times\frac{1}{6}n(n+1)(2n+1)$	Attempts to use both standard results and obtains an expression of the form $pn^2(n+1)^2 + qn(n+1)(2n+1)$ $p, q \neq 0$ Could be implied by immediate expansion	M1
	$= \frac{1}{12}n(n+1)[3n(n+1)+4(2n+1)]$ $= \frac{1}{12}n(n+1)(3n^2+11n+4)$	**d**M1: Attempts factorisation to obtain $\frac{1}{12}n(n+1)(an^2+bn+c)$ $a,b,c \neq 0$. Condone poor algebra. Could follow cubic or quartic. Allow a consistent $a = ...$, $b = ...$, $c = ...$ if quadratic never seen simplified **Requires previous M mark.** A1: Correct expression or $a=3,\ b=11,\ c=4$ Allow e.g., $\frac{1}{12}n(n+1)$ written as $\frac{n}{12}(n+1)$	**d**M1 A1
	Note: $n(n+1)(3n^2+11n+4) = 3n^4+14n^3+15n^2+4n$		**Total 4**

Question Number	Scheme	Notes	Marks
2	$2x^4-8x^3+29x^2-12x+39=0,\quad x=2+3\text{i}$ Condone work in e.g., z throughout		
(a)	2–3i	Correct conjugate	B1
			(1)
(b)	$(x-(2-3\text{i}))(x-(2+3\text{i}))$ or $(x-2+3\text{i})(x-2-3\text{i})$ $=\ldots\ \{x^2-4x+13\}$ or sum = 4, product = 13 $\Rightarrow x^2\pm4x\pm13$ or $x^2\pm13x\pm4$ or $x^2-(2+3\text{i}+2-3\text{i})x+(2+3\text{i})(2-3\text{i})$ $\Rightarrow\ldots\ \{x^2-4x+13\}$	Attempts to multiply the two correct factors to obtain a 3 term quadratic with real coefficients. Could use $(x-2)^2=(\pm3\text{i})^2$ or $x^2-2ax+a^2+b^2$ with $a=2,\ b=\pm3$ Or uses the correct sum and product of the roots to obtain an expression of the form shown (must be some minimal working – but if just a quadratic is given the next 2 marks are available) or $x^2-(\alpha+\beta)x+\alpha\beta$ to obtain a 3 term quadratic with real coefficients.	M1
	$2x^4-8x^3+29x^2-12x+39\Rightarrow(x^2-4x+13)(2x^2+3)$	Uses their 2 or 3 term quadratic factor with real coefficients to obtain a second 2 or 3 term quadratic of the form $2x^2+\ldots$ by long division, equating coefficients or inspection. Ignore any remainder from long division. Can follow M0	M1
	$2x^2+3(=0)\Rightarrow$ $x=\pm\dfrac{\sqrt{6}}{2}\text{i}$ or $\pm\text{i}\sqrt{\dfrac{3}{2}}$ or $\pm\dfrac{\sqrt{3}}{\sqrt{2}}\text{i}$ or $\sqrt{1.5}\,\text{i}$ $\sqrt{1.5\text{i}}$ is M0 1.2247...i is M1 A0	**d**M1: Solves their second quadratic factor = 0. If 2 term must get one correct non-zero root. (Usual rules if 3TQ and one correct root if no working) Could be inexact. **Requires previous method mark.** A1: Both correct exact roots with "i" **Requires all previous marks.**	**d**M1 A1
	Solving by calculator, sometimes followed by attempts to reconstruct factors. e.g., $\text{f}(x)=(x^2-4x+13)\left(x^2+\dfrac{3}{2}\right)$ is first M1 only and working for the 3TQ must be seen		**(4)**
(c)		Allow ft on their **answers to (b)** if they are of the form $\pm k\text{i}$ or $\pm k\sqrt{-1},\ k\neq0$ regardless of how they were obtained 1st B1: One of the two pairs of roots in correct positions 2nd B1: Both pairs of roots in correct positions and correct relative to each other for their k Allow any suitable indication of the roots such as vectors. Ignore all labelling and scaling but each pair should be reasonably symmetric in x-axis for any marks (for each pair -distance of one to x-axis not less than $\frac{1}{2}$ of the other)	B1 B1 (ft on (b))
			(2)
			Total 7

Question Number	Scheme	Notes	Marks
3(a)	$y=9x^{-1}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-9x^{-2}\left\{=-\frac{9}{(3t)^2}\right\}$ or $xy=9\Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{y}{x}\left\{=-\frac{\frac{3}{t}}{3t}\right\}$ or $x=3t,\ y=3t^{-1}\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}t}=3,\ \frac{\mathrm{d}y}{\mathrm{d}t}=-3t^{-2}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3t^{-2}}{3}$	Any correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ but allow e.g., $\frac{\mathrm{d}x}{\mathrm{d}y}=-9y^{-2}$ Calculus must be seen so there is no credit for just a statement e.g., $m_T=-\frac{1}{t^2}$	B1
	e.g., $m_N=\frac{(3t)^2}{9}$ or $\frac{3t}{\frac{3}{t}}$ or $\frac{3}{3t^{-2}}$ $\left\{=t^2\right\}$	Uses the perpendicular gradient rule to obtain the gradient of the normal in terms of t correct for their m_T Implied by correct use of $-\frac{\mathrm{d}x}{\mathrm{d}y}$	M1
	$y-\frac{3}{t}=t^2(x-3t)$ or $\frac{3}{t}=t^2(3t)+c\Rightarrow c=\ldots$ $\left\{c=\frac{3}{t}-3t^3\right\}$	Applies straight line method correctly with their normal (changed) gradient in terms of t. If using $y=mx+c$ coordinates must be correctly placed and $c=\ldots$ reached	M1
	$ty-t^3x=3-3t^4$ Intermediate step not required. Allow recovery from a slip.	Correct equation or f(t). Must be seen in (a). Accept equivalents for f(t) e.g., $3(1-t^4),\ -3(t^4-1)$	A1
	Allow work with $xy=c^2$ but the final mark requires use of $c^2=9$ No calculus scores a maximum of 0111 if m_T is stated and 0011 if m_N is stated		**(4)**
(b)	$xy=9,\ 2y-8x=3-3\times16$ e.g.,$\Rightarrow y=4x-\frac{45}{2}$ or $x=\frac{45}{8}+\frac{y}{4}$ $\Rightarrow x\left(4x-\frac{45}{2}\right)=9$ or $y\left(\frac{45}{8}+\frac{y}{4}\right)=9$	Uses $t=2$ in their $ty-t^3x=\mathrm{f}(t)\neq0$ and the equation of H to obtain an unsimplified three term quadratic equation in x or y (no variables in denominators). Only allow $\mathrm{f}(t)=\frac{9}{t}$ if stated first	M1
	$8x^2-45x-18=0$ or $2y^2+45y-72=0$ $\{\Rightarrow(8x+3)(x-6)=0$ or $(2y-3)(y+24)=0\}$ $\Rightarrow x=\ldots\ \left\{-\frac{3}{8},\ 6\right\}$ or $y=\ldots\ \left\{\frac{3}{2},\ -24\right\}$	Solves their 3TQ to find a value for x or y – apply usual rules. One root correct if no working. Can award for P provided it has come from quadratic. **Requires previous method mark.**	**d**M1
	$\left(-\frac{3}{8},-24\right)$ or $(-0.375,-24)$	Correct exact coordinates in simplest form from correct work. Allow $x=\ldots$, $y=\ldots$ Ignore $\left(6,\ \frac{3}{2}\right)$ but A0 for any other point shown or incorrect x or y value.	A1
	Solving in terms of t: M1: $\Rightarrow$Unsimplified 3TQ e.g., $t^2x^2+\left(\frac{3}{t}-3t^3\right)x-9=0$ M1 M1: Solves e.g, $x=\frac{-\frac{3}{t}+3t^3\pm\sqrt{\left(\frac{3}{t}-3t^3\right)^2+36t^2}}{2t^2}\left\{\Rightarrow\left(-\frac{3}{t^3},\ -3t^3\right)\right\}$ A1 $:t=2\Rightarrow\left(-\frac{3}{8},\ -24\right)$		**(3)**
	Correct final answer with no incorrect work is 111 provided f(t) was correct		**Total 7**

Question Number	Scheme	Notes	Marks
4	$\mathbf{A}=\begin{pmatrix}-3 & 8\\ -3 & k\end{pmatrix}$ $\quad\mathbf{B}=\begin{pmatrix}a & -4\\ 2 & 3\end{pmatrix}$ $\quad\mathbf{BC}=\begin{pmatrix}2 & 5 & 1\\ 1 & 4 & 2\end{pmatrix}$		
(i)	$\det\mathbf{A}=-3k-8(-3)\ \{=-3k+24\}$ Could be implied	Attempts det **A** and obtains $\pm3k\pm8(\pm3)$ or $\pm3k\pm24$	M1
	$-3k+24=3$ or $-3k+24=-3$ $\Rightarrow k=...$ May see $(-3k+24)^2=+\mathbf{9}\Rightarrow 9k^2-144k+567=0\Rightarrow...$	Equates their det **A** of form $ak+b\ \ a,b\neq0$ to 3 or -3 or equivalent work and solves for k (usual rules if quadratic and must use +9)	M1
	$k=7,\ k=9$ 1st A1: Either correct value of k from correct work. Allow e.g., $\frac{-21}{-3}$ or $\frac{-27}{3}$ 2nd A1: Both correct values of k from correct work. 7 and 9 only. No extra		A1 A1
			(4)
(ii)	$\det\mathbf{B}=1\times3a-(-4)\times2\ \{=3a+8\}$	Correct unsimplified expression for det **B**. Could be implied	B1
	$\mathbf{B}^{-1}=\dfrac{1}{"3a+8"}\begin{pmatrix}3 & 4\\ -2 & a\end{pmatrix}$	Correct $\mathbf{B}^{-1}$ with their det **B**. Adj(**B**) to be correct but allow elements to have their det **B** as denominators if incorporated.	M1
	$\mathbf{C}=\mathbf{B}^{-1}\mathbf{BC}=\dfrac{1}{3a+8}\begin{pmatrix}3 & 4\\ -2 & a\end{pmatrix}\begin{pmatrix}2 & 5 & 1\\ 1 & 4 & 2\end{pmatrix}=...$ Access to this mark is allowed if there is no determinant or if $\mathbf{B}^{-1}=$ det **B** $\times$ Adj(**B**) used	Multiplies **BC** by their $\mathbf{B}^{-1}$ (changed – and not just by incorporation of their determinant) **the correct way round.** Expect four correct elements for their matrices if the method is unclear. The incorrect order scores M0 even if the correct result is obtained.	M1
	$\mathbf{C}=\dfrac{1}{3a+8}\begin{pmatrix}10 & 31 & 11\\ a-4 & 4a-10 & 2a-2\end{pmatrix}$ Ignore any reference to inapplicable values of a $\left(a\neq-\frac{8}{3}\right)$	Correct **C** or equivalent with like terms collected and single fractions if necessary. e.g., $\begin{pmatrix}\dfrac{10}{3a+8} & \dfrac{31}{3a+8} & \dfrac{11}{3a+8}\\ \dfrac{a-4}{3a+8} & \dfrac{2(2a-5)}{3a+8} & \dfrac{2(a-1)}{3a+8}\end{pmatrix}$	A1
			(4)
Alt **Sim. equations**	$\begin{pmatrix}a & -4\\ 2 & 3\end{pmatrix}\begin{pmatrix}p & q & r\\ s & t & u\end{pmatrix}=\begin{pmatrix}2 & 5 & 1\\ 1 & 4 & 2\end{pmatrix}\Rightarrow \begin{matrix}ap-4s=2 & aq-4t=5 & ar-4u=1\\ 2p+3s=1 & 2q+3t=4 & 2r+3u=2\end{matrix}$ Multiplies in the correct order to obtain at least **three** correct equations		B1
	$(3a+8)p=10\quad(3a+8)q=31\quad(3a+8)r=11$ $p=\dfrac{10}{3a+8}\quad q=\dfrac{31}{3a+8}\quad r=\dfrac{11}{3a+8}$ $s=\dfrac{1}{3}\left(1-\dfrac{20}{3a+8}\right)\quad t=\dfrac{1}{3}\left(4-\dfrac{62}{3a+8}\right)\quad u=\dfrac{1}{3}\left(2-\dfrac{22}{3a+8}\right)\Rightarrow\begin{pmatrix}\dfrac{10}{3a+8} & \dfrac{31}{3a+8} & \dfrac{11}{3a+8}\\ \dfrac{a-4}{3a+8} & \dfrac{4a-10}{3a+8} & \dfrac{2a-2}{3a+8}\end{pmatrix}$ $s=\dfrac{a-4}{3a+8}\quad t=\dfrac{4a-10}{3a+8}\quad u=\dfrac{2a-2}{3a+8}$ M1: Solves their equations to find expressions in terms of a for three elements M1: Finds expressions in terms of a for all six elements A1: Correct matrix – like terms collected and single fractions		M1 M1 A1
			Total 8

Question Number	Scheme	Notes	Marks
5	Solutions that rely entirely on solving the equation are generally unlikely to score but there may be attempts which include some of the work below which can receive appropriate credit.		
(a)	$\alpha+\beta=6 \quad \alpha\beta=3$	Correct sum and product. Could be implied. Allow $\frac{6}{1}$ and $\frac{3}{1}$	B1
	$(\alpha^2+1)(\beta^2+1)=\alpha^2\beta^2+\alpha^2+\beta^2+1$	Multiplies $(\alpha^2+1)(\beta^2+1)$ to obtain 3 or 4 terms with 3 correct. Do not condone $\alpha\beta^2$ for $(\alpha\beta)^2$ unless implied later	M1
	$=\alpha^2\beta^2+(\alpha+\beta)^2-2\alpha\beta+1$	Uses $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	M1
	$\{=3^2+6^2-2\times3+1\}$ $=40$	Correct answer from correct work. Use of e.g., $\alpha+\beta=-6$ is A0	A1
			(4)
(b)	Allow use of their $(\alpha^2+1)(\beta^2+1)$ which could be from (a) or a first or reattempt in (b). Numerator must be correct		
	$\frac{\alpha}{(\alpha^2+1)}+\frac{\beta}{(\beta^2+1)}=\frac{\alpha(\beta^2+1)+\beta(\alpha^2+1)}{"(\alpha^2+1)(\beta^2+1)"}$	Any correct expression with their $(\alpha^2+1)(\beta^2+1)$ for the new sum as a single fraction (or two fractions both with the common denominator)	B1
	$=\frac{\alpha\beta(\beta+\alpha)+(\alpha+\beta)}{"(\alpha^2+1)(\beta^2+1)"}=\frac{"3"\times"6"+"6"}{"40"}=\ldots$	Uses a correct expression with their $(\alpha^2+1)(\beta^2+1)$ for the new sum to obtain a correct numerical expression with their denominator, $\alpha+\beta$ & $\alpha\beta$ and achieves a value.	M1
	$\frac{\alpha\beta}{"(\alpha^2+1)(\beta^2+1)"}=\frac{"3"}{"40"}$	Uses a correct expression with their $(\alpha^2+1)(\beta^2+1)$ for the new product to obtain a correct value with their denominator and $\alpha\beta$	M1
	new sum $=\frac{24}{40}\left\{=\frac{3}{5}\right\}$ **or** new product $=\frac{3}{40}$	One value for new sum or new product correct. Any equivalent fractions. Not ft. **Requires appropriate previous M mark.**	A1
	$x^2-\frac{24}{40}x+\frac{3}{40} \quad \{=0\}$	Correctly uses $x^2-(\text{sum of roots})x+(\text{product of roots})$ or equivalent work with their new sum and product. Condone use of a different variable. Allow appropriate values for p, q and r	M1
	$40x^2-24x+3=0$	Any correct equation with integer coefficients and "= 0". Condone use of a different variable. Allow e.g., $p=40$, $q=-24$, $r=3$. Requires all marks.	A1
			(6)
	Note that although $(\alpha^2+1)(\beta^2+1)$ may be attempted or reattempted in (b) there is no credit for work in (a) that is only seen in (b)		**Total 10**

Question Number	Scheme	Notes	Marks
6(a)	$\lvert z_1+z_2\rvert\{=\lvert 3+2\text{i}+2+3\text{i}\rvert=\lvert 5+5\text{i}\rvert\}=\sqrt{5^2+5^2}$	Attempts the sum (allow one slip) and uses Pythagoras correctly	M1
	$\sqrt{50}$ or $5\sqrt{2}$	Either correct exact answer	A1
	Answer only is no marks but working can be minimal e.g., $\lvert 5+5\text{i}\rvert=5\sqrt{2}$		**(2)**
(b)	$\frac{z_2z_3}{z_1}=\frac{(2+3\text{i})(a+b\text{i})}{(3+2\text{i})}=\frac{(2+3\text{i})(a+b\text{i})}{(3+2\text{i})}\times\frac{(3-2\text{i})}{(3-2\text{i})}$ or $\frac{z_2}{z_1}=\frac{2+3\text{i}}{3+2\text{i}}\times\frac{3-2\text{i}}{3-2\text{i}}$ or $\frac{z_3}{z_1}=\frac{a+b\text{i}}{3+2\text{i}}\times\frac{3-2\text{i}}{3-2\text{i}}$	Substitutes complex numbers and correct multiplier to rationalise the denominator seen or implied. See note below Could use $\times\frac{-3+2\text{i}}{-3+2\text{i}}$	M1
	$(3+2\text{i})(3-2\text{i})=13$	13 <u>obtained from</u> $(3+2\text{i})(3-2\text{i})$ Could be implied.	B1
	$\frac{z_2z_3}{z_1}=\frac{12a-5b}{13}+\frac{5a+12b}{13}\text{i}$ or $\frac{1}{13}(12a-5b)+\frac{\text{i}}{13}(5a+12b)$ or $\frac{12}{13}a-\frac{5}{13}b+\text{i}\left(\frac{5}{13}a+\frac{12}{13}b\right)$ etc. Condone $\frac{(12a-5b)+(5a+12b)\text{i}}{13}$	**d**M1: Attempts to simplify the numerator and collects terms to obtain $pa+qb+ra\text{i}+sb\text{i}$ with at least three of p, q, r and s non-zero. **Requires previous M mark.** A1: Correct answer in any form with a single "i". Correct bracketing where needed. Allow $x=...,\ y=...$	dM1 A1
	Note: The following marks are accessible if complex numbers are substituted in the wrong places: z_2 as denominator max 1010, z_3 as denominator max 1000		**(4)**
(c)	$\frac{12a-5b}{13}=\frac{4}{13},\ \frac{5a+12b}{13}=\frac{58}{13}\ \Rightarrow\ a=...,\ b=...$	Equates their x to $\frac{4}{13}$ and their y to $\frac{58}{13}$ to obtain 2 linear equations in both a and b and solves to obtain values for both a and b.	M1
	No need to check values but must be some working between equations and values. "$\frac{12a-5b}{13}=\frac{4}{13},\ \frac{5a+12b}{13}=\frac{58}{13}\quad 12a-5b=4,\ 5a+12b=58\quad a=2,\ b=4$" is M0A0 Values can immediately follow if equations are produced with coefficients of a or b of the same magnitude		
	$a=2$ and $b=4$	Correct values for a and b from correct equations with working.	A1
	SC: Allow access to both marks for the exact $a=-\frac{242}{169}$ and $b=\frac{716}{169}$ from using $w=\frac{z_1z_3}{z_2}=\frac{12a+5b}{13}+\frac{12b-5a}{13}\text{i}$ There are no marks in (c) if z_3 was used as the denominator in (b) [leads to a = b = 0]		**(2)**
(d)	$\arctan\left(\frac{\frac{58}{13}}{\frac{4}{13}}\right)\{=1.5019...\text{ or }86.05...^\circ\}$ or $\arctan\left(\frac{\frac{4}{13}}{\frac{58}{13}}\right)\{=0.068856...\text{ or }3.945...^\circ\}$	Either correct arctan or tan^{-1} seen or implied by a correct 2sf value (awrt 1.5, 86, 0.069/0.068, 3.9) Could use equivalent trig. Note: $\tan\frac{58}{4}=-2.634$ or 0.258	M1
	1.502	1.502 only (**not** awrt) Mark final answer if 1.502 is followed by e.g., $\frac{\pi}{2}-1.502=0.06880$	A1
			(2)
			Total 10

Question Number	Scheme	Notes	Marks
7(a)	$f(x)=x^{\frac{3}{2}}+x-3$ $f(1)=1+1-3=-1 \quad f(2)=\sqrt{8}+2-3=1.828...$	Calculates values for both f(1) and f(2) with one correct. Allow e.g., $f(2)=2\sqrt{2}-1$ or awrt 2	M1
	f is **continuous** and **changes sign**, so **root** or α in [1, 2]. Correct interval [1, 2] if given. Sign change can be implied by "negative, positive", "f(1) < 0, f(2) > 0" or "f(1)f(2) < 0"	**Correct values** and sight of **continuous**, **sign change** and e.g., **root/shown/QED/true/proven/✓**	A1
			(2)
(b) **Work may be seen in a table**	$f(1.5)=1.5^{\frac{3}{2}}+1.5-3 \quad \{=...0.3371...\}$	Obtains a <u>numerical expression or value</u> for f(1.5)	M1
	$f(1.25)=1.25^{\frac{3}{2}}+1.25-3=... \quad \{-0.3524...\}$	Obtains a <u>value</u> for f(1.25). **Requires previous M mark.**	dM1
	$\Rightarrow$ root/α/x/it's in/on/$\in$ [1.25, 1.5] or "**in** [1.25, 1.5]" or 1.25 $\leqslant$ root/α/x $\leqslant$1.5	Correct values (awrt 0.3 and –0.3 or –0.4) and suitable conclusion. Allow "between $\frac{5}{4}$ and $\frac{3}{2}$ **inclusive**"	A1
	Do not accept [1.5, 1.25]. Just "f(1.25) = ... **followed by** f(1.5) = ... so..." is 100 if no evidence of interval bisection. There are no marks if it is a clear attempt at interpolation.		**(3)**
(c)(i)	$f'(x)=\frac{3}{2}x^{\frac{1}{2}}+1$	Correct differentiation. Any correct equivalent e.g., $1.5\sqrt{x}+1$	B1
(ii)	$\alpha \approx 1.375-\dfrac{1.375^{\frac{3}{2}}+1.375-3}{"\frac{3}{2}\times 1.375^{\frac{1}{2}}+1"}=...$ $\left\{=1.375-\dfrac{-0.01266958256...}{2.75890591...}=1.375+0.004592248875... =1.379592249...\right\}$ $\left\{\text{exact values}: \dfrac{11}{8}-\dfrac{11\sqrt{22}-52}{32}\div\dfrac{8+3\sqrt{22}}{8}\right\}$	Correctly applies the Newton-Raphson formula with 1.375 & their $f'(x)$ **and** obtains a value. Some working must be seen unless approx. root is seen correct to 6 d.p. accuracy (1.379592) or better. Allow "$...=1.375-\dfrac{f(1.375)}{f'(1.375)}$" followed by value but formula must be **fully** substituted if just followed by value unless "x_0" defined	M1
	awrt 1.380 or "1.38" (Ignore further iterations)	No clearly incorrect work.	A1
	NB Actual root is 1.379589808. Answer only is no marks.		**(3)**
(d)	e.g., $\dfrac{\alpha-1.25}{1.5-\alpha}=\dfrac{0.3524575141...}{0.3371173071...}$ or e.g., $\dfrac{1.5-\alpha}{0.337...}=\dfrac{1.5-1.25}{0.337...+0.352...}$	Forms an equation in e.g., α with their f(1.25) and f(1.5) allowing for sign errors only but must be using differences. Allow use of "f(1.25)" and "f(1.5)"- could recover sign error	M1
	$\alpha=1.377780737...=1.378$	dM1: Solves $\Rightarrow$ value **Requires previous M mark.** A1: awrt 1.378	dM1 A1
	May use a formula. Allow work in, e.g., x for all marks. No working required for 2nd M		**(3)**
Alt (Equation of line methods)	or $y-(-0.3524...[\text{or } 0.3371...])=\dfrac{0.3371...-(-0.3524...)}{1.5-1.25}(x-1.25[\text{or}1.5])$ or $-0.3524...[\text{or } 0.3371...]=\dfrac{0.3371...-(-0.3524...)}{1.5-1.25}(1.25[\text{or}1.5])+c \Rightarrow c=...$ A full method to determine the equation of the line using their f(1.25) and f(1.5) allowing for sign errors only (but allow subsequent errors finding c if $y=mx+c$ used)		M1
	$\{\Rightarrow y=2.758...x-3.800...\}$ $\alpha=1.377780737...=1.378$	dM1: Puts $y=0$ and solves $\Rightarrow$ value **Requires previous M mark.** A1: awrt 1.378	dM1 A1 **(3)**
	May use a formula. Allow work in, e.g., x for all marks. No working required for 2nd M		**Total 11**

Question Number	Scheme	Notes	Marks
8	$y^2=8x \quad P\left(2p^2,4p\right) \quad Q\left(\dfrac{2}{p^2},\dfrac{-4}{p}\right)$		
	Each part is marked separately. For example there is no credit in (c) for work seen in (b) unless that work is referred to in (c)		
(a) **Subs. both x and y into** $y^2=8x$	LHS or $y^2\left\{=\left(\dfrac{-4}{p}\right)^2\right\}=\dfrac{16}{p^2}$ RHS or $8x\left\{=8\times\dfrac{2}{p^2}\right\}=\dfrac{16}{p^2}$ So Q lies on the parabola* Allow e.g., $\left(\dfrac{-4}{p}\right)^2=8\left(\dfrac{2}{p^2}\right)\Rightarrow\dfrac{16}{p^2}=\dfrac{16}{p^2}\Rightarrow$ true	Substitutes both coordinates of Q into the parabola equation, obtains e.g., $\dfrac{16}{p^2}$ twice and makes minimal conclusion - e.g., shown/QED/true/proven/✓ Sight of just "$y^2=8x$" is insufficient but allow "$y_Q{}^2=8x_Q$"	B1*
			(1)
Alt **Subs. x or y to find y or x**	$x=\dfrac{2}{p^2}\Rightarrow y^2=8\times\dfrac{2}{p^2}$ or $\dfrac{16}{p^2}\Rightarrow y=\dfrac{-4}{p}$ or $\pm\dfrac{4}{p}$ or $y=\dfrac{-4}{p}\Rightarrow\dfrac{16}{p^2}=8x\Rightarrow x=\dfrac{2}{p^2}$ So Q lies on the parabola*	Substitutes one coordinate of Q into the parabola equation to correctly find the other coordinate and makes minimal conclusion - e.g., - e.g., shown/QED/true/proven/✓ Sight of just "$y^2=8x$" is insufficient but allow "$y_Q{}^2=8x_Q$"	B1*
			(1)
8(b)	Focus is (2, 0) or $x=2, y=0$ Could be seen on a diagram	Correct focus seen or used. Condone (0, 2) if $x=2, y=0$ used but award final A0	B1
	gradient of $PQ=\dfrac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}$ or $\dfrac{-\frac{4}{p}-4p}{\frac{2}{p^2}-2p^2}$ $\left\{=\dfrac{4p^3+4p}{2p^4-2}=\dfrac{2p^3+2p}{p^4-1}=\dfrac{2p\left(p^2+1\right)}{p^4-1}=\dfrac{2p}{p^2-1}\right\}$	Attempts the gradient of PQ condoning one term of incorrect sign. Allow this mark is they subsequently attempt to convert it to a normal gradient. Note that m may be obtained from $4p=2mp^2+c,\ -\frac{4}{p}=\frac{2m}{p^2}+c\ \Rightarrow m=\ldots$	M1
	e.g., $y-4p=\dfrac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}\left(x-2p^2\right)$	Any correct equation for PQ. May use Q. Allow this mark to be implied if their equation would have been correct but errors were made **simplifying** a correct gradient.	A1
	If $y=mx+c$ is used, one of the following expressions oe for c must be reached following correct gradient seen: $c=4p-2p^2\left(\text{gradient}\right)$ or $c=\dfrac{-4}{p}-\dfrac{2}{p^2}\left(\text{gradient}\right)$		
	Examples with fully simplified gradient (see overleaf for a fuller list): $x=2\Rightarrow y-4p=\dfrac{2p}{p^2-1}\left(2-2p^2\right)\Rightarrow y=\dfrac{4p-4p^3+4p^3-4p}{p^2-1}=0$ or $y-4p=\dfrac{2p}{p^2-1}\left(2-2p^2\right)\Rightarrow y-4p=-4p\Rightarrow y=0$ $y=0\Rightarrow-4p=\dfrac{2p}{p^2-1}\left(x-2p^2\right)\Rightarrow x=\dfrac{-4p^3+4p+4p^3}{2p}=2$ $(2,0)\Rightarrow-4p=\dfrac{2p}{p^2-1}\left(2-2p^2\right)\Rightarrow-4p=-4p$	$\Rightarrow$ So PQ passes through the focus*	A1*

	Substitutes $x = 2$ and shows $y = 0$ or vice versa or substitutes both values and shows that the equation is true. Must have minimal conclusion e.g., shown/QED/true/proven/✓ and no incorrect work. Condone no conclusion if the mark in (a) was withheld for this reason only. The examples indicate the minimum **level** of algebra acceptable. With the exception of using (2, 0) with a fully simplified gradient, look for substitution into the line followed by a further step which shows an expression that clearly leads to 0, 2 or e.g., $-4p$ or "1=1" followed by a minimal conclusion		
	Work in "a" can only access the accuracy marks when $a = 2$ is substituted		**(4)**
Alt 1 **Grad PF = Grad QF**	Focus is (2, 0) or $x = 2$, $y = 0$ Could be seen on a diagram	Correct focus seen or used. Condone (0, 2) if $x = 2$, $y = 0$ used but award final A0	B1
	gradient PF $= \dfrac{4p}{2p^2-2}$ or $\dfrac{-4p}{2-2p^2}$ and gradient QF $= \dfrac{\frac{4}{p}}{2-\frac{2}{p^2}}$ or $\dfrac{-\frac{4}{p}}{\frac{2}{p^2}-2}$	M1: Obtains expressions for both gradients condoning one term of incorrect sign in either or both expressions A1: Both correct expressions oe	M1 A1
	Grad QF $= \dfrac{4p}{2p^2-2}$ = Grad PF So PQ passes through the focus*	Shows that the gradients are the same plus minimal conclusion e.g., shown/QED/true/proven/✓ with no incorrect work. Condone no conclusion if penalised in (a).	A1*
	Note: A variation is to show grad PF or grad QF = grad PQ – marked as Alt		**(4)**
	Alt 2 Follows (similar triangles)		

8(b) <u>Examples</u> of minimum amount of algebra required with different expressions for gradient:	
$$y-4p=\frac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}\left(x-2p^2\right)$$	
$x=2,\ y=\ldots$	$x=2\Rightarrow y-4p=\dfrac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}\left(2-2p^2\right)\Rightarrow y=\dfrac{8p+\frac{8}{p}-8p^3-8p+8p^3-\frac{8}{p}}{2p^2-\frac{2}{p^2}}=0$
$y=0,\ x=\ldots$	$y=0\Rightarrow -4p=\dfrac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}\left(x-2p^2\right)\Rightarrow x=\dfrac{-8p^3+\frac{8}{p}+8p^3+8p}{4p+\frac{4}{p}}=2$
$(2,\ 0)\Rightarrow$	$(2,0)\Rightarrow -4p=\dfrac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}\left(2-2p^2\right)\Rightarrow -4p=\dfrac{8p+\frac{8}{p}-8p^3-8p}{2p^2-\frac{2}{p^2}}\Rightarrow -4p=-4p$
$$y-4p=\frac{4p^3+4p}{2p^4-2}\left(x-2p^2\right)$$	
$x=2,\ y=\ldots$	$x=2\Rightarrow y-4p=\dfrac{4p^3+4p}{2p^4-2}\left(2-2p^2\right)\Rightarrow y=\dfrac{8p^3+8p-8p^5-8p^3+8p^5-8p}{2p^4-2}=0$ or $y-4p=\dfrac{4p^3+4p}{2p^4-2}\left(2-2p^2\right)\Rightarrow y=\dfrac{-4p^3-4p+4p^3+4p}{p^2+1}=0$
$y=0,\ x=\ldots$	$y=0\Rightarrow -4p=\dfrac{4p^3+4p}{2p^4-2}\left(x-2p^2\right)\Rightarrow x=\dfrac{-8p^5+8p+8p^5+8p^3}{4p^3+4p}=2$

$(2, 0) \Rightarrow$	$(2,0) \Rightarrow -4p = \dfrac{4p^3+4p}{2p^4-2}(2-2p^2) \Rightarrow -4p = \dfrac{8p^3+8p-8p^5-8p^3}{2p^4-2} \Rightarrow -4p = -4p$
	$y-4p = \dfrac{2p}{p^2-1}(x-2p^2)$
$x=2,\ y=\ldots$	$x=2 \Rightarrow y-4p = \dfrac{2p}{p^2-1}(2-2p^2) \Rightarrow y = \dfrac{4p-4p^3+4p^3-4p}{p^2-1} = 0$ or $y-4p = \dfrac{2p}{p^2-1}(2-2p^2) \Rightarrow y-4p = -4p \Rightarrow y = 0$
$y=0,\ x=\ldots$	$y=0 \Rightarrow -4p = \dfrac{2p}{p^2-1}(x-2p^2) \Rightarrow x = \dfrac{-4p^3+4p+4p^3}{2p} = 2$
$(2, 0) \Rightarrow$	$(2,0) \Rightarrow -4p = \dfrac{2p}{p^2-1}(2-2p^2) \Rightarrow -4p = -4p$

Note that this not an exhaustive list (for example there are all the corresponding $y = mx + c$ approaches or those using Q) and the precise choice of algebra will vary widely but with the exception of the last example above this mark requires substitution into the line followed by a further step which shows an expression that clearly leads to 0, 2 or e.g., $-4p$ or "1=1" followed by a minimal conclusion (unless B0 was given in (a) for that reason).

8(b) cont.	$y^2 = 8x \quad P(2p^2, 4p) \quad Q\left(\dfrac{2}{p^2}, \dfrac{-4}{p}\right) \quad X\left(2, \dfrac{-4}{p}\right) \quad Y\left(2p^2, \dfrac{-4}{p}\right)$		
Alt 2 **Similar triangles**	Focus is (2, 0) or $x = 2, y = 0$ Could be seen on a diagram	Correct focus seen or used. Condone (0, 2) if $x = 2, y = 0$ used but award final A0	B1
	$\dfrac{XF}{XP} = \dfrac{\frac{4}{p}}{4p+\frac{4}{p}} \quad \dfrac{QX}{QY} = \dfrac{2-\frac{2}{p^2}}{2p^2-\frac{2}{p^2}}$	M1: Obtains expressions for two ratios condoning one term of incorrect sign in either or both expressions A1: Both correct expressions oe	M1 A1
	$\dfrac{XF}{XP} = \dfrac{1}{p^2+1} = \dfrac{QX}{QY}$ So Δs XFQ & YPQ are similar $\Rightarrow PQ$ passes through the focus*	Shows that the ratios are the same, makes reference to similarity plus minimal conclusion e.g., shown/QED/true/proven/✓ with no incorrect work. Condone no conclusion if penalised in (a).	A1*
			(4)
8 cont.	$y^2 = 8x \quad P(2p^2, 4p) \quad Q\left(\dfrac{2}{p^2}, \dfrac{-4}{p}\right)$		
(c)	$y = \sqrt{8}x^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2}\sqrt{8}x^{-\frac{1}{2}}$ or $2y\dfrac{dy}{dx} = 8 \Rightarrow \dfrac{dy}{dx} = \dfrac{8}{2y}$ or $x = at^2,\ y = 2at \Rightarrow \dfrac{dx}{dt} = 2at,\ \dfrac{dy}{dt} = 2at \Rightarrow \dfrac{dy}{dx} = \dfrac{2a}{2at}$ (They may use p for t and/or $a = 2$)	Achieves an expression of the correct form (sign/coefficient errors only) for the gradient. There is no requirement for calculus so they may use e.g., $m_T = \dfrac{1}{t}$. Must go beyond $\dfrac{dx}{dy}$	M1
	m_T at $P = \dfrac{1}{2}\sqrt{8}\dfrac{1}{\sqrt{2p^2}}$ or $\dfrac{8}{2\times 4p}$ or $\dfrac{2\times 2}{2\times 2\times p}$ $\left\{=\dfrac{1}{p}\right\}$ or m_T at $Q = -\dfrac{1}{2}\sqrt{8}\dfrac{1}{\sqrt{\frac{2}{p^2}}}$ or $\dfrac{8}{2\times\left(\frac{-4}{p}\right)}$ or $x = \dfrac{2}{p^2},\ y = \dfrac{-4}{p} \Rightarrow \dfrac{dx}{dp} = \dfrac{-4}{p^3},\ \dfrac{dy}{dp} = \dfrac{4}{p^2} \Rightarrow \dfrac{dy}{dx} = -p$ Any correct unsimplified expression for the tangent gradient in terms of p at either point. If awarding for Q via explicit differentiation must choose the negative root		A1

	Eqn of tgt at P: $y-4p=\frac{1}{p}\left(x-2p^2\right)$ oe or Eqn of tgt at Q: $y+\frac{4}{p}=-p\left(x-\frac{2}{p^2}\right)$ oe	M1: Correct straight line method for either point with their tangent gradient in terms of p (but allow if "a" also present) Coordinates correctly placed. If $y=mx+c$ is used must reach $c=$ ... following correctly placed coordinates A1: Any correct unsimplified equation for either tangent	M1 A1
	Note: $y=mx+c$: At P, $y=\frac{1}{p}x+2p$ At Q, $y=-px-\frac{2}{p}$		
	$\frac{1}{p}\left(x-2p^2\right)+4p=-p\left(x-\frac{2}{p^2}\right)-\frac{4}{p}\Rightarrow x=\ldots$ or $\frac{1}{p}x+2p=-px-\frac{2}{p}\Rightarrow x=\ldots$ Eliminates y from their tangent equations and solves for x (See note below if eliminate x). Gradients must be different and no clear evidence of conversion of any line to a normal. Condone poor algebra.		M1
	$x\left(\frac{1}{p}+p\right)=-\frac{2}{p}-2p\Rightarrow x=\frac{-\left(2p^2+2\right)}{\left(p^2+1\right)}=-2$	$x=-2$ only	A1
	$y=\frac{1}{p}\left(-2-2p^2\right)+4p,\quad -p\left(-2-\frac{2}{p^2}\right)-\frac{4}{p},\quad \frac{1}{p}(-2)+2p,\quad -p(-2)-\frac{2}{p}$ **d**M1: Substitutes their x (a constant or function of p) into one of their two tangent equations to obtain an expression for y. **Requires previous M mark.**		**d**M1
	e.g., $y=2p-\frac{2}{p},\quad 2\left(p-\frac{1}{p}\right),\quad \frac{2}{p}\left(p^2-1\right),\quad \frac{2p^2-2}{p},\quad \frac{2(p+1)(p-1)}{p}$ A1: Correct y in simplest form – two terms which could be factorised in any correct way and/or written as a single fraction. Note there is no requirement for coordinate notation.		A1
	Note it is obviously possible to eliminate x. In this case, award the last 4 marks in this order: M1: Eliminates x and solves for y A1: Any correct y in simplest form **d**M1: Substitutes their y (a constant or function of p) into one of their two tangent equations to obtain an expression for x. **Requires previous M mark.** A1: $x=-2$		**(8)**
	Working which involves "a" where a is never replaced by 2 can score the Ms		**Total 13**

Question Number	Scheme	Notes	Marks
9	$\mathrm{f}(n)=4^n+6n-10 \quad n\in\mathbb{Z} \quad n\geqslant 2$		

General guidance:
Apply the way that best fits the overall approach.
Condone work in e.g., n instead of k.
Attempts with no induction e.g., not using f(k) in an equation with f(k +1) score a max of 11000.
Using e.g., f(k + 2) – f(k + 1) requires a clear indication of assuming f(k + 1) is true to access the last three marks.
Alternative explanations are unlikely to access the last three marks unless there is a fully convincing justification of divisibility, e.g., $\mathrm{f}(k+1)-\mathrm{f}(k)=3\times4^k+6$ followed by "Since 3×4^k is a multiple of both 3 and 4 and hence 12, $3\times4^k+6$ is divisible by 18" is not a sound argument. Attempts that involve further induction on different expressions must be complete methods to access the last 3 marks.
Allow use of –18 but if any different multiples of 18 are involved e.g., 36, the first A1 requires "36 is a multiple of/divisible by (but not "factor of") 18" oe for each case
B1: Any correct numerical expression that is not just "18" is sufficient for this mark
e.g., 16 + 12 –10, 28 – 10, 4^2 + 2. Starting with e.g., f(3) scores a max of 01110.
Ignore an extra evaluation of $\mathrm{f}(1)$ but a comment on $\mathrm{f}(1)$'s divisibility is final A0 since $n\geqslant2$
Final A1: There must be evidence that true for $n=k\Rightarrow$ true for $n=k+1$ but it could be minimal and be scored in a conclusion or a narrative or via both. So if e.g., "Assume true for $n=k$ ..." is seen in the work followed by "true for $n=k+1$" in a conclusion this is sufficient.
Condone "for all $n\in\mathbb{Z}$", "all $n\in\mathbb{Z}$ $n>2$", "all $\mathbb{Z}>$ (or $\geqslant$) 2" but **not** $n\in\mathbb{R}$

Way	Scheme	Notes	Marks
Way 1 $\mathrm{f}(k+1)-\mathrm{f}(k)$	$\mathrm{f}(2)=4^2+6\times2-10=18$	Obtains f(2) = 18 with substitution	B1
	$\mathrm{f}(k+1)=4^{k+1}+6(k+1)-10$	Attempts $\mathrm{f}(k+1)$	M1
	$\mathrm{f}(k+1)-\mathrm{f}(k)=4^{k+1}+6(k+1)-10-(4^k+6k-10)$ $=4^{k+1}-4^k+6=3\times4^k+6$ $=3(4^k+6k-10)-18k+36$	Attempts $\mathrm{f}(k+1)-\mathrm{f}(k)$, uses $4^{k+1}=4\times4^k$ & obtains $p\mathrm{f}(k)+\mathrm{g}(k)$ with g(k) linear (allow constant $\neq0$)	M1
	$\mathrm{f}(k+1)=4\mathrm{f}(k)+18(2-k)$ f(k) may be written in full	Correct factorised expression Allow $4\mathrm{f}(k)+18\times2-18\times k$ If f(k + 1) is not made the subject then e.g., "true for f(k + 1) – f(k)" is also required	A1
	True for n = 2, if **true for $n=k$ then true for $n=k+1$** so **true for all $n\in\mathbb{Z}$** $(n\geqslant2)$ Minimum in bold.	Full conclusion/narrative and no errors. All marks needed but allow if B0 provided this mark was only withheld for insufficient working.	A1
			(5)
Way 2 $\mathrm{f}(k+1)=\ldots$	$\mathrm{f}(2)=4^2+6\times2-10=18$	Obtains f(2) = 18 with substitution	B1
	$\mathrm{f}(k+1)=4^{k+1}+6(k+1)-10$	Attempts $\mathrm{f}(k+1)$	M1
	$=4\times4^k+6k-4$ $=4(4^k+6k-10)-18k+36$	Uses $4^{k+1}=4\times4^k$ & obtains $p\mathrm{f}(k)+\mathrm{g}(k)$ with g(k) linear (allow constant $\neq0$)	M1
	$=4\mathrm{f}(k)+18(2-k)$ f(k) may be written in full	Correct factorised expression Allow $4\mathrm{f}(k)+18\times2-18\times k$	A1
	True for n = 2, if **true for $n=k$ then true for $n=k+1$** so **true for all $n\in\mathbb{Z}$** $(n\geqslant2)$ Minimum in bold.	Full conclusion/narrative and no errors. All marks needed but allow if B0 provided this mark was only withheld for insufficient working.	A1
			(5)

Question Number	Scheme	Notes	Marks
9 cont.	$\text{f}(n) = 4^n + 6n - 10 \quad n \in \mathbb{Z} \quad n \geqslant 2$		
Way 3 $\text{f}(k+1) - m\text{f}(k)$	$\text{f}(2) = 4^2 + 6\times 2 - 10 = 18$	Obtains f(2) = 18 with substitution	B1
	$\text{f}(k+1) = 4^{k+1} + 6(k+1) - 10$	Attempts $\text{f}(k+1)$	M1
	$\text{f}(k+1) - m\text{f}(k) = 4^{k+1} + 6(k+1) - 10 - m(4^k + 6k - 10)$ $= (4-m)4^k + (6-6m)k - 4 + 10m$ e.g. $m = -14 \Rightarrow 18\times 4^k + 90k - 144$ e.g. $m = 4 \Rightarrow -18k + 36$	Attempts $\text{f}(k+1) - m\text{f}(k)$ and uses a value of m to obtain $c\times 4^k + \ldots$ where c is a multiple of their 18 or uses $m = 4$	M1
	e.g., $\text{f}(k+1) = -14\text{f}(k) + 18(4^k + 5k - 8)$ $\text{f}(k+1) = 4\text{f}(k) + 18(2-k)$ f(k) may be written in full	A correct factorised expression Allow $-14\text{f}(k) + 18\times 4^k + 18\times 5k - 18\times 8$ If f(k + 1) is not made the subject then e.g., "true for f(k + 1) – mf(k)" is also required	A1
	True for n = 2, if **true for n = k then true for $n = k+1$** so **true for all $n \in \mathbb{Z}$** $(n \geqslant 2)$ Minimum in bold.	Full conclusion/narrative and no errors. All marks needed but allow if B0 provided this mark was only withheld for insufficient working.	A1
			(5)
Way 4 $\text{f}(k) = 18M$	$\text{f}(2) = 4^2 + 6\times 2 - 10 = 18$	Obtains f(2) = 18 with substitution	B1
	$\text{f}(k+1) = 4^{k+1} + 6(k+1) - 10$	Attempts $\text{f}(k+1)$	M1
	$\text{f}(k) = 18M, \quad \text{f}(k+1) = 4\times 4^k + 6k - 4$ $= 4\times 18M - 18k + 36$	Sets $\text{f}(k) = 18M$, uses $4^{k+1} = 4\times 4^k$ & obtains $p\text{f}(k) + \text{g}(k)$ with g(k) linear (allow constant $\neq 0$)	M1
	$\text{f}(k+1) = 18(4M + 2 - k)$	A correct factorised expression Allow $18\times 4M + 18\times 2 - 18\times k$	A1
	True for n = 2, if **true for n = k then true for $n = k+1$** so **true for all $n \in \mathbb{Z}$** $(n \geqslant 2)$ Minimum in bold.	Full conclusion/narrative and no errors. All marks needed but allow if B0 provided this mark was only withheld for insufficient working.	A1
			(5)
	PAPER TOTAL: 75		

Question	Scheme	Marks
1.(a)	$\det \mathbf{M} = (2k+1)(k+4) - k(k+7)$	M1
	$\det \mathbf{M} = k^2 + 2k + 4 \Rightarrow b^2 - 4ac = 4 - 16$ or $\det \mathbf{M} = k^2 + 2k + 4 = (k+1)^2 + 3$ or $\dfrac{\mathrm{d}(\det \mathbf{M})}{\mathrm{d}k} = 2k + 2 = 0 \Rightarrow k = -1$	M1
	$b^2 - 4ac < 0 \Rightarrow k^2 + 2k + 4 > 0$ or $\det \mathbf{M} = (k+1)^2 + 3 \ldots 3$ or $k = -1$ at minimum so $\det \mathbf{M} \ldots 3$ Hence $\mathbf{M}$ is non-singular for all real values of k	A1
		(3)
(b)	$\mathbf{M}^{-1} = \dfrac{1}{k^2 + 2k + 4}\begin{pmatrix} k+4 & -k \\ -k-7 & 2k+1 \end{pmatrix}$	M1 A1
		(2)
		(Total 5 marks)

Notes

(a)

M1: Attempts the determinant of **M**. Must see evidence of the attempt at subtracting but allow e.g. minor sign slips inside the brackets. Must be seen in (a).

M1: Begins a correct strategy for attempting to establish that the determinant is non-zero, must follow a valid attempt at the determinant involving all four terms.. May find the discriminant, complete the square (usual rules) on the determinant, attempt to solve the quadratic via formula, or attempt a minimisation process. There must be an attempt at a calculation.

A1: Full and correct reasoning and conclusion. Must see consideration of the sign or non-zero oe, but accept e.g. $\ldots 3$ (condone >3) as being sufficient to show det **M** cannot be zero for the reason. Showing the roots $\left(-1 \pm \mathrm{i}\sqrt{3}\right)$ are not real is acceptable The final deduction must refer to non-singular, but no need to mention **M** and condone det**M** is non-singular as conclusion.

(b)

M1: For applying $\mathbf{M}^{-1} = \dfrac{1}{\det \mathbf{M}} \times \operatorname{adj}(\mathbf{M})$ with their determinant. At least three entries in adj(**M**) must be correct initially.

A1: Correct matrix, and isw after a correct answer.

Question	Scheme	Marks
2.	$\mathrm{f}(z) = 2z^3 + pz^2 + qz - 41$	
(a)	$(z =) 5 + 4\mathrm{i}$	**B1**
		(1)
(b)	$z = 5 \pm 4\mathrm{i} \Rightarrow (z-(5+4\mathrm{i}))(z-(5-4\mathrm{i})) = \ldots$ Or e.g. Sum of roots = 10, Product of roots = 41	**M1**
	$z^2 - 10z + 41$	**A1**
	$\mathrm{f}(z) = (z^2 - 10z + 41)(2z + \ldots)$	**M1**
	$\Rightarrow z = \frac{1}{2}, (5 \pm 4\mathrm{i})$	**A1**
		(4)
(c)	$\mathrm{f}(z) = (z^2 - 10z + 41)(2z - 1) = \ldots$	**M1**
	$p = -21,\ q = 92$	**A1**
		(2)
(d)	Area $= \frac{1}{2} \times 8 \times \left(5 - "\frac{1}{2}"\right)$	**M1**
	$= 18$	**A1ft**
		(2)
		(Total 9 marks)

Notes Mark (b) and (c) together

(a)
B1: Correct complex number
(b)
M1: Correct strategy to find the quadratic factor using the conjugate pair.
A1: Correct quadratic factor.
M1: Attempts to find the linear factor. Look for $2z + k$ where k is number (or allow if k is in terms of p as long as a value of p is also found at some stage). May arise from attempts at long division.
A1: Correct real root (condone if labelled x). The complex roots do not have to be stated. Must be seen in (b) (or (c) if done together).
(c)
M1: Multiplies out to obtain values for p and q. May have been found as part of a long division process in (b).
A1: Correct values. May be seen embedded in the cubic.
(d)
M1: For $\frac{1}{2} \times 8 \times \left|5 - \text{their}"\frac{1}{2}"\right|$ where their real root is non-zero.
A1ft: Correct area (follow through their non-zero real root).

Alt 1 **(b)**	Product of complex roots = 41, Product of all roots = $\frac{\pm 41}{2}$	**M1**
	Product of complex roots = 41, Product of all roots = $\frac{41}{2}$	**A1**
	$z = \frac{\text{Product of roots of f}(z)}{\text{Product of complex roots}} = \frac{41}{2} \div 41$	**M1**
	$\Rightarrow z = \frac{1}{2}, (5 \pm 4\text{i})$	**A1**
		(4)
(c)	$\frac{p}{2} = -\sum \alpha_i \quad \frac{q}{2} = \sum \alpha_i \alpha_j \Rightarrow p = ..., q = ...$	**M1**
	$p = -21, q = 92$	**A1**
		(2)

Notes

Alt 1 (b)
M1: Identifies the product of roots of f(z) up to sign error, and the product of complex roots.
A1: Correct products seen or implied.
M1: Attempts to find the third root of f(z).
A1: Correct real root. The complex roots do not have to be stated. Must be seen in (b) (or (c) if done together)
(c)
M1: Applies sum and pair sum properties, or multiplies out to obtain values for p and q.
A1: Correct values. May be seen embedded in the cubic.

Alt 2 **(b)**	$\text{f}(5 \pm 4\text{i}) = -230 \pm 472\text{i} + p(9 \pm 40\text{i}) + q(5 \pm 4\text{i}) - 41 = 0$ $\Rightarrow -271 + 9p + 5q = 0, 472 + 40p + 4q = 0$	**M1** **A1**
	$\Rightarrow p = ..., q = ... \Rightarrow \text{f}(z) = 2z^3 + "p"z^2 + "q"z - 41 \Rightarrow z = ...$	**M1**
	$\Rightarrow z = \frac{1}{2}, (5 \pm 4\text{i})$	**A1**
		(4)
(c)	$\Rightarrow p = ..., q = ...$	**M1**
	$p = -21, q = 92$	**A1**
		(2)

Notes Alt (b)+(c) together

Alt 2 (b)
M1: Attempts factor theorem with one of the complex roots **and** equates real and imaginary terms to produce simultaneous equations.
A1: Correct equations.
M1: Uses their p and q from solving the simultaneous equations in f(z) and solves the cubic (may just see roots).
A1: Correct real root. The complex roots do not have to be stated. Must be seen in (b) (or (c) if done together)
(c)
M1: Awarded before previous M. Solves their simultaneous equations to obtain values for p and q.
A1: Correct values. May be seen embedded in the cubic.

Question	Scheme	Marks
3.(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{t^2}$	B1
	$y-\frac{c}{t}=-\frac{1}{t^2}(x-ct)$	M1
	$\Rightarrow x=\ldots, y=\ldots$	M1
	$A(2ct,0)$ **and** $B\left(0,\ \frac{2c}{t}\right)$	A1
		(4)
(b)	$\frac{1}{2}\times 2ct\times\frac{2c}{t}=90\Rightarrow c=\ldots$	M1
	$c^2=45\Rightarrow c=3\sqrt{5}$	A1cso
		(2)
		(Total 6 marks)

Notes

(a)

B1: Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. May be implied at the point of substitution into the equation if only found in terms of x and/or y initially. Allow if just stated, no working needed (may have been memorised). You may see attempts where this is derived but they must get to or imply the correct derivative in terms of t.

M1: Correctly forms the equation of the tangent. If no working for the gradient is shown accept $m=\pm\frac{1}{t^2}$ for this mark. If using $y=mx+c$ they must proceed at least as far as finding c.

M1: Uses their "tangent" (which must be a straight line equation) to find the non-zero coordinates of A and B. Both must be attempted.

A1: Both correct. Accept A, $x=2ct$, B, $y=\frac{2c}{t}$ or list as separate coordinate as long as it is clear, but just $\left(2ct,\frac{2c}{t}\right)$ is A0. If coordinates are labelled the wrong way award A0 but allow both marks in (b).

(b)

M1: Uses their coordinates and the "90" correctly to form and solve an equation for c.

A1cso: Correct value for c (must be simplified surd so $3\sqrt{5}$). Must have come from correct coordinates. A0 if the negative root is also given (and not rejected).

Question	Scheme	Marks
4.(a)	Stretch – SF 3 parallel to the y-axis	M1 A1
		(2)
(b)	$\mathbf{B}=\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	B1
		(1)
(c)	$\mathbf{C}=\mathbf{BA}=\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$	M1
	$\mathbf{C}=\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{3}{2} \\ -\frac{1}{2} & -\frac{3\sqrt{3}}{2} \end{pmatrix}$	A1
		(2)
(d)	$\det\mathbf{C}=-\frac{\sqrt{3}}{2}\times-\frac{3\sqrt{3}}{2}-\frac{3}{2}\left(-\frac{1}{2}\right)=3$ So area of $H¢$ is $5\times\det\mathbf{C}=\ldots$	M1
	$=15$	A1
		(2)
		(Total 7 marks)

Notes

(a)

M1: Identifies one correct aspect, EITHER **stretch** (or allow scaling)
OR scale factor 3 **and** correct direction indicated (need not be precise).

A1: Fully correct description with both aspects correct. Must mention stretch or scaling, but be tolerant with the description of direction and scale factor as long as both are clear. Accept e.g. parallel to the y-axis, y direction, in y axis, or vertically for direction, but not "about y" (reflection implied). Accept e.g. scale factor 3, by 3, ×3 or three times for the scale factor.
Ignore references to "about origin" or additional references to stretch of factor 1 parallel to the x-axis.

Some examples:

- "stretched by 3 in direction of y axis".
- "stretch the y for scale factor of 3."

  Both the above score M1A1. Stretch stated, direction and scale factor 3 both indicated.
- "enlargement scale factor 3 for y-axis."
- "A enlarge by three times paralleled to y-axis."

  Both score M1A0 Indicates direction and scale factor but does not mention stretch or scaling.
- "the y-axis will be enlarged by three times, whereas the x-axis stay the same."

  M1A0 Indicates direction and scale factor but does not mention stretch or scaling, the reference to the x-axis is ignored as not incorrect.

(b)
B1: Correct matrix. Must be exact (trig terms evaluated) and seen in (b).
(c)
M1: Attempts to multiply matrices the right way around. Implied by 3 correct entries if no product shown.
A1: Correct matrix. Must be exact.
(d)
M1: Attempts determinant of **C** (or deduces area scale factor is 3) and multiplies by 5. Implied by a correct answer if no incorrect working is seen.
A1: Cao. Allowed if scored from a **C** arising from multiplication the wrong way round in (c) or an incorrect **B** that has determinant ± 1

Question	Scheme	Marks
5.(a)	$2x^2 - 3x + 7 = 0$	
	$\alpha + \beta = \frac{3}{2},\quad \alpha\beta = \frac{7}{2}$	B1
		(1)
(b)	$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$	M1
	$= \left(\frac{3}{2}\right)^2 - 2\left(\frac{7}{2}\right) = -\frac{19}{4}$	A1
		(2)
(c)	$\text{Sum} = \alpha - \frac{1}{\beta^2} + \beta - \frac{1}{\alpha^2} = \alpha + \beta - \frac{\alpha^2+\beta^2}{\alpha^2\beta^2} = \frac{3}{2} - \frac{-\frac{19}{4}}{\left(\frac{7}{2}\right)^2} = \frac{185}{98}$	M1A1
	$\text{Prod} = \left(\alpha - \frac{1}{\beta^2}\right)\left(\beta - \frac{1}{\alpha^2}\right) = \alpha\beta - \frac{\alpha+\beta}{\alpha\beta} + \frac{1}{\alpha^2\beta^2} = \frac{7}{2} - \frac{3}{7} + \frac{4}{49} = \frac{309}{98}$	M1A1
	$x^2 - \frac{185}{98}x + \frac{309}{98}(=0)$	M1
	$98x^2 - 185x + 309 = 0$	A1
		(6)
		(Total 9 marks)

Notes

(a)
B1: Both values correct.
(b)
M1: Attempts to use a correct identity to find the sum of square of roots.
A1: Correct value. Note do not allow recovery from $\alpha + \beta = -\frac{3}{2}$ for this mark.
(c)
M1: Attempts sum for the new roots using their values from (a) and (b). They must be substituting into a correct identity for this mark. If substitution not seen allow for any value appearing after a suitable combined identity is seen.
A1: Correct value.
M1: Attempts product for the new roots using their values from (a). Must be substituting into an expression of the correct form, but allow if a sign slip occurs when expanding. If substitution not seen allow for any value appearing after a suitable expanded identity is seen.
A1: Correct value.
M1: Applies x^2 – (their sum)x + their prod (= 0). May be implied by suitable values for p, q and r stated if no quadratic seen.
A1: Allow any integer multiple. Must include the "= 0", and must be an equation, not just values for p, q and r.
Note: Answers from solving the quadratic will gain no credit for (a) and (b) and only score in (c) if the method marks as described are earned.

Question	Scheme	Marks
6.(i)	$\mathrm{f}(x) = x - 4 - \cos\left(5\sqrt{x}\right) \qquad x > 0$	
(a)	$\mathrm{f}\left(2.5\right) = -1.44...,\ \mathrm{f}\left(3.5\right) = 0.497...$	**M1**
	Sign change (negative, positive) and f(x) is **continuous** therefore (a **root**) α is between x = 2.5 and x = 3.5	**A1**
		(2)
(b)	E.g. $\dfrac{\alpha - 2.5}{\left\lvert \mathrm{f}\left(2.5\right) \right\rvert} = \dfrac{3.5 - \alpha}{\mathrm{f}\left(3.5\right)} \Rightarrow \alpha = ...$ or $\dfrac{\alpha - 2.5}{0 - \mathrm{f}\left(2.5\right)} = \dfrac{3.5 - 2.5}{\mathrm{f}\left(3.5\right) - \mathrm{f}\left(2.5\right)} \Rightarrow \alpha = ...$	**M1**
	$\alpha = \text{awrt}\, 3.24$	**A1**
		(2)
(ii)	$\mathrm{g}(x) = \dfrac{1}{10}x^2 - \dfrac{1}{2x^2} + x - 11 \qquad x > 0$	
(a)	$\mathrm{g}'(x) = \dfrac{1}{5}x + \dfrac{1}{x^3} + 1$	**M1** **A1**
		(2)
(b)	$x_1 = 6 - \dfrac{\mathrm{g}\left(6\right)}{\mathrm{g}'\left(6\right)} = 6 - \dfrac{-1.41388...}{2.20462...}$	**M1**
	$= 6.641$	**A1cao**
		(2)
		(Total 8 marks)

Notes

(i)(a)

M1: Attempts both f(2.5) and f(3.5) with at least one correct in either radians or degrees. Note that in degrees f(2.5) = −2.49… and f(3.5) = −1.4867…

A1: Both f (2.5) = awrt −1 and f(3.5) = awrt 0.5, sign change (accept f(2.5)f(3.5) <0), continuous and conclusion all given but be forgiving with exact language. Use of degrees will be A0 as there is no change in sign.

(b)

M1: Uses a correct interpolation method to find a value for α. There are other alternative versions but look for a correct full process. E.g. may attempt the equation of the line through the two end points, then substitute y = 0 to find x. Allow if using degrees so long as a correct interpolation statement is clear.

A1: Correct value, accept awrt 3.24.

(ii)(a)

M1: $x^n \to x^{n-1}$ in at least two of the first 3 terms.

A1: All correct simplified or unsimplified.

(b)

M1: Correct application of Newton-Raphson. If no expression is seen, the method may be implied by a correct answer. (Look for the process rather than labelling if they write f but use g.)

A1cao: Correct value. Must be to 3d.p.. ISW if they try a second application of N-R.

Note: If correct answers for (b) appear after an incorrect derivative then please send to review.

Question	Scheme	Marks
7.(a)	$x=\frac{1}{3}t^2, y=\frac{2}{3}t \Rightarrow \frac{dy}{dx}=\frac{2}{3}\div\frac{2}{3}t=\frac{1}{t}$ or $y^2=\frac{4}{3}x \Rightarrow 2y\frac{dy}{dx}=\frac{4}{3} \Rightarrow \frac{dy}{dx}=\frac{2}{3y}=\frac{1}{t}$ or $y^2=\frac{4}{3}x \Rightarrow y=\frac{2}{\sqrt{3}}\sqrt{x} \Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{3}\sqrt{x}}=\frac{1}{t}$	B1
	$y-\frac{2}{3}t=-t\left(x-\frac{1}{3}t^2\right)$	M1
	$3tx+3y=t^3+2t$ *	A1*
		(3)
(b)	$t=9 \Rightarrow 27x+3y=747$	B1
	$y^2=\frac{4}{3}x \Rightarrow x=\frac{3y^2}{4} \Rightarrow 3y+3\times 9\times\frac{3y^2}{4}=729+18$ or $y^2=\frac{4}{3}x \Rightarrow \frac{1}{9}(747-27x)^2=\frac{4}{3}x \Rightarrow 729x^2-40350x+558009=0$	M1
	$27y^2+4y-996=0 \Rightarrow y=\ldots$ **or** $729x^2-40350x+558009=0 \Rightarrow x=\ldots$	M1
	$y=-\frac{166}{27},\ x=\frac{6889}{243}$	A1
		(4)
(b) ALT	$t=9 \Rightarrow 27x+3y=747$	B1
	$9x+y=249 \Rightarrow 3t^2+\frac{2}{3}t=249$	M1
	$9t^2+2t-747=0 \Rightarrow t=\ldots\left(-\frac{83}{9}\right)$	M1
	$y=-\frac{166}{27},\ x=\frac{6889}{243}$	A1
		(Total 7 marks)

Notes

(a)

B1: Correct $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of t from a calculus method. Must have seen a derivative used.

M1: Correct straight line method for the normal – must be using $\frac{-1}{\text{their } m_T}$ (or other correct approach). If using $y=mx+c$ they must proceed at least as far as finding c.

A1: cso – must have seen the evidence of use of calculus.

(b) + Alt (b)

B1: Correct equation for the normal at $t=9$

M1: Solves normal and equation of C simultaneously to obtain a quadratic equation in x or y or substitutes the parametric form to obtain a quadratic in t.

M1: Solves 3TQ in y to obtain a value (other than 6) or in x to obtain a value (other than 27) or in t to obtain a value (other than 9)

A1: Both coordinates correct and no incorrect ones (but ignore (27,6))

Question	Scheme	Marks
8.(a)	$\sum_{r=1}^{n} r\left(2r^2-3r-1\right) = \sum_{r=1}^{n}\left(2r^3-3r^2-r\right)$ $=2\times\frac{1}{4}n^2(n+1)^2-3\times\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)$	M1 A1
	$\frac{1}{2}n^2(n+1)^2-\frac{1}{2}n(n+1)(2n+1)-\frac{1}{2}n(n+1)$ $=\frac{1}{2}n(n+1)\left[n(n+1)-(2n+1)-1\right]$	M1
	$=\frac{1}{2}n(n+1)\left[n^2-n-2\right]=\frac{1}{2}n(n+1)(n+1)(n-2)$ $=\frac{1}{2}n(n+1)^2(n-2)$*	A1*
		(4)
(b)	$\sum_{r=n}^{2n} r\left(2r^2-3r-1\right)=\sum_{r=1}^{2n} r\left(2r^2-3r-1\right)-\sum_{r=1}^{n-1} r\left(2r^2-3r-1\right)$ $=\frac{1}{2}(2n)(2n+1)^2(2n-2)-\frac{1}{2}(n-1)(n)^2(n-3)$	M1
	$=\frac{1}{2}n(n-1)\left[4(2n+1)^2-n(n-3)\right]$	M1
	$=\frac{1}{2}n(n-1)\left[15n^2+19n+4\right]$	A1
	$=\frac{1}{2}n(n-1)(15n+4)(n+1)$	A1
		(4)
		(Total 8 marks)

Notes

(a) Note – attempts at induction score no marks.

M1: Expands the bracket and attempt to use at least one of the standard formulae correctly.

A1: Fully correct expression

M1: Attempts to factorise out at least $n(n+1)$ - both terms must have been common factors in the terms of their expression. If expanded to a quartic, there must be a clear attempt at factorisation in stages, directly to the given answer will be M0A0. (Note if they try to find roots there needs to be evidence that −1 is a repeated root before going direct to the given answer from these.)

A1*: cso Must have achieved a suitable correct intermediate stage with a quadratic in their working.

(b)

M1: Applies f(2n) – f(k) where k is n – 1 or n with the formula from (a) or allow from restarts using the standard formulae.

dM1: Attempts to factorise out $n(n-1)$ - which must be factors of their expression, so use of f(2n) −f(n) will score dM0. Accept for this mark if they expand from a correct expression and achieve the correct answer.

A1: Correct quadratic factor. May be implied if expansion to a quartic achieves the correct answer without intermediate factorisation shown. **A1:** Correct expression		

Question	Scheme	Marks
9.(a)	$\frac{3z-1}{2}=\frac{\lambda+5\text{i}}{\lambda-4\text{i}}\times\frac{\lambda+4\text{i}}{\lambda+4\text{i}}$	**M1**
	$=\frac{\lambda^2+9\lambda\text{i}-20}{\lambda^2+16}$	**M1**
	$\frac{3z-1}{2}=\frac{\lambda^2+9\lambda\text{i}-20}{\lambda^2+16}\Rightarrow z=\frac{2\left(\frac{\lambda^2+9\lambda\text{i}-20}{\lambda^2+16}\right)+1}{3}$	**ddM1**
	$=\frac{\lambda^2-8}{\lambda^2+16}+\frac{6\lambda}{\lambda^2+16}\text{i}$	**A1**
		(4)
(a) **Way 2**	$\frac{3z-1}{2}=\frac{\lambda+5\text{i}}{\lambda-4\text{i}}\Rightarrow 3z=\frac{2\lambda+10\text{i}}{\lambda-4\text{i}}+1=\frac{3\lambda+6\text{i}}{\lambda-4\text{i}}$ $\Rightarrow 3z=\frac{3\lambda+6\text{i}}{\lambda-4\text{i}}\times\frac{\lambda+4\text{i}}{\lambda+4\text{i}}$ or $z=\frac{\lambda+2\text{i}}{\lambda-4\text{i}}\times\frac{\lambda+4\text{i}}{\lambda+4\text{i}}$	**M1**
	$3z=\frac{3\lambda^2+18\lambda\text{i}-24}{\lambda^2+16}$ or $z=\frac{\lambda^2+6\lambda\text{i}-8}{\lambda^2+16}$	**M1**
	$3z=\frac{3\lambda^2+18\lambda\text{i}-24}{\lambda^2+16}\Rightarrow z=\ldots$	**ddM1**
	$=\frac{\lambda^2-8}{\lambda^2+16}+\frac{6\lambda}{\lambda^2+16}\text{i}$	**A1**
(a) **Way 3**	$\frac{3z-1}{2}=\frac{\lambda+5\text{i}}{\lambda-4\text{i}}\Rightarrow(3x+3y\text{i}-1)(\lambda-4\text{i})=2\lambda+10\text{i}$ $\Rightarrow 3\lambda x-\lambda+12y+(4+3\lambda y-12x)\text{i}=2\lambda+10\text{i}$	**M1**
	$\Rightarrow 3\lambda x+12y=3\lambda,\ 3\lambda y-12x=6$	**M1**
	$\Rightarrow x=\ldots, y=\ldots$	**ddM1**
	$z=\frac{\lambda^2-8}{\lambda^2+16}+\frac{6\lambda}{\lambda^2+16}\text{i}$	**A1**
(b)	$\arg z=\frac{\pi}{4}\Rightarrow \text{Re}\,z=\text{Im}\,z\ (>0)\Rightarrow\lambda^2-6\lambda-8=0\Rightarrow\lambda=\ldots$ or $\arg z=\frac{\pi}{4}\Rightarrow\frac{6\lambda}{\lambda^2-8}=\tan\frac{\pi}{4}=1\Rightarrow\lambda^2-6\lambda-8=0\Rightarrow\lambda=\ldots$	**M1**
	(Also need $\text{Re}(z),\text{Im}(z)>0$, so $\lambda>0$) $\lambda=3+\sqrt{17}$	**A1**

		(2)
		(Total 6 marks)

Notes

(a)

M1: Multiplies rhs by $\dfrac{\lambda+4\mathrm{i}}{\lambda+4\mathrm{i}}$

M1: Applies $\mathrm{i}^2=-1$ in both numerator and denominator and obtains a real number in the denominator.

ddM1: Rearranges to $z = \ldots$

A1: Correct and in the required form, but accept $\dfrac{\lambda^2-8+6\lambda\mathrm{i}}{\lambda^2+16}$. Need not be fully simplified. Accept e.g $\dfrac{3\lambda^2-24}{3\lambda^2+48}+\dfrac{18\lambda}{3\lambda^2+48}\mathrm{i}$

(a) Way 2

M1: Rearranges to $3z = \ldots$ (or $z = \ldots$) and multiplies numerator and denominator by the complex conjugate of their denominator.

M1: Applies $\mathrm{i}^2=-1$ in both numerator and denominator and obtains a real number in the denominator.

ddM1: Rearranges to $z = \ldots$ if not already done so. If rearranged to z initially **M1ddM1** will be scored together.

A1: As per main scheme.

There may be variations on the rearrangement, but the key steps will remain the same.

(a) Way 3

M1: Cross multiplies, applies $z = x+\mathrm{i}y$, expands and applies $\mathrm{i}^2=-1$ to achieve Cartesian form terms.

M1: Equates real and imaginary parts to form two equations with real coefficients.

ddM1: Solves the equations simultaneously to find x and y in terms of λ.

A1: As per main scheme.

(b)

M1: Sets the imaginary part of z equal to their real part of z, or divides these and sets equal to 1, and forms and solves the resulting quadratic in λ. (Need not be real roots for the M.)
Watch for answers to (a) with a negative imaginary component that do not consider the minus sign, as these should score M0 as they have not set real and imaginary parts equal.

A1: Correct exact answer only. The negative solution must have been rejected. Allow if both numerators were correct in (a) if there was a slip in the denominator only (e.g. λ^2+4 or $\lambda+16$) or if they were only out by a positive scale factor (e.g. lost the 3).

Question	Scheme	Marks
10.(i)	$n=1 \Rightarrow 3^{n-1}\begin{pmatrix}2n+3 & -n\\ 4n & 3-2n\end{pmatrix}=3^0\begin{pmatrix}2(1)+3 & -1\\ 4(1) & 3-2(1)\end{pmatrix}=\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}=\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}^1$	**B1**
	Assume true for $n=k$ so that $\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}^k=3^{k-1}\begin{pmatrix}2k+3 & -k\\ 4k & 3-2k\end{pmatrix}$	
	$\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}^{k+1}=3^{k-1}\begin{pmatrix}2k+3 & -k\\ 4k & 3-2k\end{pmatrix}\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}$ or $\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}^{k+1}=3^{k-1}\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}\begin{pmatrix}2k+3 & -k\\ 4k & 3-2k\end{pmatrix}$	**M1**
	$=3^{k-1}\begin{pmatrix}10k+15-4k & -2k-3-k\\ 20k+12-8k & -4k+3-2k\end{pmatrix}$ or $3^{k-1}\begin{pmatrix}10k+15-4k & -5k-3+2k\\ 8k+12+4k & -4k+3-2k\end{pmatrix}$ or $3^{k-1}\begin{pmatrix}6k+15 & -3k-3\\ 12k+12 & -6k+3\end{pmatrix}$	**A1**
	$\left[=3^k\begin{pmatrix}2k+5 & -k-1\\ 4k+4 & -2k+1\end{pmatrix}\right]=3^k\begin{pmatrix}2(k+1)+3 & -(k+1)\\ 4(k+1) & 3-2(k+1)\end{pmatrix}$	**A1**
	If the result is true for $n=k$ then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n.	**A1cso**
		(5)
(ii)	$\text{f}(1)=8^3+6=518=74\times7$ (so true for $n=1$)	**B1**
	Assume true for $n=k$ so that $8^{2k+1}+6^{2k-1}$ is divisible by 7	
	$\text{f}(k+1)=8^{2k+3}+6^{2k+1}$	**M1**
	$=64\times\left(8^{2k+1}+6^{2k-1}\right)+\ldots$ or $36\times\left(8^{2k+1}+6^{2k-1}\right)+\ldots$	**dM1**
	$=64\times\left(8^{2k+1}+6^{2k-1}\right)-28\times6^{2k-1}$ or $36\times\left(8^{2k+1}+6^{2k-1}\right)+28\times8^{2k+1}$	**A1**
	So if the result is true for $n=k$ then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n.	**A1cso**
		(5)
(ii) Alt	$\text{f}(1)=8^3+6=518=74\times7$ (so true for $n=1$)	**B1**
	Assume true for $n=k$ so that $8^{2k+1}+6^{2k-1}$ is divisible by 7	
	$\text{f}(k+1)-M\text{f}(k)=8^{2k+3}+6^{2k+1}-M\left(8^{2k+1}+6^{2k-1}\right)$	**M1**
	$=(64-M)\left(8^{2k+1}+6^{2k-1}\right)+\ldots$ or $(36-M)\left(8^{2k+1}+6^{2k-1}\right)+\ldots$	**dM1**
	$=(64-M)\left(8^{2k+1}+6^{2k-1}\right)-28\times6^{2k-1}$ or $(36-M)\left(8^{2k+1}+6^{2k-1}\right)+28\times8^{2k+1}$	**A1**
	$\Rightarrow \text{f}(k+1)-M\text{f}(k)$ divisible by 7 $\Rightarrow \text{f}(k+1)$ divisible by 7. So if the result is true for $n=k$ then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n.	**A1cso**
		(Total 10 marks)

Notes
(i) **B1:** Shows the result is true for $n = 1$. The LHS may just be stated, for the RHS accept as a minimum either one unsimplified term or the 3^0 seen. (Conclusion not needed here if both sides have been found correctly.) **M1:** Attempts $\begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}^{k+1}$ either way round using the result for $n = k$. **A1:** Correct unsimplified matrix. The coefficients inside may be simplified but the common factor 3 not taken out directly. **A1:** Achieves this result with no errors, via $3^{k-1}\begin{pmatrix} 6k+15 & -3k-3 \\ 12k+12 & -6k+3 \end{pmatrix}$ or $3^k\begin{pmatrix} 2k+5 & -k-1 \\ 4k+4 & -2k+1 \end{pmatrix}$ (oe with simplified linear terms). **A1cso:** Suitable conclusion following fully correct work. Must include in some form the points "true for n =1", "true for $n = k$ implies true for $n = k + 1$" and conclude true for all n in the conclusion. Depends on the preceding MAA marks and at least stating the correct matrix for n = 1in the initial base case check (So B0M1A1A1A1 is possible). (ii) **B1:** Shows the result is true for $n = 1$, Must express as a multiple of 7 or clearly show the factor. **M1:** Attempts f(k + 1) **dM1:** Attempts to express f(k + 1) in terms of f(k). Note they may let f(k) = 7m where m is an integer and use this in the working. **A1:** Correct expression for f(k + 1) in terms of f(k) (or m) **A1cso:** Suitable conclusion following fully correct work. Must include in some form the points "true for n =1", "true for $n = k$ implies true for $n = k + 1$" and conclude true for all n in the conclusion. Depends on the preceding MdMA marks and finding at least f(1) = 518 (So B0M1dM1A1A1 is possible if all that is missing is showing the factor 7 in f(1)). (ii) Alt **B1:** Shows the result is true for $n = 1$. Must express as a multiple of 7 or clearly show the factor. **M1:** Attempt f(k +1) – Mf(k) for any integer M. If M = 0 this is the main scheme. M = 1 may be seen frequently, but other value are possible. **dM1:** Attempts to express f(k +1) – Mf(k) in terms of f(k) or otherwise show a common factor of 7. **A1:** Correct expression for f(k +1) – Mf(k) in terms of f(k) or with clear common factor of 7 shown. Note if M = 1 is used, the expression becomes $63\times8^{2k+1}+35\times6^{2k-1}=7\left(9\times8^{2k+1}+5\times6^{2k-1}\right)$ which is fine for dM1A1. **A1cso:** Refers to divisibility of f(k +1) and makes suitable conclusion following fully correct work. Must include in some form the points "true for n =1", "true for $n = k$ implies true for $n = k + 1$" and conclude true for all n in the working. Depends on the preceding MdMA marks and finding at least f(1) = 518 (So B0M1dM1A1A1 is possible if all that is missing is showing the factor 7 in f(1)).