<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td rowspan="5">1</td><td colspan="3">$f(z)=4z^3-5z^2+14z+15$
Note: A minimum acceptable solution would be:
$z_1=1-2\text{i},\ z_2=1+2\text{i}$
$z_1+z_2=2$
$z_1z_2=1+4=5$
$f(z)=(4z+3)(z^2-2z+5)$ [by inspection]
$z_3=-\frac{3}{4}$</td><td></td></tr>
<tr><td colspan="2">Another root is $1+2\text{i}$</td><td>$z=1+2\text{i}$
Award B1 for sight of $1+2\text{i}$ anywhere in the response</td><td>B1</td></tr>
<tr><td colspan="2">$(z-(1-2\text{i}))(z-(1+2\text{i}))$
or sum $=1+2\text{i}+1-2\text{i}=2$, product $=(1+2\text{i})(1-2\text{i})=5$
$\Rightarrow z^2-2z+5$</td><td>M1: e.g. $(z\pm1\pm2\text{i})(z\pm1\pm2\text{i})$
or an attempt at the sum and product of roots. Working may be minimal.
A1: Correct quadratic factor</td><td>M1
A1</td></tr>
<tr><td colspan="3">$f(z)=(4z+3)(z^2-2z+5)$
M1: Uses their 3-term quadratic expression with real coefficients to attempt a linear factor, obtaining $az+b \quad a, b\neq 0$. May be via long division, inspection or comparing coefficients. Do not accept a fully factorised cubic without seeing a valid non-calculator method for the quadratic factor at least. If the linear factor appears in the working before the quadratic factor, this probably indicates calculator use.</td><td>M1</td></tr>
<tr><td colspan="2">$z=-\frac{3}{4}$</td><td>Correct real root following correct work. Errors in long division, for example, would be A0, even if two correct roots are found.
Condone work in x</td><td>A1</td></tr>
<tr><td rowspan="2">ALT for M1A1 M1A1</td><td colspan="2">$1+2\text{i}+1-2\text{i}+z=\frac{5}{4}$</td><td>M1: Considers the sum of all 3 roots
A1: Correct equation or equivalent working</td><td>M1
A1</td></tr>
<tr><td colspan="2">e.g. $\Rightarrow z=\frac{5}{4}-2$
$z=-\frac{3}{4}$</td><td>M1: Attempts to solves their equation, or uses equivalent work involving the sum of the three roots, reaching a value for the third root.
A1: Correct real root following correct work. Note that it is possible to obtain a third root of $-\frac{3}{4}$ despite having an incorrect quadratic factor. In this case both A marks are lost.
Ignore labelling, e.g. z, z_3, or may work in x.</td><td>M1
A1</td></tr>
<tr><td colspan="5" align="right">(5)</td></tr>
<tr><td colspan="5" align="right">Total 5</td></tr>
</table>

Question Number	Scheme	Notes	Marks
2(a)	$x=3y-3, xy=36 \Rightarrow y(3y-3)=36 \Rightarrow 3y^2-3y-36=0$ or $y=\dfrac{x}{3}+1 \Rightarrow x\left(\dfrac{x}{3}+1\right)=36 \Rightarrow \dfrac{x^2}{3}+x=36$ or $6t-\dfrac{18}{t}+3=0 \Rightarrow 6t^2+3t-18=0$	Eliminates x or y to form an equation in a single variable or Substitutes $x=6t$ and $y=\dfrac{6}{t}$ into l and obtains an equation in t only Condone numerical slips that lead to incorrect coefficients in the 3TQ, or to e.g. $xy=6$.	M1
	$y^2-y-12=(y-4)(y+3)=0 \Rightarrow y=4,\ -3$ or $x^2+3x-108=(x-9)(x+12)=0 \Rightarrow x=9,\ -12$ or $2t^2+t-6=(2t-3)(t+2)=0 \Rightarrow t=\dfrac{3}{2},\ -2$ $\Rightarrow (9,\ 4),\ (-12,\ -3)$	**Note: Both A marks require a non-calculator solution to a 3TQ. Usual rules, but any factorisation that is inconsistent with their 3TQ is A0A0.** A1: Any two of the four correct coordinates. A1: All four correct coordinates, obtained, paired correctly. Condone $x=\ldots, y=\ldots$ if clearly paired correctly.	A1 A1
			(3)
(b)	$\{y=36x^{-1}\}$ $\dfrac{dy}{dx}=-\dfrac{36}{x^2}$ $\{xy=36\}$ $y+x\dfrac{dy}{dx}=0$ $\dfrac{dy}{dx}=-\dfrac{y}{x}$ $\left\{x=ct \quad y=\dfrac{c}{t}\right\}$ $\dfrac{dx}{dt}=c \quad \dfrac{dy}{dt}=-\dfrac{c}{t^2}$ $\dfrac{dy}{dx}=-\dfrac{1}{t^2}$ May use $c=6$	Differentiates the equation of the curve to obtain a correct expression for $\dfrac{dy}{dx}$ No calculus is B0, but allow the next two marks.	B1
	$m_T=-\dfrac{1}{4} \Rightarrow y-(-3)=-\dfrac{1}{4}(x-(-12))$ or $-3=-\dfrac{1}{4}(-12)+c \Rightarrow c=-6$ {leading to $y=-\dfrac{1}{4}x-6$ } $0=-\dfrac{1}{4}x-6 \Rightarrow x=\ldots$	Forms equation of tangent with the numerical value for their $\dfrac{dy}{dx}$ and their coordinates of P (where $x<0$) correctly placed. Must set $y=0$ and obtain $x=\ldots$ M0 for attempts at a normal.	M1
	$x=-24$	Correct x coordinate	A1
			(3)
			Total 6

Question Number	Scheme	Notes	Marks
3(a)	det $\mathbf{A} = 5k(k-1)-(k+2)(k-3)$	Correct method for determinant. Allow one copying/bracketing or sign slip	**M1**
	$5k^2-5k-\left(k^2-k-6\right)=5$ $4k^2-4k+1=0 \Rightarrow (2k-1)^2=0 \Rightarrow k=\frac{1}{2}$	M1: Sets expression in k equal to 5, and solves a quadratic equation in k. If solving a 3TQ, usual rules apply. A1: Correct value for k	**M1** **A1**
			(3)
(b)	$\Rightarrow \mathbf{A}=\begin{pmatrix} \frac{5}{2} & -\frac{5}{2} \\ \frac{5}{2} & -\frac{1}{2} \end{pmatrix} \Rightarrow \mathbf{A}^{-1}=\frac{1}{5}\begin{pmatrix} -\frac{1}{2} & \frac{5}{2} \\ -\frac{5}{2} & \frac{5}{2} \end{pmatrix}$ $=\frac{1}{10}\begin{pmatrix} -1 & 5 \\ -5 & 5 \end{pmatrix}$	M1: Uses their value of k to find $\mathbf{A}$ and uses an acceptable method to find $\mathbf{A}^{-1}$. Accept any matrix that is not $\mathbf{A}$ multiplied by $\frac{1}{5}$. Condone matrix $\mathbf{A}$ not being written down, but must see any matrix that is not $\mathbf{A}$ multiplied by $\frac{1}{5}$. Alternatively finds $\mathbf{A}^{-1}$ in terms of k and then substitutes their k. Both methods require using det $\mathbf{A}$ = 5. A1: Correct inverse in required form	**M1** **A1**
	Note that in terms of k, $\mathbf{A}^{-1}=\frac{1}{4k^2-4k+6}\begin{pmatrix} k-1 & 3-k \\ -k-2 & 5k \end{pmatrix}$		
			(2)
(c)	$1.3^2 \times 5 = 8.45$	8.45 or exact equivalent and no other answers given.	**B1**
			(1)
(d)	**Note:** If no working is seen, a correct answer can be given M1A1 For A1 using either method, also accept a column vector or $x=\ldots, y=\ldots$		
	$\frac{1}{10}\begin{pmatrix} -1 & 5 \\ -5 & 5 \end{pmatrix}\begin{pmatrix} -8 \\ 6 \end{pmatrix}=\frac{1}{10}\begin{pmatrix} 8+30 \\ 40+30 \end{pmatrix}=\frac{1}{10}\begin{pmatrix} 38 \\ 70 \end{pmatrix}$ $\Rightarrow \left(\frac{19}{5},\ 7\right)$	M1: Correct method to multiply their inverse and $\begin{pmatrix} -8 \\ 6 \end{pmatrix}$. May need to be checked if $\mathbf{A}^{-1}$ is incorrect. A1: $\left(\frac{19}{5},\ 7\right)$ or $\left(3\frac{4}{5},\ 7\right)$ or $(3.8,\ 7)$	**M1** **A1**
Alt	$\frac{1}{2}\begin{pmatrix} 5 & -5 \\ 5 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} -8 \\ 6 \end{pmatrix} \Rightarrow \begin{matrix} 5x-5y=-16 \\ 5x-y=12 \end{matrix}$ $\Rightarrow x=\frac{19}{5},\ y=7$	M1: Uses $(\text{their } \mathbf{A})\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} -8 \\ 6 \end{pmatrix}$ appropriately to form simultaneous equations and solves A1: $\left(\frac{19}{5},\ 7\right)$ or $\left(3\frac{4}{5},\ 7\right)$ or $(3.8,\ 7)$	**M1** **A1**
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
4	$\mathrm{f}(x)=\dfrac{3}{2}x^3-6\sqrt{x}+\dfrac{1}{x}$		
(a)	$\mathrm{f}'(x)=\dfrac{9}{2}x^2-3x^{-\frac{1}{2}}-\dfrac{1}{x^2}$	M1: One term differentiated correctly A1: Fully correct differentiation (any form). Condone $-1x^{-2}$ for final term.	**M1** **A1**
			(2)
(b)	$x_1=0.25-\dfrac{\mathrm{f}(0.25)}{\mathrm{f}'(0.25)}$ $=0.25-\dfrac{\frac{3}{2}(0.25)^3-6\sqrt{0.25}+\frac{1}{0.25}}{\frac{9}{2}(0.25)^2-3(0.25)^{-\frac{1}{2}}-\frac{1}{(0.25)^2}}$ $=0.25-\dfrac{1.0234375}{-21.71875}$ or $0.25-\dfrac{\frac{131}{128}}{-\frac{695}{32}}$ or $\dfrac{1}{4}+\dfrac{131}{2780}\left\{=\dfrac{413}{1390}\right\}$ $=0.25-(-0.04712230216...)$ $=0.2971223022...=0.297$ (3 d.p.)	M1: Attempts to apply the Newton-Raphson formula with their $\mathrm{f}'(x)$ and obtains a value. Condone a copying/sign error in $\mathrm{f}(x)$ and/or their $\mathrm{f}'(x)$ if intention clear. Implied by awrt 0.297. Must be working if value or $\mathrm{f}'(x)$ incorrect. Allow as minimum $x_0=0.25, x_1=x_0-\dfrac{\mathrm{f}(x_0)}{\mathrm{f}'(x_0)}\Rightarrow x_1=...$ A1: awrt 0.297. Must be decimal. Note: actual root is 0.305 or 0.3054852282... isw if x_2 is attempted, etc.	**M1** **A1**
			(2)
(c)	$\mathrm{f}(1.65)=-0.3628914411...$ $\mathrm{f}(1.7)=0.1346924079...$	Attempts both $\mathrm{f}(1.65)$ and $\mathrm{f}(1.7)$ with at least one correct to 2 d.p.	**M1**
	e.g., $\dfrac{\beta-1.65}{1.7-\beta}=\dfrac{0.3628914411...}{0.1346924079...}\Rightarrow\beta=...$ May see $\dfrac{1.7-\beta}{0.134...}$ or $\dfrac{\beta-1.65}{0.362...}=\dfrac{1.7-1.65}{0.362...+0.134...}$ Forms a correct equation in β/x with their $\mathrm{f}(1.65)$ and $\mathrm{f}(1.7)$ values and solves. If e.g., A is used for $\beta-1.65$ then 1.65 + A must be seen later Implied by awrt 1.686. If value incorrect must see an equation but allow $\dfrac{1.65(0.134...)-1.7(-0.362...)}{0.134...-(-0.362...)}$ from use of $\dfrac{a\mathrm{f}(b)-b\mathrm{f}(a)}{\mathrm{f}(b)-\mathrm{f}(a)}$. If a correct interpolation formula is quoted, allow slips in the substitution if the intention is clear.		**M1**
	SC for M1M1: Some candidates are not evaluating $\mathrm{f}(1.65)$ and $\mathrm{f}(1.7)$. Award M1M1 if $\mathrm{f}(1.65)$ and $\mathrm{f}(1.7)$ are used correctly within a correct interpolation formula. For example:		

	1) $\dfrac{1.7-\beta}{\beta-1.65}=\dfrac{\frac{3}{2}(1.7)^3-6(1.7)^{1/2}+(1.7)^{-1}}{-\left[\frac{3}{2}(1.65)^3-6(1.65)^{1/2}+(1.65)^{-1}\right]}=0.3711...\Rightarrow\beta=1.686$. This scores M1M1A1. 2) $\dfrac{1.7-\beta}{\beta-1.65}=0.3711...\Rightarrow\beta=...$ This scores M1M1, and if their solution β is correct, also scores A1. 3) As a minimal solution: $\dfrac{x-1.65}{\lvert f(1.65)\rvert}=\dfrac{1.7-x}{\lvert f(1.7)\rvert}\Rightarrow x=1.686$ This scores M1M1A1. The correct answer implies that correct values have been used for $f(1.65)$ and $f(1.7)$.		
	$\beta=1.686465356...=1.686$ (3 d.p.)	Awrt 1.686. Allow "*x*=" (Actual root is 1.687 or 1.686860129...)	**A1**
Alt	$y-(-0.362...[\text{or } 0.134...])=\dfrac{0.134...-(-0.362...)}{1.7-1.65}(x-(1.65[\text{or } 1.7]))\Rightarrow y=0,\ x=...$ M1: As main scheme M1: Forms correct straight line equation, sets y = 0 and solves		**M1** **M1**
	$\beta=1.686465356...=1.686$ (3 d.p.)	Awrt 1.686	**A1**
			(3)
			Total 7

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>5</td><td colspan="2">$3z+\mathrm{i}w=-1+\mathrm{i} \qquad kz-w=-9\mathrm{i}$</td><td></td></tr>
<tr><td>(a)</td><td>$w=kz+9\mathrm{i}\Rightarrow 3z+\mathrm{i}(kz+9\mathrm{i})=-1+\mathrm{i}$; or
$kz-w=-9\mathrm{i}\Rightarrow \mathrm{i}kz-\mathrm{i}w=9\Rightarrow \mathrm{i}kz-(-1+\mathrm{i}-3z)=9$
$\Rightarrow z(3+k\mathrm{i})=8+\mathrm{i}\Rightarrow z=\frac{8+\mathrm{i}}{3+k\mathrm{i}}$</td><td>Uses appropriate algebra to eliminate w and reaches $z=\frac{a+ib}{c+\mathrm{i}d}=\mathrm{f}(k)$</td><td>M1</td></tr>
<tr><td></td><td>$z=\frac{8+\mathrm{i}}{3+k\mathrm{i}}\times\frac{3-k\mathrm{i}}{3-k\mathrm{i}}$</td><td>Appropriate multiplier seen. Condone "$\times(3-k\mathrm{i})$" if subsequently both numerator and denominator are multiplied.</td><td>dM1</td></tr>
<tr><td></td><td>$z=\frac{24+k+(3-8k)\mathrm{i}}{9+k^2}$ *</td><td>Fully correct proof. Condone invisible brackets if recovered.</td><td>A1*</td></tr>
<tr><td>Alt
Sim equations</td><td>$z=a+b\mathrm{i}, w=c+d\mathrm{i}$
Substitute into given equations:
$3(a+b\mathrm{i})+\mathrm{i}(c+d\mathrm{i})=-1+\mathrm{i}$
$k(a+b\mathrm{i})-(c+d\mathrm{i})=-9\mathrm{i}$
Equate real and imaginary parts, leading to:
$3a-d=-1,\ \ 3b+c=1$
$ka-c=0, \quad kb-d=-9$
Eliminate c and d leading to:
$a=\frac{24+k}{9+k^2}$ and $b=\frac{3-8k}{9+k^2}$
So $z=\frac{24+k+(3-8k)\mathrm{i}}{9+k^2}$ *</td><td>M1: Obtains four simultaneous equations in the real and imaginary coefficients of z and w

dM1: Eliminates real and imaginary coefficients of w

A1*: Fully correct proof.</td><td></td></tr>
<tr><td colspan="4" align="right">(3)</td></tr>
<tr><td>(b)</td><td>$k=2\Rightarrow z=\frac{26-13\mathrm{i}}{13}=2-\mathrm{i}$
$\Rightarrow w=...$</td><td>Substitutes $k=2$ to find z and then substitutes to find w. If they substitute $k=2$ into simultaneous equations, they must reach $w=\ldots$</td><td>M1</td></tr>
<tr><td></td><td>$w=2(2-\mathrm{i})+9\mathrm{i}=4+7\mathrm{i}$ *</td><td>Correct proof with sufficient working</td><td>A1*</td></tr>
<tr><td colspan="4" align="right">(2)</td></tr>
<tr><td>(c)(i)</td><td>$\frac{|w|}{|z|}=\frac{\sqrt{4^2+7^2}}{\sqrt{2^2+1^2}}\left\{=\frac{\sqrt{65}}{\sqrt{5}}\right\}=\sqrt{13}$; or
$\frac{|w|}{|z|}=\left|\frac{w}{z}\right|=\left|\frac{4+7\mathrm{i}}{2-\mathrm{i}}\right|=\left|\frac{1}{5}+\frac{18}{5}\mathrm{i}\right|=\sqrt{\left(\frac{1}{5}\right)^2+\left(\frac{18}{5}\right)^2}=\sqrt{13}$</td><td>M1: Correct use of modulus formula. Award when a correct numerical expression is seen for either modulus.
Note: may see $\sqrt{zz^*}$ or $\sqrt{ww^*}$
A1: $\sqrt{13}$ only</td><td>M1
A1</td></tr>
<tr><td>(c)(ii)</td><td>$\arg(z+w)=\arg(6+6\mathrm{i})$
$=\arctan(1)=\frac{\pi}{4}$</td><td>$\frac{\pi}{4}$ or $45\{°\}$
Isw if $\frac{\pi}{4}$ is written as a decimal (rounded or truncated to 0.78 or 0.79 to 2 d.p.), but not if a correct answer is followed by e.g. $-\frac{\pi}{4},\frac{3\pi}{4},-45°,135°$</td><td>B1</td></tr>
<tr><td colspan="4" align="right">(3)</td></tr>
<tr><td colspan="4" align="right">Total 8</td></tr>
</table>

Question Number	Scheme	Notes	Marks
6(a)	$n=1\Rightarrow \text{LHS}=\begin{pmatrix}3&-2\\2&-1\end{pmatrix}^1=\begin{pmatrix}3&-2\\2&-1\end{pmatrix}$ $\text{RHS}=\begin{pmatrix}2(1)+1&-2(1)\\2(1)&1-2(1)\end{pmatrix}=\begin{pmatrix}3&-2\\2&-1\end{pmatrix}$	Shows that LHS and RHS $=\begin{pmatrix}3&-2\\2&-1\end{pmatrix}$ or **A** for n = 1 At minimum, condone only substitution into RHS, reaching $\begin{pmatrix}3&-2\\2&-1\end{pmatrix}$ and statement "true" o.e. **Note:** it is possible to use $n=0$ in the basis case. In this case they would prove LHS=RHS=**I** **Note:** the conclusion for the basis case may be seen as a part of the final conclusion of proof.	**B1**
	Assume true for $n=k$, i.e., $\begin{pmatrix}3&-2\\2&-1\end{pmatrix}^k=\begin{pmatrix}2k+1&-2k\\2k&1-2k\end{pmatrix}$		
	$\begin{pmatrix}3&-2\\2&-1\end{pmatrix}^{k+1}=\begin{pmatrix}3&-2\\2&-1\end{pmatrix}^k\begin{pmatrix}3&-2\\2&-1\end{pmatrix}=\begin{pmatrix}2k+1&-2k\\2k&1-2k\end{pmatrix}\begin{pmatrix}3&-2\\2&-1\end{pmatrix}$ or $\begin{pmatrix}3&-2\\2&-1\end{pmatrix}\begin{pmatrix}2k+1&-2k\\2k&1-2k\end{pmatrix}$ Forms appropriate product of matrices for $\mathbf{A}^{k+1}$		**M1**
	$=\begin{pmatrix}6k+3-4k&-4k-2+2k\\6k+2-4k&-4k-1+2k\end{pmatrix}$ or $\begin{pmatrix}6k+3-4k&-6k-2+4k\\4k+2-2k&-4k-1+2k\end{pmatrix}\Rightarrow\begin{pmatrix}2k+3&-2k-2\\2k+2&-2k-1\end{pmatrix}$ **d**M1: Correct method for multiplication. Look for 3 correct elements unsimplified. A1: Correct simplified matrix with k terms collected. This must be seen, not implied, before being written in terms of $k+1$.		**dM1** **A1**
	$=\begin{pmatrix}2(k+1)+1&-2(k+1)\\2(k+1)&1-2(k+1)\end{pmatrix}$ True for n = 1, true for $n=k+1$ when assumed true for $n=k$, hence true for $n\in\mathbb{Z}^+$	Correct working leading to correct matrix in terms of $k+1$ and suitable conclusion. The conclusion must have 3 parts: 1) "True for $n=1$", but this need not be repeated if seen earlier. 2) True for $n=k\Rightarrow$ true for $n=k+1$, but if "Assume true for $n=k$" is seen earlier, then this need not be repeated, i.e. can conclude with "Hence true for $n=k+1$" o.e. 3) True for $n\in\mathbb{Z}^+$, but accept any of these: true for $n\in\mathbb{Z}$; true for all n; true for all integers; true for all numbers; true for $n\in\mathbb{N}$; true for all; always true. A1 can follow B0. A0 if work is in terms of n throughout.	**A1**
	Note: A meet in the middle approach is possible. For example, reaching $\begin{pmatrix}2k+3&-2k-2\\2k+2&-2k-1\end{pmatrix}$ for dM1 A1 and expanding $\begin{pmatrix}2(k+1)+1&-2(k+1)\\2(k+1)&1-2(k+1)\end{pmatrix}$ to reach the same.		
			(5)

(b)	$\mathbf{A}^5 = \begin{pmatrix} 11 & -10 \\ 10 & -9 \end{pmatrix}$	Correct $\mathbf{A}^5$, but also give B1 if their calculation of $\mathbf{A}^5$ is unsimplified and/or embedded in their calculation of **C**.	**B1**
	$\mathbf{C} = \mathbf{A}^5 - \mathbf{AB} + 7\mathbf{I} = \begin{pmatrix} 11 & -10 \\ 10 & -9 \end{pmatrix} - \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} p & -2 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$ $= \begin{pmatrix} 11 & -10 \\ 10 & -9 \end{pmatrix} - \begin{pmatrix} 3p+8 & -10 \\ 2p+4 & -6 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} = \ldots$ A full attempt at **C** with their $\mathbf{A}^5$ including correct method of multiplication for **AB** (with 3 correct elements unsimplified). Must reach a single matrix **C** in terms of p.		**M1**
	$\begin{pmatrix} 10-3p & 0 \\ 6-2p & 4 \end{pmatrix}$	Correct matrix	**A1**
			(3)
(c)	$\left\{ p = 3 \Rightarrow \mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \right\}$ Stretch of scale factor 4 parallel to the y–axis o.e.	M1: If a matrix **C** is written down, it must be of the form $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & q \end{pmatrix}, \lvert q \rvert \neq 1$. States "stretch". Condone "enlarge", but not "multiply". Must be a single transformation. M0 if alternatives also given. A1ft: Fully correct description. Requires "scale factor 4", "4 times" o.e., not e.g. "by 4 units". Accept "y-direction" or "vertically". FT on their $\lvert q \rvert \neq 1$	**M1** **A1ft**
			(2)
			Total 10

Question Number	Scheme	Notes	Marks
7(a)	$2x^2-3x+5=0 \Rightarrow \alpha+\beta=\frac{3}{2},\ \alpha\beta=\frac{5}{2}$	Either correct value. Award this mark if the value(s) are only seen embedded in calculations.	**B1**
	$(\alpha+1)^3(\beta+1)^3=(\alpha\beta+\alpha+\beta+1)^3$	States or uses $(\alpha+1)^3(\beta+1)^3=(\alpha\beta+\alpha+\beta+1)^3$	**M1**
	$=\left(\frac{5}{2}+\frac{3}{2}+1\right)^3=5^3=125*$	Achieves result via correct work with substitution seen	**A1***
Alt Working backwards	$2x^2-3x+5=0 \Rightarrow \alpha+\beta=\frac{3}{2},\ \alpha\beta=\frac{5}{2}$	As above	**B1**
	$(\alpha+1)^3(\beta+1)^3=125$ $\Rightarrow(\alpha+1)(\beta+1)=5$ $\alpha\beta+\alpha+\beta+1=5$	Cube roots and expands brackets to give an expression in terms of $\alpha+\beta$ and $\alpha\beta$	**M1**
	$\frac{5}{2}+\frac{3}{2}+1=5$ ✓	Verification that LHS=RHS. Must be some minimal conclusion, e.g. ✓ , “shown”, “QED”, “proved”, etc.	**A1***
Alt Changing variable	Let $z=x+1 \Rightarrow x=z-1$	$x=z-1$	**B1**
	$2x^2-3x+5=0$ $\Rightarrow 2(z-1)^2-3(z-1)+5=0$ $\Rightarrow 2z^2-7z+10=0$	Substitutes $x=z-1$ and obtains 3TQ in z	**M1**
	Product of roots $=\frac{10}{2}=5$ $\Rightarrow(\alpha+1)^3(\beta+1)^3\{=5^3\}=125*$	Correct completion	**A1***
Alt Expanding brackets for the last two marks	It is possible (but tricky) to complete this proof by expanding brackets. $(\alpha+1)^3(\beta+1)^3=(\alpha^3+3\alpha^2+3\alpha+1)(\beta^3+3\beta^2+3\beta+1)$ $=(\alpha\beta)^3+(\alpha^3+\beta^3)+3(\alpha\beta)^2(\alpha+\beta)+3\alpha\beta(\alpha^2+\beta^2)+9(\alpha\beta)^2$ $+9\alpha\beta(\alpha+\beta)+9\alpha\beta+3(\alpha^2+\beta^2)+3(\alpha+\beta)+1$ Then using $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ and $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$: $=(\alpha\beta)^3+(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)+3(\alpha\beta)^2(\alpha+\beta)+3\alpha\beta\left[(\alpha+\beta)^2-2\alpha\beta\right]+9(\alpha\beta)^2$ $+9\alpha\beta(\alpha+\beta)+9\alpha\beta+3\left[(\alpha+\beta)^2-2\alpha\beta\right]+3(\alpha+\beta)+1$ M1: Reaches an expression they can evaluate, involving terms that they have been given or have evaluated using correct identities. A1*: Substitutes values (e.g. for $\alpha\beta$ and $(\alpha+\beta)$) and reaches 125 with no errors.		
			(3)

(b)	$\text{new sum} = \dfrac{(\alpha+1)^3+(\beta+1)^3}{(\alpha+1)^3(\beta+1)^3}$ $\quad$ $\text{new product} = \dfrac{1}{(\alpha+1)^3(\beta+1)^3}$		
	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)\left\{=-\dfrac{63}{8}\right\}$ **or** $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta\left\{=-\dfrac{11}{4}\right\}$ **or** $(\alpha+1)^3+(\beta+1)^3=(\alpha+1+\beta+1)^3-3(\alpha+1)(\beta+1)(\alpha+1+\beta+1)$	Evaluates, states or uses at least one of these identities. May be seen or implied by working.	**B1 (M1 on epen)**
	Example 1 (expanding brackets): $(\alpha+1)^3+(\beta+1)^3$ $=\alpha^3+3\alpha^2+3\alpha+1+\beta^3+3\beta^2+3\beta+1$ $=\alpha^3+\beta^3+3(\alpha^2+\beta^2)+3(\alpha+\beta)+2$ $=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)+3\left((\alpha+\beta)^2-2\alpha\beta\right)+3(\alpha+\beta)+2$	Manipulates $(\alpha+1)^3+(\beta+1)^3$ to reach an expression involving only terms that have been evaluated using correct identities. Accept equivalents forms and condone sign errors and slips with coefficients only. Be aware that substitution of values may take place alongside the algebraic manipulation.	**M1**
	Example 2 (sum of two cubes): $(\alpha+1)^3+(\beta+1)^3$ $=(\alpha+\beta+2)(\alpha^2+\beta^2-\alpha\beta+\alpha+\beta+1)$ $=(\alpha+\beta+2)\left((\alpha+\beta)^2-3\alpha\beta+\alpha+\beta+1\right)$		
	Example 3 (working in $\alpha+1$ and $\beta+1$): $(\alpha+1)^3+(\beta+1)^3=(\alpha+1+\beta+1)^3-3(\alpha+1)(\beta+1)(\alpha+1+\beta+1)$		
	E.g. using line 4 of **Example 1**: $\left(\dfrac{3}{2}\right)^3-3\left(\dfrac{5}{2}\right)\left(\dfrac{3}{2}\right)+3\left(\left(\dfrac{3}{2}\right)^2-2\left(\dfrac{5}{2}\right)\right)+3\left(\dfrac{3}{2}\right)+2=-\dfrac{77}{8}$ Or line 3: $\left(-\dfrac{63}{8}\right)+3\left(-\dfrac{11}{4}\right)+3\left(\dfrac{3}{2}\right)+2=-\dfrac{77}{8}$ For **Example 2**: $\left(\dfrac{3}{2}+2\right)\left(\left(\dfrac{3}{2}\right)^2-3\left(\dfrac{5}{2}\right)+\dfrac{3}{2}+1\right)=-\dfrac{77}{8}$ For **Example 3**: $\left(\dfrac{3}{2}+2\right)^3-3(5)\left(\dfrac{3}{2}+2\right)=-\dfrac{77}{8}$ $\left\{\text{new sum}=\dfrac{\text{"}-77/8\text{"}}{125}=-\dfrac{77}{1000}\right\}$	Substitutes all relevant values correctly and reaches $-\dfrac{77}{8}$ o.e. for the numerator, or $-\dfrac{77}{1000}$ o.e. for the new sum. Note that $(\alpha^3+\beta^3)$, $(\alpha^2+\beta^2)$ and/or $(\alpha+1)(\beta+1)$ may have been evaluated in part (a).	**A1 (M1 on epen)**
	$\text{new product}=\dfrac{1}{125}\Rightarrow x^2+\dfrac{\text{"}77/8\text{"}}{125}x+\dfrac{1}{125}\{=0\}$	Forms a new quadratic correctly for their new sum and the product of $\dfrac{1}{125}$	**M1**
	$1000x^2+77x+8=0$	Any correct equation with integer coefficients	**A1**
			(5)
			Total 8

Question Number	Scheme	Notes	Marks
8(a)	$n=1 \Rightarrow \text{LHS}=\sum_{r=1}^{1} r^3 = 1^3 = 1$ $\text{RHS}=\frac{1}{4}\times 1^2 \times (1+1)^2 = \frac{1}{4}\times 1 \times 2^2 = 1$	Shows that LHS and RHS $=1$ for $n=1$. Must be some substitution into RHS. Gives conclusion "true", "shown", "proved", "✓" o.e. **Note:** the conclusion for the basis case may be seen as a part of the final conclusion of proof.	**B1**
	Assume true for $n=k$, i.e., $\sum_{r=1}^{k} r^3 = \frac{1}{4}k^2(k+1)^2$		
	$\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$	Uses $\sum_{r=1}^{k+1} r^3 = \left(\sum_{r=1}^{k} r^3\right) + (k+1)\text{th term}$	**M1**
	$=(k+1)^2\left(\frac{1}{4}k^2+k+1\right)=\frac{1}{4}(k+1)^2(k^2+4k+4)$	Reaches $\frac{1}{4}(k+1)^2$(quadratic in k) Must reach this intermediate step; it cannot be implied by $\frac{1}{4}(k+1)^2(k+2)^2$	**dM1**
	$=\frac{1}{4}(k+1)^2(k+2)^2$ or $\frac{1}{4}(k+1)^2((k+1)+1)^2$	Either expression	**A1**
	True for $n=1$, true for $n=k+1$ when assumed true for $n=k$, hence true for $n \in \mathbb{Z}^+$	Fully correct proof with correct conclusion. The conclusion must have 3 parts: 1) "True for $n=1$", but this need not be repeated if seen earlier. 2) True for $n=k \Rightarrow$ true for $n=k+1$, but if "Assume true for $n=k$" is seen earlier, then this need not be repeated, i.e. can conclude with "Hence true for $n=k+1$" o.e. 3) True for $n \in \mathbb{Z}^+$, but accept any of these: true for $n \in \mathbb{Z}$; true for all n; true for all integers; true for all numbers; true for $n \in \mathbb{N}$; true for all; always true. A1 can follow B0. A0 if work is in terms of n throughout.	**A1**
	Note: A meet in the middle approach is possible. For example, after reaching $\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$ for M1, expanding brackets to give $\frac{1}{4}k^4+\frac{3}{2}k^3+\frac{13}{4}k^2+3k+1$. They must then expand the target expression $\frac{1}{4}(k+1)^2(k+2)^2$ to a quartic (dM1) to reach the same expression (A1), followed by conclusion as above (A1).		
			(5)

(b)	$\sum_{r=2n}^{4n-1} r^3 = \frac{1}{4}(4n)^2(4n-1)^2 - \frac{1}{4}(2n)^2(2n-1)^2$	Attempts $f(4n-1)-f(2n-1)$	**M1**
	$\sum_{r=1}^{n} 1 = n$	Uses $\sum_{r=1}^{n} 1 = n$	**B1**
	$4n^2(16n^2-8n+1)-n^2(4n^2-4n+1)=1363n^2$ $\Rightarrow 60n^2-28n-1360=0$ $\Rightarrow 15n^2-7n-340=0$ $\Rightarrow (15n+68)(n-5)=0 \Rightarrow n=5$	M1: Reaches 3TQ in n that arises from at least one application of the sum of cubes formula, and solves (usual rules). Alternatively, solves a quartic to reach a value for n. In both cases, must reach a positive value of n. A1: $n=5$ only	**M1** **A1**
			(4)
			Total 9

Question Number	Scheme	Notes	Marks
9	$P\left(3p^2, 6p\right)$ $Q\left(3q^2, 6q\right)$		
(a)	$y=\sqrt{12}x^{\frac{1}{2}}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sqrt{12}}{2}x^{-\frac{1}{2}}=\frac{\sqrt{3}}{\sqrt{3p^2}}=\frac{1}{p}$; $y^2=12x$, $2y\frac{\mathrm{d}y}{\mathrm{d}x}=12$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6}{y}=\frac{1}{p}$; $y=2at$ $x=at^2$, $\frac{\mathrm{d}y}{\mathrm{d}t}=2a$ $\frac{\mathrm{d}x}{\mathrm{d}t}=2at$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{t}=\frac{1}{p}$. May use $a=3$ and/or $t=p$. Uses calculus to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{p}$. This may be implied by subsequent work.		**B1**
	$y-6p=-p\left(x-3p^2\right)$ or $6p=-p\times 3p^2+c\Rightarrow c=6p+3p^3\Rightarrow y=-px+6p+3p^3$	Forms equation of line with a changed gradient in terms of p and places coordinates correctly	**M1**
	$y+px=6p+3p^3$ *	Correct proof with evidence of calculus. Note that B0M1A1 is not possible.	**A1***
			(3)
(b) Using equation of line PQ	**Note:** There are many variations on the below, e.g. using (3, 0) to form the equation of the line (first M1), then using $\left(3p^2,6p\right)$ or $\left(3q^2,6q\right)$ (second M1)		
	gradient $PQ=\frac{6p-6q}{3p^2-3q^2}$ $\left\{=\frac{6(p-q)}{3(p+q)(p-q)}=\frac{2}{p+q}\right\}$	A correct expression for the gradient of the chord	**B1**
	e.g. $y-6p=\frac{2}{p+q}\left(x-3p^2\right)$ or $6p=\frac{2}{p+q}\times 3p^2+c\Rightarrow c=6p-\frac{6p^2}{p+q}\Rightarrow y=\frac{2}{p+q}x+6p-\frac{6p^2}{p+q}$	Forms equation of line with their chord gradient in terms of p and q and places coordinates correctly. The example uses $y-y_1=m\left(x-x_1\right)$ with $\left(x_1,y_1\right)=\left(3p^2,6p\right)$, but (3, 0) or $\left(3q^2,6q\right)$ could be used instead.	**M1**
	$-6p=\frac{6-6p^2}{p+q}$	Uses (3, 0) with their line to form an equation in p and q	**M1**
	$p\left(p+q\right)=p^2-1\Rightarrow p^2+pq=p^2-1\Rightarrow pq=-1$*	Fully correct proof with sufficient working	**A1***
	Note: If the gradient is not simplified you may see e.g., $-6p=\frac{6p-6q}{3p^2-3q^2}\left(3-3p^2\right)\Rightarrow -18p^3+18pq^2=18p-18q-18p^3+18p^2q$ $\Rightarrow -pq\left(p-q\right)=p-q\Rightarrow pq=-1$*		
			(4)
Alt 1 Using gradients of PF and QF	grad $PF=\frac{6p}{3p^2-3}$, grad $QF=\frac{6q}{3q^2-3}$	A correct expression for the gradient of PF **or** QF	**B1**
		Correct expressions for gradients of **both** PF **and** QF	**M1**
	$\frac{6p}{3p^2-3}=\frac{6q}{3q^2-3}$	Equates to form an equation in p and q	**M1**
	$pq^2-p=p^2q-q$; $-p+q=p^2q-pq^2$; $-\left(p-q\right)=pq\left(p-q\right)\Rightarrow pq=-1$*	Fully correct proof with sufficient working	**A1***

Alt 2 Similar triangles	$\dfrac{3p^2-3}{3-3q^2}=\dfrac{6p}{-6q}$	Finds **either** the ratio of the widths $\dfrac{3p^2-3}{3-3q^2}$ **or** the heights $\dfrac{6p}{-6q}$	**B1**
		Finds **both** the ratio of the widths $\dfrac{3p^2-3}{3-3q^2}$ **and** the heights $\dfrac{6p}{-6q}$	**M1**
	$\dfrac{p^2-1}{1-q^2}=\dfrac{p}{-q}$	Equates to form an equation in p and q	**M1**
	$-p^2q+q=p-pq^2$ $-pq(p-q)=p-q$ $pq=-1^*$	Fully correct proof with sufficient working	**A1***
Alt 3 Using gradients of PQ and PF	gradient $PQ=\dfrac{6p-6q}{3p^2-3q^2}$, gradient $PF=\dfrac{6p}{3p^2-3}$	A correct expression for the gradient of PQ **or** PF	**B1**
		Correct expressions for the gradients of **both** PQ **and** PF	**M1**
	$\dfrac{6p-6q}{3p^2-3q^2}=\dfrac{6p}{3p^2-3}$	Equates to form an equation in p and q	**M1**
	$\Rightarrow\dfrac{1}{p+q}=\dfrac{p}{p^2-1}\Rightarrow p^2-1=p^2+pq\Rightarrow pq=-1^*$	Fully correct proof with sufficient working	**A1***
Alt 4 Using lengths: $PQ=PF+QF$	$PF=3+3p^2$ $QF=3+3q^2$ $PQ=\sqrt{(3p^2-3q^2)^2+(6p-6q)^2}$	Correct expression for one of the lengths PQ, PF or QF, or the square of any of these	**B1**
		Attempts all three lengths, or their squares	**M1**
	e.g. $\sqrt{(3p^2-3q^2)^2+(6p-6q)^2}=3(p^2+q^2)+6$	Uses $PQ=PF+QF$ to form an equation in p and q	**M1**
	Completes the proof with no errors $(3p^2-3q^2)^2+(6p-6q)^2=\left(3(p^2+q^2)+6\right)^2$ Multiplying out and dividing both sides by 9 leads to: $p^4-2(pq)^2+q^4+4p^2-8(pq)+4q^2=p^4+2(pq)^2+q^4+4p^2+4q^2+4$ $\Rightarrow(pq)^2+2(pq)+1=0\Rightarrow(pq+1)^2=0\Rightarrow pq=-1^*$		**A1***
Alt 5 Sim equations	Line PQ has equation $y=mx+c$ Sub in $(3p^2,6p),(3q^2,6q),(3,0)$: $6p=3mp^2+c$ (1) $6q=3mq^2+c$ (2) $0=3m+c$ (3)	Forms 3 simultaneous equations in p,q,m,c	**B1**
	$(1)-(3)\Rightarrow 6p=3m(p^2-1)$ (4) $(2)-(3)\Rightarrow 6q=3m(q^2-1)$ (5)	Eliminates c or m	**M1**
	$\dfrac{6p}{p^2-1}=\dfrac{6q}{q^2-1}$	Eliminates both c and m	**M1**
	$pq^2-p=p^2q-q$ $-p+q=p^2q-pq^2$ $-(p-q)=pq(p-q)$	Fully correct proof with sufficient working	**A1***

	$pq=-1$*		
9(c) Way 1	$y+px=6p+3p^3,\quad y+qx=6q+3q^3$ $\Rightarrow 6p+3p^3-px=6q+3q^3-qx$	M1: Eliminates y without sign errors – must form a correct equation.	**M1**
	$6(p-q)+3(p^3-q^3)=(p-q)x$ $x=6+3(p^2+pq+q^2)$ $=3+3(p^2+q^2)$ $=3+3(p+q)^2-6pq$ $=3(p+q)^2+9$	M1: Uses appropriate algebra with sign/coefficient errors only to reach an expression for x of the correct form A1: Correct expression	**M1** **A1**
	$y=6p+3p^3-p\left(3(p+q)^2+9\right)$ $=6p+3p^3-3p^3-6p^2q-3pq^2-9p$ $=-3p-3pq(2p+q)$ $=-3p+6p+3q$ $=3(p+q)$	dM1: Substitutes their expression for x of the correct form to obtain an expression for y of the correct form condoning sign/coefficient errors only A1: Correct expression	**dM1** **A1**
(c) Way 2	$y+px=6p+3p^3,\quad y+qx=6q+3q^3$ $\dfrac{6p+3p^3-y}{p}=\dfrac{6q+3q^3-y}{q}$	M1: Eliminates x without sign errors – must form a correct equation.	**M1**
	$6pq+3p^3q-qy=6pq+3pq^3-pq$ $(p-q)y=3pq(q^2-p^2)$ $(p-q)y=3(p^2-q^2)$ $y=3(p+q)$	M1: Uses appropriate algebra with sign/coefficient errors only to reach an expression for y of the correct form A1: Correct expression	**M1** **A1**
	$x=\dfrac{6p+3p^3-3(p+q)}{p}$ $=6+3p^2+\dfrac{3pq}{p}(p+q)$ $=6+3p^2+3pq+3q^2$ $=9+3(p^2+2pq+q^2)$ $=3(p+q)^2+9$	dM1: Substitutes their expression for y of the correct form to obtain an expression for x of the correct form condoning sign/coefficient errors only A1: Correct expression	**dM1** **A1**
			(5)
(d)	$P(12, 12)$ $\Rightarrow p=2,\ q=-\dfrac{1}{2},\ p+q=\dfrac{3}{2}$ $x_R=3\left(\dfrac{3}{2}\right)^2+9,\ y_R=3\left(\dfrac{3}{2}\right)$ $\Rightarrow R\left(\dfrac{63}{4},\dfrac{9}{2}\right)$	M1: Correct method to find one coordinate of R. Substitutes $p=2,\ q=-\dfrac{1}{2}$ or $p+q=\dfrac{3}{2}$ into an expression for x or y from part (c). A1: Correct coordinates of R in simplest form, i.e. $\left(\dfrac{63}{4},\dfrac{9}{2}\right)$ or $\left(15\dfrac{3}{4},4\dfrac{1}{2}\right)$ or $(15.75, 4.5)$ accept $x=...,y=...$	**M1** **A1**
			(2)
			Total 14
			PAPER TOTAL: 75