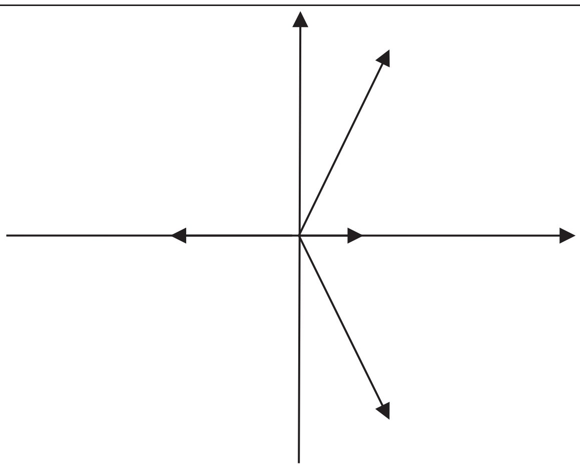
Question Number	Scheme	Notes	Marks
1	$\mathbf{C}=3\mathbf{A}+2\mathbf{B}=3\begin{pmatrix}2&4\\3&1\end{pmatrix}+2\begin{pmatrix}1&k\\5&3k\end{pmatrix}$ Condone any type of brackets (or even missing brackets) for matrices		
(a)	$\mathbf{C}=3\mathbf{A}+2\mathbf{B}=\begin{pmatrix}6&12\\9&3\end{pmatrix}+\begin{pmatrix}2&2k\\10&6k\end{pmatrix}$ $=\begin{pmatrix}8&12+2k\\19&3+6k\end{pmatrix}$	B1: Any 2 correct elements (terms collected but could be factorised) **d**B1: Fully correct **C** (terms collected but could be factorised)	**B1** **dB1**
			(2)
(b)	$\lvert\mathbf{C}\rvert=8(3+6k)-19(12+2k)$ $\{=24+48k-228-38k=10k-204\}$	**Correct** determinant explicitly seen (may be unsimplified but if only seen with the brackets multiplied out this must be done correctly) **for their C** of the correct form i.e., $\begin{pmatrix}a&b+ck\\d&e+fk\end{pmatrix}$ $a,b,c,d,e,f\in\mathbb{Z}^+$ Note that this mark cannot be scored in (c). If parts are unlabelled mark in the order presented. If there is any doubt look for using the determinant to find k	**M1**
	$\lvert\mathbf{C}\rvert=0\Rightarrow 8(3+6k)-19(12+2k)=0\Rightarrow k=\ldots$ Finds a value for k following an attempt to apply $\lvert\mathbf{C}\rvert=0$ with their **C** which must have had at least 1 term in k. **Allow slips here** (e.g., errors multiplying out brackets which they may do immediately) provided there is a recognisable attempt at $ad-bc=0$ Note this mark is not dependent on the previous method mark.		**M1**
	$\{k=\}\dfrac{102}{5},\ 20\dfrac{2}{5},\ 20.4$	Correct value. Any equivalent e.g., $\dfrac{204}{10}$	**A1**
			(3)
(c)	$\mathbf{C}^{-1}=\dfrac{1}{10k-204}\begin{pmatrix}3+6k&-12-2k\\-19&8\end{pmatrix}$ oe e.g., $\begin{pmatrix}\dfrac{3(1+2k)}{2(5k-102)}&\dfrac{-6-k}{5k-102}\\\dfrac{-19}{2(5k-102)}&\dfrac{8}{2(5k-102)}\end{pmatrix}$	M1: Award for a completed attempt at $\dfrac{1}{their\lvert\mathbf{C}\rvert}\times(\text{A }\mathbf{changed}\ their\ \mathbf{C})$ $\lvert\mathbf{C}\rvert$ must be a 2 term linear function of k although there may have been slips when simplifying and they may reattempt the determinant **<u>OR</u>** sight of a correct ft Adj(their **C**) ignoring any determinant (it could be missing) provided **C** had at least one term in k A1: Fully correct inverse in simplest form (terms collected). Not ft. Elements/determinant may be factorised. Isw once a correct inverse is seen. Remember that ISW applies throughout this paper.	**M1** **A1**
			(2)
			(Total 7 marks)

Question Number	Scheme	Notes	Marks
2	$y^2 = 10x$ $\quad y^2 = 4ax,\ x = at^2,\ y = 2at,$ focus : $(a, 0)$, directrix : $x = -a$		
(a) (i) & (ii)	Focus is $\left(\frac{5}{2}, 0\right)$ **or** directrix is $x = -\frac{5}{2}$ oe e.g., $2x + 5 = 0$	Correct focus (may be given as $x = \frac{5}{2},\ y = 0$) **OR** any correct **equation** for directrix	**B1**
	Focus is $\left(\frac{5}{2}, 0\right)$ **and** directrix is $x = -\frac{5}{2}$ oe	Correct focus (may be given as $x = \frac{5}{2},\ y = 0$) **and** directrix. Allow $\frac{10}{4}$ or $2\frac{1}{2}$ or 2.5 for both marks	**dB1**
	SC: Allow B1dB0 for $(k, 0)$ & $x = -k \quad k > 0,\ k \neq 4$ Just $a = \frac{5}{2}$ is 00 unless $(a, 0),\ x = -a$ are seen		
			(2)
(b)	20	Correct distance	**B1**
			(1)
(c)	$x_P = 20 - "\frac{5}{2}" = \ldots \left\{= \frac{35}{2}\right\}$	Correct ft value for the x coordinate of P with 20 and their a where $0 < a < 20$. If $a = \ldots$ is not seen or implied and their focus and directrix are not consistent then look for $20 - a = \ldots$ from a directrix of $x = -a,\ a > 0$. Ignore any extra x-coordinates	**B1ft**
	$y_P = (\pm)\sqrt{20^2 - (17.5 - 2.5)^2}$ or $y = (\pm)\sqrt{10 \times \frac{35}{2}}$	A correct numerical expression or value for one of the y coordinates of P. Allow awrt $(\pm)13.2$ **Not an ft mark**	**M1**
	$\left(\frac{35}{2}, \pm 5\sqrt{7}\right)$ or e.g., $\left(17.5, \sqrt{175}\right), \left(17.5, -\sqrt{175}\right)$	Fully correct exact coordinates of P May be given as $x = \ldots \ y = \ldots$ Answer only is acceptable. No extra coordinates unless rejected	**A1**
	Using a parameter (scheme above still applies): $P\left(at^2, 2at\right) = P\left(\frac{5}{2}t^2, 5t\right) \Rightarrow \frac{5}{2}t^2 = 20 - \frac{5}{2} \Rightarrow t = \pm\sqrt{\frac{2}{5} \times \frac{35}{2}} = \sqrt{7} \Rightarrow P\left(\frac{35}{2}, \pm 5\sqrt{7}\right)$ Allow B1ft for a consistent $x = \ldots$ from $"\frac{5}{2}"t^2 + "\frac{5}{2}" = 20$ provided $0 < "\frac{5}{2}" < 20$ May also see e.g., $20^2 = \left(\frac{5}{2}t^2 - \frac{5}{2}\right)^2 + (5t)^2 \Rightarrow t^4 + 2t^2 - 63 = \left(t^2 - 7\right)\left(t^2 + 9\right) = 0 \Rightarrow t = \pm\sqrt{7} \Rightarrow P\left(\frac{35}{2}, \pm 5\sqrt{7}\right)$ Calculators may be used to solve equations in solutions such as: $\left(x - \frac{5}{2}\right)^2 + y^2 = 20^2 \Rightarrow x^2 + 5x - \frac{1575}{4} = 0 \Rightarrow 4x^2 + 20x - 1575 = (2x - 35)(2x + 45) = 0 \Rightarrow x = \frac{35}{2}$		
			(3)
			(Total 6 marks)

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td>3(a)</td><td colspan="2">$$\sum_{r=1}^{n}(2r-3)(3r+2)=\sum_{r=1}^{n}6r^2-5r-6=6\times\frac{1}{6}n(n+1)(2n+1)-\frac{5}{2}n(n+1)-6n$$
M1: Expands to a quadratic and uses at least one of the standard formulae correctly for that term. If quadratic has two terms in r must correctly use the formula twice if scoring for using the sum of r
A1: Correct expression - no need to simplify it</td><td>M1
A1</td></tr>
<tr><td></td><td>$=n\left[(n+1)(2n+1)-\frac{5}{2}(n+1)-6\right]$
or
$=\frac{n}{2}\left[2(n+1)(2n+1)-5(n+1)-12\right]$</td><td>Completes an attempt to factorise out n or $\frac{1}{2}n$. May expand completely first.
Sign/coefficient errors only.
Requires an expression of the correct form i.e.,
3TC with no contant $\pm$ 2TQ with no constant $\pm kn$</td><td>dM1</td></tr>
<tr><td></td><td>$=\frac{1}{2}n\left[2(2n^2+3n+1)-5n-5-12\right]$
$=\frac{1}{2}n(4n^2+n-15)$</td><td>Correct expression in this form.
Not just values for the constants unless $\frac{1}{2}n(an^2+bn+c)$ is seen in the work</td><td>A1</td></tr>
<tr><td></td><td colspan="2">Alternative for the last 2 marks - expands expression and given form and compares coefficients:
$$6\times\frac{1}{6}n(n+1)(2n+1)-\frac{5}{2}n(n+1)-6n=2n^3+3n^2+n-\frac{5}{2}n^2-\frac{5}{2}n-6n=2n^3+\frac{1}{2}n^2-\frac{15}{2}n$$
$$\frac{1}{2}n(an^2+bn+c)=\frac{1}{2}an^3+\frac{1}{2}bn^2+\frac{1}{2}cn$$
$$\Rightarrow a=4,\ b=1,\ c=-15$$
dM1: Expands both sides. LHS must have been of the form 3TC with no contant $\pm$ 2TQ with no constant $\pm kn$ and must achieve 2 3TCs with no constants
A1: Correct values. $\frac{1}{2}n(an^2+bn+c)$ must be explicitly seen in the work.</td><td></td></tr>
<tr><td colspan="4" align="right">(4)</td></tr>
<tr><td>(b)</td><td colspan="2">$$21\times38+23\times41+...+97\times152$$
$$=\frac{1}{2}\times50(4\times50^2+50-15)-\frac{1}{2}\times11(4\times11^2+11-15)$$
Attempts $\sum_{r=1}^{50}(2r-3)(3r+2)-\sum_{r=1}^{11}(2r-3)(3r+2)$ explicitly using their result from part (a) and obtains a numerical expression. Allow slips but it must clearly be an attempt at f(50) - f(11). There must be evidence of use of the formula for both summations so accept e.g., $\frac{1}{2}\times501750-\frac{1}{2}\times5280$ as a minimum but not $250875-2640$</td><td>M1</td></tr>
<tr><td></td><td>= 248 235</td><td>Correct answer from correct work</td><td>A1</td></tr>
<tr><td colspan="4" align="right">(2)</td></tr>
<tr><td colspan="4" align="right">(Total 6 marks)</td></tr>
</table>

Question Number	Scheme	Notes	Marks
4	$f(x)=\dfrac{1}{10}x-\dfrac{(x-4)^2}{\sqrt{x}}=\dfrac{1}{10}x-\dfrac{x^2-8x+16}{\sqrt{x}}=\dfrac{1}{10}x-x^{\frac{3}{2}}+8x^{\frac{1}{2}}-16x^{-\frac{1}{2}} \qquad x>0$		
(a)	$f(5)=0.05278... \qquad f(5.1)=-0.02579...$ Obtains values for both f (5) and f (5.1) with one correct (1 sf as shown below) if just values are given. Accept $f(5)=\frac{5-2\sqrt{5}}{10}$ for the M mark but the A mark requires a convincing argument that this is > 0		**M1**
	Sign change (or > 0, < 0, or positive, negative or product negative) and f(x) is **continuous** therefore a **root** α is between x = 5 and x = 5.1	f (5) = awrt 0.05 and f (5.1) = awrt −0.03 (or truncated: -0.02), sign change, any reference to continuity (which could be poor e.g., "x/it/the graph is continual") and a minimal conclusion.	**A1**
			(2)
(b)	$f(5.05)=0.01439... \quad (\Rightarrow 5.05<\alpha<5.1)$	For f (5.05) = awrt 0.01	**B1**
	$f(5.075)=... \qquad \{-0.00547...\}$	Obtains a value for f (5.075) following a positive value for f (5.05)	**M1**
	$[5.05,\ 5.075]$ or e.g., $5.05<\alpha<5.075$ Correct interval following no clearly incorrect values for f(5.05) and f(5.075). Allow any equivalent notation but must be a **mathematical** interval. May use x or a etc for α . Note real root is 5.068164... Ignore further bisections and additional work/comment		**A1**
			(3)
(c)	$f'(x)=\dfrac{1}{10}-\dfrac{3}{2}x^{\frac{1}{2}}+4x^{-\frac{1}{2}}+8x^{-\frac{3}{2}}$ M1: For $...x^n \to ...x^{n-1}$ once, e.g., $\frac{1}{10}x \to \frac{1}{10}$ & allow unprocessed indices e.g., $\frac{1}{10}x^{1-1}$ A1: An expression of the correct form (sign and coefficient slips only). No unprocessed indices nor x^0 A1: All correct. Accept $...+\frac{8}{2}x^{-\frac{1}{2}}+\frac{16}{2}x^{-\frac{1}{2}}$		**M1** **A1** **A1**
	If quotient/product rule is used on the fraction you may see: $\dfrac{1}{10}-\dfrac{2x^{\frac{1}{2}}(x-4)-\frac{1}{2}x^{-\frac{1}{2}}(x-4)^2}{x}$ or $\dfrac{1}{10}-\dfrac{x^{\frac{1}{2}}(2x-8)-\frac{1}{2}x^{-\frac{1}{2}}(x^2-8x+16)}{x}$ or $\dfrac{1}{10}-\left(2x^{-\frac{1}{2}}(x-4)-\dfrac{1}{2}x^{-\frac{3}{2}}(x-4)^2\right)$ or $\dfrac{1}{10}-\left(x^{-\frac{1}{2}}(2x-8)-\dfrac{1}{2}x^{-\frac{3}{2}}(x^2-8x+16)\right)$ and remember to ISW once a correct answer is seen		
			(3)
(d)	$x_1=3.3-\dfrac{f(3.3)}{f'(3.3)}=3.3-\dfrac{0.06026387754...}{0.9115437451...}=... \quad \{3.23388811...\}$ Attempts to apply a correct Newton-Raphson formula with 3.3, obtaining a value. Implied by awrt 3.234 (i.e., correct to 3 dp) and could just give this value which scores both marks. Accept as minimal evidence e.g., "$\{x_1=\}x_0-\dfrac{f(x_0)}{f'(x_0)}=...$" and no need to check value. Note: real root is 3.2368768...		**M1**
	= awrt 3.234 Ignore how the value is labelled and it can follow an incorrect f '(x)		**A1**
			(2)
			(Total 10 marks)

Question Number	Scheme	Notes	Marks
5	$2x^2 - x + 3 = 0$ Note that solving the equation and clearly using the values $\alpha = \frac{1+\sqrt{23}\mathrm{i}}{4}$ and $\beta = \frac{1-\sqrt{23}\mathrm{i}}{4}$ throughout i.e. for $\alpha^3+\beta^3$, $\alpha^4\beta - 2 + \alpha\beta^4 - 2$ & $(\alpha^4\beta-2)(\alpha\beta^4-2)$ is likely to only be able to access the final M mark in (c). However, if identities are used in (b) and (c) allow all the marks to be accessible.		
(a)	$\alpha+\beta = \frac{1}{2}, \quad \alpha\beta = \frac{3}{2}$	Both values correct. B0 if these have clearly come from solving the equation.	**B1**
			(1)
(b)	$\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = \left(\frac{1}{2}\right)^3 - 3\times\frac{3}{2}\times\frac{1}{2}$ Obtains a numerical expression for $\alpha^3+\beta^3$ with a **correct** identity. Condone substitution slips e.g., mixing up "$\frac{1}{2}$" and "$\frac{3}{2}$" and allow M1 if a correct identity is seen or implied. Sight of a correct identity followed by a value is sufficient but if only a value is seen it must be $-\frac{17}{8}$		**M1**
	$= -\frac{17}{8}$ oe, -2.125	Correct value. Must not be fortuitous but allow all part (c) marks.	**A1**
			(2)
(c)	Work for the new sum/product may be embedded within a quadratic		
	new sum $= \{\alpha^4\beta - 2 + \alpha\beta^4 - 2 =\}\ \alpha\beta(\alpha^3+\beta^3) - 4 = \frac{3}{2}\left(-\frac{17}{8}\right) - 4 \quad \left\{= -\frac{115}{16}\right\}$ Obtains a numerical expression for the new sum with a **correct** identity. Condone substitution slips e.g., mixing up "$\frac{1}{2}$" and "$\frac{3}{2}$" and allow M1 if a correct identity is seen or implied. Also ft their $\alpha^3+\beta^3$ which may be recalculated wrongly and may be 0. Sight of a correct identity followed by a value is sufficient but if only a value is given it must be $-\frac{115}{16}$		**1st M1 (sum)**
	new product $= \{(\alpha^4\beta-2)(\alpha\beta^4-2) =\}\ (\alpha\beta)^5 - 2\alpha\beta(\alpha^3+\beta^3) + 4 = \left(\frac{3}{2}\right)^5 - 2\times\left(\frac{3}{2}\right)\left(-\frac{17}{8}\right) + 4 \quad \left\{= \frac{575}{32}\right\}$ Obtains a numerical expression for the new product with a **correct** identity. Condone substitution slips e.g., mixing up "$\frac{1}{2}$" and "$\frac{3}{2}$" and allow M1 if a correct identity is seen or implied. Also ft their $\alpha^3+\beta^3$ which may be recalculated wrongly and may be 0. Sight of a correct identity followed by a value is sufficient but if only a value is given it must be $\frac{575}{32}$		**2nd M1 (prod.)**
	new sum $= -\frac{115}{16}$ **or** new product $= \frac{575}{32}$	One correct value. Allow -7.1875, 17.96875 **Requires one previous M mark.**	**A1** **(M1 on ePen)**
	$x^2 - \left(-\frac{115}{16}\right)x + \frac{575}{32} (= 0)$ Correctly applies x^2 – (their new sum)x + their new product. If e.g., the "x" is missing it must be recovered. Allow a different variable e.g., z. May use rounded decimals. Accept values for p, q, r		**M1**
	$32x^2 + 230x + 575 = 0$	Correct **equation in x**. Three terms in any order on same side. Allow any integer multiple. If just values are given $px^2 + qx + r = 0$ must be seen in the work	**A1**
			(5)
			(Total 8 marks)

Question Number	Scheme	Notes	Marks
6	$\mathrm{f}(z) = 12z^4 + Az^3 + Bz^2 + Cz - 306$	Allow work in x	
(a)	$(z =) 1 - 4\mathrm{i}$	Correct conjugate seen anywhere	**B1**
			(1)
(b)	$z = 1 \pm 4\mathrm{i} \Rightarrow (z-(1+4\mathrm{i}))(z-(1-4\mathrm{i})) = \ldots \quad \{z^2 - 2z + 17\}$ or e.g., sum of roots $= \ldots\ \{2\}$, product of roots $= (1+4\mathrm{i})(1-4\mathrm{i}) = \ldots\ \{17\} \Rightarrow \ldots\ \{z^2 - 2z + 17\}$ M1: Applies an appropriate strategy and finds a 3 term quadratic factor with real coefficients. Condone poor algebra provided there is a completed attempt using $1+4\mathrm{i}$ and $1-4\mathrm{i}$ A1: A correct 3 term quadratic factor. Could be just written down. Ignore if "= 0" or f(z) =...		**M1** **A1**
			(2)
	Mark parts (c) and (d) together		
(c)	The other factor is $(4z \pm k)$	Sight of $(4z \pm k)$ as the other linear factor. See note below. No requirement to find k but do not award for just $4z$	**B1**
	e.g., $k \times -2 \times "17" = -306 \Rightarrow k = \ldots(9)$ $(Dz+E)("3z^3 - 8z^2 + 55z - 34") = f(z) \Rightarrow D = 4,\ E = \ldots$ Any appropriate method using $(4z \pm k)$ to obtain k seen. See note below		**M1**
	Note: If $\left(z \pm \frac{j}{4}\right)$ is used then a multiplier of 4 must be recovered later to score marks		
	$\{\mathrm{f}(z) =\}(z^2 - 2z + 17)(3z-2)(4z+9)$ oe e.g., $(z-(1+4\mathrm{i}))(z-(1-4\mathrm{i}))(3z-2)(4z+9)$ $12(z^2 - 2z + 17)\left(z - \frac{2}{3}\right)\left(z + \frac{9}{4}\right)$	All correct on one line. Factors in any order. Ignore irrelevant errors. Answer could just be written down.	**A1**
			(3)
(d)	$\mathrm{f}(z) = (z^2 - 2z + 17)(3z-2)(4z+9) = \ldots$	Expands and collects terms or obtains values for all 3 constants via any credible method. If method unclear look for one of the correct values below	**M1**
	$\mathrm{f}(z) = (12z^4) - 5z^3 + 148z^2 + 359z(-306)$ or $A = -5,\ B = 148,\ C = 359$	A1: 2 correct (with 3 found) A1: All 3 correct	**A1** **A1**
			(3)
(e)		B1: Conjugate pair (broadly symmetrical but anywhere in Q1 and Q4) **or** real pair (positive root closer to origin). **d**B1: All plotted such that the distance to the origin for the real positive root < the shortest distance to the y-axis for the complex roots **and** the positive real root positioned closer to the origin than the real negative root. Points or vectors/lines. Ignore root/axes labelling. These are not ft marks so do not award if their work clearly indicates they are using any roots which are not $1 \pm 4\mathrm{i}$, $\frac{2}{3}$, $-\frac{9}{4}$ but ignore the labelling on the diagram	**B1** **dB1**
			(2)
			(Total 11 marks)

Question Number	Scheme	Notes	Marks
7	$\mathbf{M}=\begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix}$ $\quad\mathbf{Q}=\begin{pmatrix}5 & -1\\ 5 & 3\end{pmatrix}$	Allow any (or no) brackets for matrices	
(i)(a)	A stretch, (scale) factor 3, parallel to the/in the/on the/up the/down the/along the/across the y-axis/ y direction/of the y-coordinates or vertically/ perpendicular to x-axis M1: Stretch. Must be a single transformation and no alternatives also offered. Ignore additional "and parallel to the x-axis scale of factor 1" & similar May be mis-spelt but must be an attempt at "stretch" - not e.g., "expansion". A1: Fully correct description. Just "in y" is insufficient. Accept surrogates for "factor" e.g., "scale" but not just "stretch of/by 3". Ignore reference to "about O"		**M1** **A1**
			(2)
(b)	$\{\mathbf{N}=\}\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}$	Correct matrix. No elements as trig expressions	**B1**
			(1)
(c)	$\{\mathbf{P}=\mathbf{NM}=\}\ "\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}"\begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix}$ Indicates intention to multiply the right way round. If this is set up as a grid multiplication then they must get a consistent answer. M0 if written the wrong way around even if the correct answer results. The **NM** must be seen if **N** is incorrect and their **N** must have at least two non-zero elements and cannot be **I**		**M1**
	$\begin{pmatrix}0 & -3\\ -1 & 0\end{pmatrix}$	Correct matrix. No incorrect work. Accept answer only.	**A1**
			(2)
(ii)	$\left\{\det\mathbf{Q}=\begin{vmatrix}5 & -1\\ 5 & 3\end{vmatrix}=\right\}\ 5\times3-5\times(-1)\ \{=20\}$	Correct value or numerical expression for det **Q** Could be seen in an attempt at $\mathbf{Q}^{-1}$	**B1**
	e.g., Area of H is $\dfrac{200}{"20"}$ $\{=10\}$ $\quad A_H\times"20"=200$ M1: A correct ft equation or statement linking the area of H and 200 ignoring any incorrect expression for A_H A1: Correct statement/equation but ignore any incorrect expression for A_H		**M1** **A1**
	Area of $H=6\times\dfrac{1}{2}\times x\times x\sin 60^\circ$ or e.g., $\dfrac{3\sqrt{3}}{2}x^2$ or e.g., $2\left(\frac{1}{2}x^2\sin 120^\circ\right)+\sqrt{3}\,x.x$	Any correct expression for the area of H in terms of x. Trig expressions may be unevaluated. Not dependent.	**M1**
	$\{x=\}$ awrt 1.96	Not $\pm$. Must be decimal so A0 for e.g., $\sqrt{\frac{20\sqrt{3}}{9}}$	**A1**
	Alternative via x' : B1 and 2nd M1 as before $6\times\frac{1}{2}\times x'\times x'\sin 60^\circ=200\Rightarrow\frac{3\sqrt{3}}{2}(x')^2=200\Rightarrow(x')^2=\frac{400}{3\sqrt{3}}$ or $\frac{400\sqrt{3}}{9}\Rightarrow x'=\sqrt{\frac{400}{3\sqrt{3}}}=\frac{20\sqrt{\sqrt{3}}}{3}=8.7738...$ $x'=\frac{"8.7738..."}{\sqrt{20}}(1\text{st M1 A1})=1.961887...\approx1.96\,(\text{A1})$		
			(5)
			(Total 10 marks)

Question Number	Scheme		Notes	Marks
8	$f(n) = 3^{2n+4} - 2^{2n}$ May see subtle variations e.g., replacing f(k) with "5α" Accept a proof that is done in terms of another variable including n			
Way 1 **f(k+1)=...**	$f(1) = 3^6 - 2^2 = 725$		Correct value (or 5 x 145) for $n = 1$	**B1**
Way 2 **overleaf**	Assume true for $n = k$ so that $3^{2k+4} - 2^{2k}$ is divisible by 5			
	$\{f(k+1) =\} 3^{2(k+1)+4} - 2^{2(k+1)}$		Attempts f (k + 1).	**M1**
	$\{f(k+1) =\} 3^{2k+6} - 2^{2k+2} = 9\times 3^{2k+4} - 9\times 2^{2k} + 5\times 2^{2k}$ or $4\times 3^{2k+4} - 4\times 2^{2k} + 5\times 3^{2k+4}$		Obtains a 3 term expression which includes terms of the form $\ldots\times 3^{2k+4}$ & $\ldots\times 2^{2k}$ which might be implied by introduction of f(k). 3rd term must include a function of k as a power	**dM1**
	$f(k+1) = 9f(k) + 5\times 2^{2k}$ or $f(k+1) = 4f(k) + 5\times 3^{2k+4}$ Accept e.g., $f(k+1) = 4f(k) + 405\times 3^{2k}$ or $f(k+1) = 5(9\alpha + 2^{2k})$ Correctly expresses f (k + 1) in terms of f (k) with f (k) isolated. Allow $f(k+1) = 9(3^{2k+4} - 2^{2k}) + 5\times 2^{2k}$ or $f(k+1) = 4(3^{2k+4} - 2^{2k}) + 5\times 3^{2k+4}$ f(k + 1) must be explicitly seen at some point. Allow recovery from poor bracketing.			**A1**
	If the result is **true for *n* = *k* then** it is **true for *n* = *k* + 1**. As the result has been shown to be **true for *n* = 1**, then the result is **true for** (all) ***n*** Correct conclusion or narrative. Minimum in bold. Accept surrogates for "true". Accept e.g.,"$n \in \mathbb{Z}$" for "$n \in \mathbb{Z}^+$" but not "$n \in \mathbb{R}$". If "assume true for $n = k$" is seen the "if...then" is not required. There is no requirement to say or demonstrate that e.g., 725, 405, 45 etc are divisible by 5. Requires all previous marks.			**A1**
				(Total 5 marks)

Question Number	Scheme	Notes	Marks
8 **Way 2**	$\mathrm{f}(n)=3^{2n+4}-2^{2n}$ May see subtle variations e.g., replacing f(k) with "5α" Accept a proof that is done in terms of another variable including n		
	$\mathrm{f}(1)=3^6-2^2=725$	Correct value (or 5 x 145) for $n = 1$	**B1**
f(k+1) -mf(k)=...	$\mathrm{f}(k+1)-m\mathrm{f}(k)=3^{2(k+1)+4}-2^{2(k+1)}-m\left(3^{2k+4}-2^{2k}\right)$ If $m=1:\mathrm{f}(k+1)-\mathrm{f}(k)=3^{2(k+1)+4}-2^{2(k+1)}-\left(3^{2k+4}-2^{2k}\right)$ If $m=-1:\mathrm{f}(k+1)+\mathrm{f}(k)=3^{2(k+1)+4}-2^{2(k+1)}+3^{2k+4}-2^{2k}$	Attempts f (k + 1), can be scored as Way 1 but is likely to be within an attempt at f (k + 1) – m f (k)	**M1**
Note: It is not essential to choose a value for m	$\mathrm{f}(k+1)-m\mathrm{f}(k)=(9-m)\times\left(3^{2k+4}-2^{2k}\right)+5\times2^{2k}$ or $\mathrm{f}(k+1)-m\mathrm{f}(k)=(4-m)\left(3^{2k+4}-2^{2k}\right)+5\times3^{2k+4}$ If $m=1:\mathrm{f}(k+1)-\mathrm{f}(k)=8\times\left(3^{2k+4}-2^{2k}\right)+5\times2^{2k}$ or $3\left(3^{2k+4}-2^{2k}\right)+5\times3^{2k+4}$ If $m=-1:\mathrm{f}(k+1)+\mathrm{f}(k)=10\times\left(3^{2k+4}-2^{2k}\right)+5\times2^{2k}$ or $5\left(3^{2k+4}-2^{2k}\right)+5\times3^{2k+4}$ If $m=4:\mathrm{f}(k+1)-4\mathrm{f}(k)=5\times3^{2k+4}$ or $5\times\left(3^{2k+4}-2^{2k}\right)+5\times2^{2k}$ If $m=9:\mathrm{f}(k+1)-9\mathrm{f}(k)=5\times2^{2k}$ or $-5\times\left(3^{2k+4}-2^{2k}\right)+5\times3^{2k+4}$ Obtains a 3 term expression (**but see note below**) which includes terms of the form $\ldots\times3^{2k+4}$ or $\ldots\times2^{2k}$ but allow only seeing 1 of these if 4 or 9 is used for m and allow e.g., $5\times3^{2k+4}$ as 405×3^{2k}. 3rd term must include a function of k as a power. **Note** that it is possible that the expression on the RHS could be written as a multiple of 5 without involving f(k) e.g., $\mathrm{f}(k+1)+\mathrm{f}(k)=3^{2k+6}-2^{2k+2}+3^{2k+4}-2^{2k}=9\times3^{2k+4}-4\times2^{2k}+3^{2k+4}-2^{2k}=10\times3^{2k+4}-5\times2^{2k}$ In such cases the work must be correct.		**dM1**
	$\mathrm{f}(k+1)=9\mathrm{f}(k)+5\times2^{2k}$ or $\mathrm{f}(k+1)=4\mathrm{f}(k)+5\times3^{2k+4}$ Accept e.g., $\mathrm{f}(k+1)=4\mathrm{f}(k)+45\times3^{2k+2}$ Correctly expresses f (k + 1) in terms of f (k) with f (k) isolated. Must make f (k + 1) the subject unless there is mention of e.g., "RHS is divisible by 5 so $\Rightarrow$ f (k + 1) – m f (k) is divisible by 5" oe. Allow $\mathrm{f}(k+1)=9\left(3^{2k+4}-2^{2k}\right)+5\times2^{2k}$ or $\mathrm{f}(k+1)=4\left(3^{2k+4}-2^{2k}\right)+5\times3^{2k+4}$ $\mathrm{f}(k+1)$ must be explicitly seen at some point. Allow recovery from poor bracketing.		**A1**
	If the result is **true for $n = k$ then** it is **true for $n = k + 1$**. As the result has been shown to be **true for $n = 1$**, then the result is **true for** (all) n Correct conclusion or narrative. Minimum in bold. Accept surrogates for "true". Accept e.g.,"$n\in\mathbb{Z}$" for "$n\in\mathbb{Z}^+$" but not "$n\in\mathbb{R}$". If "assume true for $n = k$" is seen the "if...then" is not required. There is no requirement to say or demonstrate that e.g., 725, 405, 45 etc are divisible by 5. Requires all previous marks.		**A1**
			(Total 5 marks)

Question Number	Scheme	Notes	Marks
9(a)	$xy=c^2 \Rightarrow y=c^2x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-c^2x^{-2}$ or $xy=c^2 \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{y}{x}$ or $x=cp, y=\frac{c}{p} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-cp^{-2}}{c}$ Any correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$		B1
	$m_T=-\frac{c^2}{c^2p^2} \Rightarrow m_N=p^2$ or $m_T=-\frac{c/p}{cp} \Rightarrow m_N=p^2$ or $m_T=-p^{-2} \Rightarrow m_N=p^2$ Correct use of the perpendicular gradient rule and the point P to obtain a normal gradient in terms of p (and possibly c)		M1
	$y-\frac{c}{p}=p^2(x-cp)$ or $y=p^2x+k,\ \frac{c}{p}=p^2\times cp+k \Rightarrow k=...\left\{\frac{c}{p}-cp^3\right\}$ Forms the equation of the normal correctly for their changed gradient in terms of p (and possibly c). Only reaching $k=$ ... requires requires sight of $y=$ "p^2"$x+k$		M1
	$py-c=p^3x-cp^4$ or $y=p^2x+\frac{c}{p}-cp^3$ $\Rightarrow p^3x-py=c(p^4-1)$*	Obtains the given answer with an intermediate step and no errors. Last line: p^3x and $-py$ on one side, $c(p^4-1)$ on other	A1*
			(4)
(b)	Treat the M and dM marks like a B mark and a dB mark - so correct work is required. The first mark is for a significant correct first step and the second is for reaching an equation from which the given result can clearly be easily obtained. **Use this guidance for any potentially creditworthy alternative approaches but use Review if you are in doubt.**		
Way 1 **grad PQ**	gradient $PQ=\frac{\frac{c}{p}-\frac{c}{q}}{cp-cq}$	Correct expression for the gradient of the chord PQ	M1
	$\frac{\frac{c}{p}-\frac{c}{q}}{cp-cq}=p^2 \Rightarrow \frac{\frac{q-p}{pq}}{p-q}=p^2$ or $\frac{c(q-p)}{\frac{pq}{c(p-q)}}=p^2$	Equates the 2 gradients (may repeat the work in (a)) and correctly makes the numerator a single fraction or otherwise manipulates correctly to reach an equation from which the given result can clearly be easily obtained - all correct work	dM1
	$\Rightarrow p^3q=-1$* Allow e.g., $-1=qp^3$	Fully correct proof. Can proceed directly to the given answer from the expressions shown for the **d**M1	A1* (3)
Way 2 **Sub.** ***Q* into normal**	Substituting Q into the normal: $Q\left(cq, \frac{c}{q}\right)$ in $p^3x-py=c(p^4-1) \Rightarrow cp^3q-\frac{cp}{q}=c(p^4-1)$ (M1: correct equation) e.g., $p^3q+1-\frac{p}{q}-p^4=p^3q+1-\frac{p}{q}(1+p^3q)=(p^3q+1)\left(1-\frac{p}{q}\right)=0$ (**d**M1) $p^3q=-1$(A1*)		
Way 3 **Sub.** $xy=c^2$ **into normal**	$y=\frac{c^2}{x}$ in $p^3x-py=c(p^4-1) \Rightarrow p^3x-\frac{pc^2}{x}=c(p^4-1)$ (M1: correct equation) $p^3x^2-c(p^4-1)x-pc^2=(p^3x+c)(x-pc)=0 \Rightarrow x=-\frac{c}{p^3}=cq$ (dM1) $p^3q=-1$* (A1*) OR $x=\frac{c^2}{y}$ in $p^3x-py=c(p^4-1) \Rightarrow \frac{p^3c^2}{y}-py=c(p^4-1)$ (M1: correct equation) $py^2+cy(p^4-1)-p^3c^2=(py+cp^4)\left(y-\frac{c}{p}\right)=0 \Rightarrow y=-cp^3=\frac{c}{q}$ (dM1) $p^3q=-1$* (A1*)		

Question Number	Scheme	Notes	Marks
9(c)	$q^3x - qy = c(q^4-1)$ and $xy = c^2 \Rightarrow$ $q^3x - q\dfrac{c^2}{x} = c(q^4-1)$ or $q^3\dfrac{c^2}{y} - qy = c(q^4-1)$ $\Rightarrow q^3x^2 - c(q^4-1)x - c^2q = 0$ or $qy^2 + c(q^4-1)y - q^3c^2 = 0$ Or with $q = -\dfrac{1}{p^3} \Rightarrow -\dfrac{1}{p^9}x + \dfrac{1}{p^3}.\dfrac{c^2}{x} = c\left(\dfrac{1}{p^{12}}-1\right) \Rightarrow \dfrac{1}{p^9}x^2 + cx\left(\dfrac{1}{p^{12}}-1\right) - \dfrac{c^2}{p^3} = 0$ or $-\dfrac{c^2}{p^9y} + \dfrac{y}{p^3} = c\left(\dfrac{1}{p^{12}}-1\right) \Rightarrow \dfrac{y^2}{p^3} - cy\left(\dfrac{1}{p^{12}}-1\right) - \dfrac{c^2}{p^9} = 0$ Substitutes the equation of the hyperbola into the equation of the normal (which they may obtain from scratch) to obtain a 3 term quadratic equation in x or y Could be uncollected terms in x (or y) and allow slips if the intention is clear		**M1**
	$\Rightarrow (x-cq)(q^3x+c) = 0 \Rightarrow x = -\dfrac{c}{q^3}$ or $\Rightarrow (qy-c)(y+cq^3) = 0 \Rightarrow y = -cq^3$ or $p^3x^2 + cx(1-p^{12}) - c^2p^9 = (p^3x+c)(x-cp^9) = 0 \Rightarrow x = cp^9$ $p^9y^2 - cy(1-p^{12}) - p^3c^2 = (p^9y-c)(y+cp^3) = 0 \Rightarrow y = \frac{c}{p^9}$ Completes a credible attempt to find the x or y coordinate of R (not Q) Note that this may result from work in (b) but this must be restated or used in (c)		**dM1**
	R is $\left(-\dfrac{c}{q^3}, -cq^3\right)$ or $\left(cp^9, \dfrac{c}{p^9}\right)$	Correct coordinates of R May be seen as $x = \ldots,\ y = \ldots$	**A1**
	Gradient PR is $\dfrac{\frac{c}{p}+cq^3}{cp+\frac{c}{q^3}}$ or $\dfrac{\frac{c}{p}-\frac{c}{p^9}}{cp-cp^9}$	Forms a **correct** gradient of PR	**ddM1**
	gradient $PR = \dfrac{\frac{c}{p}+cq^3}{cp+\frac{c}{q^3}} = \dfrac{\frac{1+pq^3}{p}}{\frac{pq^3+1}{q^3}} = \dfrac{q^3}{p}$ and $q = -\dfrac{1}{p^3} \Rightarrow -\dfrac{1}{p^9}\times\dfrac{1}{p} = -\dfrac{1}{p^{10}}$ or gradient $PR = \dfrac{\frac{c}{p}-\frac{c}{p^9}}{cp-cp^9} = \dfrac{\frac{c(p^8-1)}{p^9}}{cp(1-p^8)} = -\dfrac{1}{p^{10}}$ Correct answer. Accept $k = 10$ from appropriate work		**A1**
			(5)
			(Total 12 marks)
Alt	Let R have coordinates $\left(cr, \dfrac{c}{r}\right)$ and $q^3r = -1$	Introduces a 3rd parameter for R e.g. r and states/uses $q^3\,"r" = -1$ somewhere	**M1**
	Gradient PR is $\dfrac{\frac{c}{p}-\frac{c}{r}}{cp-cr} = -\dfrac{1}{pr}$	**dM1**: Correctly forms the gradient of PR A1: Correct simplified gradient in terms of p and "r"	**dM1** **A1**
	$r = -\frac{1}{q^3} \Rightarrow$ gradient $= \frac{q^3}{p}$	Eliminates "r"	**ddM1**

	and $q=-\frac{1}{p^3} \Rightarrow$ gradient $PR=-\frac{1}{p^9}\times\frac{1}{p}=-\frac{1}{p^{10}}$	Correct answer	**A1**