Question Number	Scheme	Notes	Marks
1	$\mathbf{M}=\begin{pmatrix}1 & a\\ 3 & -5\end{pmatrix} \qquad \mathbf{M}^{-1}=2\mathbf{M}+8\mathbf{I}$ Condone any brackets (or missing brackets) for matrices throughout. Allow **clear** miscopying slips (e.g., $-5-3a$ becomes $-3-5a$) to access M marks.		
(a)	$\{\det\mathbf{M}=\}-5-3a$	Correct determinant. Allow unsimplified	B1
	$\{\mathbf{M}^{-1}=\}\dfrac{1}{-5-3a}\begin{pmatrix}-5 & -a\\ -3 & 1\end{pmatrix}$ or $\begin{pmatrix}\dfrac{-5}{-5-3a} & \dfrac{-a}{-5-3a}\\ \dfrac{-3}{-5-3a} & \dfrac{1}{-5-3a}\end{pmatrix}$ May see e.g., $\dfrac{-1}{5+3a}\begin{pmatrix}-5 & -a\\ -3 & 1\end{pmatrix}$, $\dfrac{1}{5+3a}\begin{pmatrix}5 & a\\ 3 & -1\end{pmatrix}$	M1: For $\dfrac{1}{\pm5\pm3a}\times$ a **changed** matrix or a correct Adj(**M**) seen i.e., $\ldots\begin{pmatrix}-5 & -a\\ -3 & 1\end{pmatrix}$ A1: Any correct inverse	M1 A1
			(3)
(b)	$\left\{\dfrac{1}{-5-3a}\begin{pmatrix}-5 & -a\\ -3 & 1\end{pmatrix}=\right\}2\begin{pmatrix}1 & a\\ 3 & -5\end{pmatrix}+8\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}2 & 2a\\ 6 & -10\end{pmatrix}+\begin{pmatrix}8 & 0\\ 0 & 8\end{pmatrix}=\begin{pmatrix}10 & 2a\\ 6 & -2\end{pmatrix}$ Substitutes **M** into the RHS of the equation and combines matrices appropriately obtaining one correct element which may be unsimplified e.g., 2 + 8. Apply BOD if only see e.g., $-5=2(-5-3a)+8$ but next mark is M0. Allow equivalent work if writes as e.g, $\mathbf{M}^{-1}-2\mathbf{M}=+8\mathbf{I}$ so $\dfrac{-5}{-5-3a}-2=8$ implies the 2 + 8. Condone use of $\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$ for **I**. If $\mathbf{I}=\mathbf{0}$ then M0. Note that it is fine to just use 1 element and proceed directly to an equation.		M1
	e.g., $\Rightarrow -5=10(-5-3a)$ or $-5=-50-30a$ or $5=10(5+3a)\Rightarrow a=\ldots \quad \left\{-\tfrac{3}{2}\right\}$ Obtains a non-zero value for a from a consistent "extracted" equation (with no fraction). If the fraction is not dealt with (going straight to a value for a) it must be consistent. Equation must come from $\dfrac{1}{\pm5\pm3a}\times$ a changed matrix = credible attempt at $2\mathbf{M}+8\mathbf{I}$ M0 for equating to a zero element of their $2\mathbf{M}+8\mathbf{I}$. Accept a value following an appropriate equation. If no equation must correctly get $-\tfrac{3}{2}$ oe. Ignore usual rules if equation is quadratic. If RHS is only seen multiplied out it must be consistent with the work. Condone use of $\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$. If $\mathbf{I}=\mathbf{0}$ then M0. May see: $-a=2a(-5-3a),\ -3=6(-5-3a),\ 1=-2(-5-3a)\Rightarrow a=\ldots$ $a=2a(5+3a),\ 3=6(5+3a),\ -1=-2(5+3a)\Rightarrow a=\ldots$		M1
	A1: $-\dfrac{3}{2}$ or $-1\dfrac{1}{2}$ or -1.5 and allow equivalent fractions e.g., $-\dfrac{45}{30}$ Requires both previous marks and no sight of incorrect **I.** Must come from correct work **for the element used**. Ignore any extra equations or checks if they are wrong but unused. But do not isw if any extra incorrect unrejected solutions are offered e.g., a = 0 from $6a^2+9a=0$		A1
			(3)
	Some alternatives for (b) are shown overleaf		**Total 6**

1(b)	$\mathbf{M}=\begin{pmatrix}1 & a\\ 3 & -5\end{pmatrix} \qquad \mathbf{M}^{-1}=2\mathbf{M}+8\mathbf{I}$	
Alt 1	$\mathbf{M}^{-1}=2\mathbf{M}+8\mathbf{I}\Rightarrow\mathbf{I}=2\mathbf{M}^2+8\mathbf{M}\Rightarrow\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}\underline{6a+10} & 0\\ 0 & 6a+10\end{pmatrix}\Rightarrow 6a+10=1\Rightarrow a=-\frac{3}{2}$ M1: Achieves $\underline{pa+q}$ M1: Solves $pa+q=1$ A1: Correct value (no others or incorrect **I**)	
Alt 2	$\det\mathbf{M}^{-1}=\det(2\mathbf{M}+8\mathbf{I})\Rightarrow\frac{1}{-5-3a}=\underline{-20-12a}\Rightarrow 36a^2+120a+99=12a^2+40a+33=(2a+3)(6a+11)=0\Rightarrow a=-\frac{3}{2}$ M1: Achieves $\underline{pa+q}$ M1: Solves $\frac{1}{\pm5\pm3a}=pa+q$ A1: Correct value and no others	
Others	Determinants may also be used to form the equation in Alt 1 i.e., $1=(6a+10)^2$ Another (unlikely) possibility is to equate the traces of the matrices: $\frac{1}{-5-3a}(-5+1)=10+(-2)\Rightarrow -4=-40-24a\Rightarrow a=-\frac{3}{2}$	

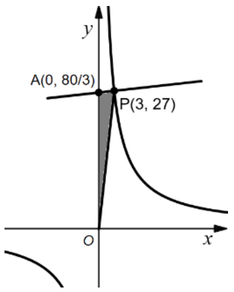
Question Number	Scheme	Notes	Marks
2	$\mathbf{P}=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$ $\mathbf{Q}=\begin{pmatrix}5 & 0\\ 0 & 5\end{pmatrix}$ Condone any brackets (or missing brackets) for matrices throughout		
(a)	Rotation...	Identifies the transformation as a rotation. Allow any reasonable attempt at this word e.g., "rotate". M0 for a combination of transformations or if any different types of transformation are given as alternatives. Note that giving an angle does not imply "rotation".	M1
	...(anti-clockwise) of $\frac{3\pi}{2}$ $(270°)$ or **clockwise** of $\frac{\pi}{2}$ $(90°)$ about/around/from (etc.) O Correct full description for the rotation, including angle & direction (if no direction is stated then assume anticlockwise is meant) and **any** mention of origin or O or (0, 0). Condone "original" for "origin". Accept in degrees (symbol not required) or exact radians. {Note also that $-\frac{\pi}{2}$ $(-90°)$ or $-\frac{3\pi}{2}$ $(-270°)$ **clockwise** are acceptable}		A1
			(2)
(b)	Enlargement...	Identifies the transformation as an enlargement. Allow any reasonable attempt at this word e.g., "inlarge", "large". **Nothing else** so e.g. "stretch" is M0. M0 for a combination of transformations or if any different types of transformation are given as alternatives.	M1
	...of scale factor/factor/scale/size 5, centre/from (etc.) O	Correct full description for the enlargement including scale factor and centre. Allow **any** mention of "5" and **any** mention of origin, O or (0, 0) Condone "original" for "origin".	A1
			(2)
(c)	$\{\mathbf{R}=\}\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}$	Correct matrix	B1
			(1)
(d)	$\left\{\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}\begin{pmatrix}4\\ 3\end{pmatrix}=\right\}$ $(-3, -4)$	Correct coordinates for A. Brackets may be missing. May be stated as $x = \ldots, y = \ldots$ Condone answer given as a vector. Isw if necessary. Score B0 if an incorrect **R** has clearly been used in this part. Allow if correct answer comes from multiplying the wrong way round. No ft.	B1
			(1)

Question Number	Scheme	Notes	Marks
2(e)	$\mathbf{P}=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$ $\quad \mathbf{Q}=\begin{pmatrix}5 & 0\\ 0 & 5\end{pmatrix}$		
	For either mark there must be no clear evidence of use of an incorrect method or matrix in this part e.g., using their matrix **R** for the reflection in part (c), i.e, sight of their $\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}$ even if this is labelled as **P**. **Do not allow confusion between their $A(-3,-4)$ and the given point $(4, 3)$ and there are no marks if a matrix or point has been miscopied/misread even if it is restated in this part. This also applies if there is an incorrect A in (d) but it is miscopied into the correct $(-3,-4)$.** Both marks can be scored from minimal or no working.		
	$\left(\frac{4}{5}, ...\right)$ or $(0.8, ...)$ or $\left(..., -\frac{3}{5}\right)$ or $(..., -0.6)$ or $\left(-\frac{"-4"[y]}{5}, \frac{"-3"[x]}{5}\right)$ For **one** correct coordinate or ft non-zero coordinate on their $A(-3,-4)$ but must not use $(4, 3)$ even if that is their answer for A from part (d). Examples of follow throughs are: $(-4,-3)\to\left(\frac{3}{5},-\frac{4}{5}\right)$ $\quad (-4, 3)\to\left(-\frac{3}{5},-\frac{4}{5}\right)$ $\quad (4,-3)\to\left(\frac{3}{5},\frac{4}{5}\right)$ $(-3, 4)\to\left(-\frac{4}{5},-\frac{3}{5}\right)$ $\quad (3,-4)\to\left(\frac{4}{5},\frac{3}{5}\right)$ $\quad (3, 4)\to\left(-\frac{4}{5},\frac{3}{5}\right)$ May be stated as $x = \ldots$ / $y = \ldots$ and condone if given without brackets or as a vector but do not condone a correct value seen within e.g., a 2 x 2 matrix		M1
	$\{B:\}\left(\frac{4}{5}, -\frac{3}{5}\right)$ or $(0.8, -0.6)$ only Both correct coordinates. Not ft. May be stated as $x = \ldots, y = \ldots$ and condone if given without brackets or as a vector. Isw if necessary.		A1
	Examples of working: Do not allow confusion between their $A(-3,-4)$ and the given point $(4, 3)$ Using matrices (Note that the matrices can be applied in either order with these two transformations): $\begin{pmatrix}5 & 0\\ 0 & 5\end{pmatrix}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0 & 5\\ -5 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}-3\\ -4\end{pmatrix}$ or $\begin{pmatrix}5 & 0\\ 0 & 5\end{pmatrix}\begin{pmatrix}y\\ -x\end{pmatrix}=\begin{pmatrix}-3\\ -4\end{pmatrix}\Rightarrow 5y=-3,\ -5x=-4\Rightarrow\left(\frac{4}{5}, -\frac{3}{5}\right)$ $\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}5 & 0\\ 0 & 5\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0 & 5\\ -5 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}-3\\ -4\end{pmatrix}$ or $\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}5x\\ 5y\end{pmatrix}=\begin{pmatrix}-3\\ -4\end{pmatrix}\Rightarrow 5y=-3,\ -5x=-4\Rightarrow\left(\frac{4}{5}, -\frac{3}{5}\right)$ Using inverse: $\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0 & 5\\ -5 & 0\end{pmatrix}^{-1}\begin{pmatrix}-3\\ -4\end{pmatrix}=\frac{1}{25}\begin{pmatrix}0 & -5\\ 5 & 0\end{pmatrix}\begin{pmatrix}-3\\ -4\end{pmatrix}$ or $\begin{pmatrix}0 & -\frac{1}{5}\\ \frac{1}{5} & 0\end{pmatrix}\begin{pmatrix}-3\\ -4\end{pmatrix}\Rightarrow\left(\frac{4}{5}, -\frac{3}{5}\right)$ Using the actual transformations: $(-3,-4)\Rightarrow\left(-\frac{3}{5}, -\frac{4}{5}\right)\Rightarrow\left(\frac{4}{5}, -\frac{3}{5}\right)$ or $(-3,-4)\Rightarrow(4,-3)\Rightarrow\left(\frac{4}{5}, -\frac{3}{5}\right)$		
			(2)
			Total 8

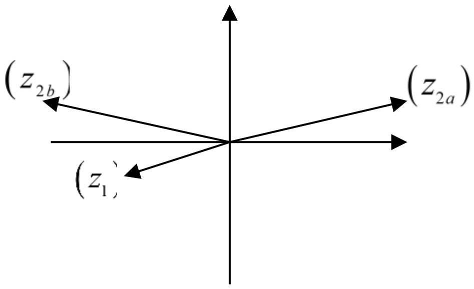
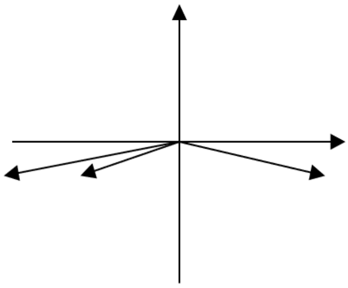
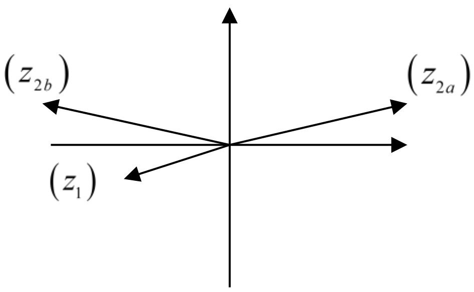
Question Number	Scheme	Notes	Marks
3(i)	There is no credit if only signs instead of values are used		
	$\mathrm{f}(x)=x^2+5-8^{5x}$ $\mathrm{f}(0)=4,\ \mathrm{f}(0.5)=-175.769...,\ [\mathrm{f}(1)=-32762]$	Obtains a **value** for f(0.5) and at least one of f(0) and f(1) with at least one correct: $\mathrm{f}(0)=4,\ \mathrm{f}(0.5)=\text{awrt}-180,\ [\mathrm{f}(1)=\text{awrt}\ -33000]$	M1
	{f(0) > 0, f(0.5) < 0 so root in [0, 0.5]} $\Rightarrow \mathrm{f}(0.25)=...$ Having obtained different signs of values for f(0.5) and f(0), obtains a **value** for f(0.25). If by error the sign change occurs with f(0.5) and f(1), allow for attempting f(0.75) $\{=-2429.93...\approx-2400\}$. Accept using a full list/table of values e.g., $\mathrm{f}(0)=4,\ \mathrm{f}(0.25)=-8.39184...,\ \mathrm{f}(0.5)=-175.769...,\ \mathrm{f}(0.75)=-2429.93...,\mathrm{f}(1)=-32762$ $\mathrm{f}(0)=4,\ \mathrm{f}(0.25)=\text{awrt}-8.4\ \ \mathrm{f}(0.5)=\text{awrt}-180,\ \mathrm{f}(0.75)=\text{awrt}-2400,\mathrm{f}(1)=\text{awrt}-33000$ Allow the values to be calculated in any order. **Previous mark required.**		**d**M1
	$\mathrm{f}(0.25)=-8.39184...\ \Rightarrow\ (\alpha\in)\ [0,\ 0.25]$ f(0.25) = awrt -8.4 **and all** other values used correct to 2 sf (ignore values from further bisections e.g., f(0.125), f(0.375), f(0.625), f(0.875)) **and** concludes required interval is [0, 0.25]. Accept e.g., (0, 0.25). Condone e.g., $0\leqslant x\leqslant 0.25,\ 0<"\beta"<0.25$ but do not allow e.g., (0.25, 0). Must give an interval so e.g., "It's between 0 & 0.25" is A0. Ignore further bisections provided correct interval has been seen. Allow 2 sf truncated values.		A1
			(3)

Question Number	Scheme	Notes	Marks
3(ii)(a)	$g(x)=3^{\sin x}-3\cos x$ $g(4)=2.396...,\ \ g(5)=-0.502...$	Attempts both g(4) and g(5) with at least one correct (using radians): $g(4)=$ awrt 2.4 (or 2.3 truncated), $g(5)=$ awrt -0.5	M1
	Sign change oe and g (x) is **continuous** therefore a **root oe** e.g., β is between x = 4 and x = 5	Both g(4) = awrt 2.4 (or 2.3 truncated) and g(5) = awrt −0.5, sign change (**accept equivalents** e.g., g(4) > 0 & g(5) < 0 or g(4)g(5) <0 or "positive, negative"), continuous and conclusion all given. Be generous with attempt at "continuous" and condone e.g., "x is continuous". Minimum in bold. Condone a wrong interval following e.g., "There is a solution in..." May use f for g.	A1
			(2)
(ii)(b)	E.g. $\dfrac{\beta-4}{"2.396..."}=\dfrac{5-\beta}{-"-0.502..."}\Rightarrow\beta=...$ $\dfrac{5-\beta}{\beta-4}=\dfrac{-"-0.502..."}{"2.396..."}\Rightarrow\beta=...$ $\dfrac{\beta-4}{5-4}=\dfrac{"2.396..."}{"2.396"-"-0.502"}\Rightarrow\beta=...$ $\dfrac{4("-0.502...")-5("2.396...")}{"-0.502..."-"2.396..."}=...$ or $m=\dfrac{"-0.502"-"2.396"}{5-4}\{=-2.8986...\}$ $\Rightarrow$ e.g., $y-"2.396..."=m(x-4),\ y=0$ $\Rightarrow x$ or $\beta=...$ If only unsubstituted "$\dfrac{ag(b)-bg(a)}{g(b)-g(a)}$" oe is seen followed by value it must round to 4.83 unless $a,\ b,\ g(a)\ \&\ g(b)$ are identified. May use f for g.	Uses a correct interpolation equation/expression for their values (allow for any value <u>provided g(4) positive, g(5) negative</u> and condone clear miscopying) **and** finds a value for β or x etc. Accept any correct statement for their values followed by any attempt to solve. If not fully substituted, values for g(4) & g(5) may be seen separately here or in part (a). May use f for g. Alternatively finds the gradient and then the line joining the endpoints and sets y = 0 to get β. Straight line equation must be correct for their values but allow a correct unsimplified gradient seen which is miscalculated later and allow errors finding c from a consistent equation with a correctly substituted point. Ignore how the value is labelled (may be unlabelled or x or α used). If their variable denotes e.g., the distance between (4, 0) and $(\beta,\ 0)$ then 4 must be added later. Implied by awrt 4.83 **provided no evidence of incorrect equation** but must not clearly be using the actual root of 4.8245... from solving by calculator. Note that failure to change the sign of g(5) leads to 5.265...	M1
	= 4.827 (4 s.f.)	awrt 4.827 Accept answer only. Ignore further interpolations.	A1
			(2)
			Total 7

Question Number	Scheme	Notes	Marks
4(a)	For the first two marks if any **ambiguous** fractions within fractions are seen e.g., $\frac{\frac{-9}{t^2}}{9}$ then marks must be confirmed by appropriate later processing.		
	$xy=81,\quad P\left(9t,\frac{9}{t}\right)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{81}{x^2}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{-9}{t^2}}{9}$ or $x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0\left\{\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{y}{x}\right\}$ Any correct equation involving $\frac{\mathrm{d}y}{\mathrm{d}x}\left(\text{or }\frac{\mathrm{d}x}{\mathrm{d}y}\right)$. Accept just $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $m_T=-\frac{1}{t^2}$. May see e.g. $x=\frac{81}{y}\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=-\frac{81}{y^2}$ or $-\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{81}{y^2}$		B1
	e.g., $-\frac{81}{x^2}\to\frac{x^2}{81},\quad -\frac{81}{(9t)^2}\to\frac{(9t)^2}{81},\quad \frac{-\frac{9}{t^2}}{9}\to\frac{9}{\frac{9}{t^2}},\quad -\frac{y}{x}\to\frac{x}{y},\quad -\frac{1}{t^2}\to-\frac{1}{-\frac{1}{t^2}}\quad\{\Rightarrow t^2\}$ Applies correct perpendicular gradient rule to obtain a gradient for the normal. Allow in terms of t, x, y, or x and y. $\frac{\mathrm{d}y}{\mathrm{d}x}$ may be incorrect - just look for clear use of $-\frac{1}{m_T}$. Could just change the sign of $\frac{\mathrm{d}x}{\mathrm{d}y}$. If starts with just $m_N=t^2$ then 0110 max		M1
	$y-\frac{9}{t}=t^2(x-9t)$ or $\frac{9}{t}=t^2\times 9t+c\Rightarrow c=\ldots\left\{\frac{9}{t}-9t^3\right\}$	Correct straight line method with any **changed** gradient **in terms of *t***. "Changed" gradient may just be the negative or reciprocal instead of negative reciprocal. Condone (for all marks) late substitution if gradient not initially in terms of t (but no "x"s or "y"s can be combined before this substitution)	M1
	$\Rightarrow ty-9=t^3x-9t^4$ or $y=t^2x+\frac{9}{t}-9t^3$ $\Rightarrow ty=t^3x+9(1-t^4)$ *	Correct equation reached with an intermediate step and no errors seen. Allow recovery of poor bracketing provided it is before the final answer. Allow e.g., $yt=9(1-t^4)+xt^3$ (yt must be on its own on one side, +9 must have been factorised out)	A1*
			(4)
(b)	$\{x=0\Rightarrow\}\,ty=9(1-t^4)\Rightarrow y=\ldots$	Sets $x=0$ in **given** equation of normal and obtains an expression in t for y (which may be incorrect)	M1
	$A\left(0,\ \frac{9}{t}\times(1-t^4)\right)$ or oe e.g., $\left(0,\ \frac{9}{t}-9t^3\right),\left(0,\ \frac{9-9t^4}{t}\right),\left(0,\ 9\left(\frac{1}{t}-t^3\right)\right)$	Correct answer - coordinates or $x=0,\ y=\ldots$ stated separately. Brackets may be missing. The $x=0$ must be seen at some point in this part. Isw once a correct y is seen.	A1
			(2)

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td rowspan="4">4(c)</td><td>
$A:\left(0,\ \frac{9}{\frac{1}{3}}\left(1-\frac{1}{3^4}\right)\right)$ or $\left(0,\ \frac{80}{3}\right)$
$\Rightarrow \{\text{Area } OPA =\}$
$\frac{1}{2}\times 9\times\frac{1}{3}\times "\frac{9}{\frac{1}{3}}\left(1-\frac{1}{3^4}\right)"$ or $\frac{1}{2}\times 9\times\frac{1}{3}\times "\frac{80}{3}"$
or $\frac{1}{2}\times 3\times "\frac{80}{3}"$
May see alternatives e.g.,
$27\times 3-\frac{1}{2}\times\left(27-"\frac{80}{3}"\right)-\frac{1}{2}\times 3\times 27$</td><td>Uses $t=\frac{1}{3}$ to obtain a positive value/expression for the y coordinate of A using their result from (b) provided it is a function of t and obtains a consistent exact numerical expression or value for the area of OPA. If there is no work calculating y_A then the value must be correct and exact. Note that subsequent work could recover an exact expression. The coordinate/coordinates of P that are used must be correct. Do not allow if any length is negative.
May use "shoelace" algorithm - award once multiplications are set up e.g., $\frac{1}{2}\left|3\times"\frac{80}{3}"-0\right|$. If modulus is missing and expression within is negative, it must be corrected to positive.
M0 if the x-axis intercept (-240) of the normal is used.
Allow if correct method for area in terms of t followed by substitution to obtain a numerical expression or value e.g.,
$\left[\frac{1}{2}\times 9t\times"\frac{9}{t}(1-t^4)"\right]_{t=\frac{1}{3}} \Rightarrow ...$</td><td>M1</td></tr>
<tr><td>= 40</td><td>40 only. No equivalents.
Allow if e.g., the y-coordinate of P is incorrect but not used</td><td>A1</td></tr>
<tr><td colspan="2">If other alternatives for finding the area of the triangle are used work must be exact (or lead to exactly 40 - exactness can be recovered) to score any marks</td><td></td></tr>
<tr><td colspan="3" align="right">(2)</td></tr>
<tr><td></td><td colspan="2"></td><td>Total 8</td></tr>
</table>

Question Number	Scheme	Notes	Marks
5	$z_1 = r\left(\cos\frac{7\pi}{6} + \mathrm{i}\sin\frac{7\pi}{6}\right)$, $\lvert z_1 z_2\rvert = 15$ $\lvert z_2\rvert = 5$ $\left\{z_1 = -\frac{3\sqrt{3}}{2} - \frac{3}{2}\mathrm{i} = 3\mathrm{e}^{\frac{7\pi}{6}\mathrm{i}}\right\}$		
(a)	$r = 3$ Allow $z_1 = 3\left(\cos\frac{7\pi}{6} + \mathrm{i}\sin\frac{7\pi}{6}\right)$	Correct value. No others and not ± 3. Accept just "3" but $\lvert z_1\rvert = 3$ is insufficient unless later work implies $r = 3$	B1
			(1)
(b)	$z_2 = a + b\mathrm{i},\ z_1 + z_2 = c + 0\mathrm{i} \Rightarrow r\sin\frac{7\pi}{6} + b = 0$ $\Rightarrow b = -3\sin\frac{7\pi}{6}$ $\left\{= -3\times -\frac{1}{2} = \frac{3}{2}\right\}$	Obtains $"b" = \pm"3"\sin\frac{7\pi}{6}$ or $\pm"3"\times\pm\frac{1}{2}$ with their real "3" from (a) which may have been negative or $\pm$. The $"b"$ may be implied by e.g., $z_2 = x + \frac{3}{2}\mathrm{i}$ or later work	M1
	$a^2 + b^2 = 25$ or e.g., $\sqrt{a^2 + \left("\tfrac{3}{2}"\right)^2} = 5$ Uses the modulus of z_2 to form a correct equation linking real and imaginary parts. Allow even if equation has no real solution. **See SC below if** $r = 5$ Using $\lvert z_1 z_2\rvert = 15$ leads to $\sqrt{\left(-\tfrac{3\sqrt{3}}{2}a + \tfrac{3}{2}b\right)^2 + \left(-\tfrac{3}{2}a - \tfrac{3\sqrt{3}}{2}b\right)^2} = 15$ oe Must see a correct equation ft their *r* and value for *b* if necessary.		M1
	$a = \sqrt{25 - \left("\frac{3}{2}"\right)^2}$ $\left\{= \ldots\ \pm\frac{\sqrt{91}}{2}\right\}$ Allow $a = (\pm)\sqrt{25 - \left("3"\sin\tfrac{7\pi}{6}\right)^2}$	Substitutes *b* into a correct equation and finds at least one value or expression for *a*. Pythagoras must be used correctly and expression must be real.	M1
	SC: If $r = 5$ in (a) we will allow access to the M marks (and the M only in (c)): $"b" = \pm 5\sin\frac{7\pi}{6}$ or $\pm 5\times\pm\frac{1}{2} \Rightarrow a^2 + b^2 = 9$ or e.g., $\sqrt{a^2 + \left("\tfrac{5}{2}"\right)^2} = 3 \Rightarrow a = \sqrt{9 - \left("\frac{5}{2}"\right)^2}$ $\left\{= \ldots\ \pm\frac{\sqrt{11}}{2}\right\}$		
	$\{z_{2a} =\}\ \frac{\sqrt{91}}{2} + \frac{3}{2}\mathrm{i},\ \ \{z_{2b} =\}\ -\frac{\sqrt{91}}{2} + \frac{3}{2}\mathrm{i}$ or exact equivalents e.g., $\pm\sqrt{\frac{91}{4}} + \frac{3}{2}\mathrm{i}$		
	A1: One correct answer: $(\pm\text{awrt } 4.8) + 1.5\mathrm{i}$ and allow $-3\sin\frac{7\pi}{6}$ for 1.5 (but final A0 and *a* must be a **value** for any marks)		A1
	A1: Both correct **exact** answers. Accept e.g., $\frac{\pm\sqrt{91} + 3\mathrm{i}}{2}$ Allow both marks for e.g., $a = \pm\frac{\sqrt{91}}{2},\ b = \frac{3}{2}$ (if *a* + *b*i not seen there should be no subsequent evidence of *a* or *b* wrongly defined) or e.g., $z_2 = x + \frac{3}{2}\mathrm{i}$ seen and $x = \pm\frac{\sqrt{91}}{2}$ No additional answers. Ignore any labelling of the answers. Isw if necessary.		A1
			(5)
	A possible variation is: $z_1 = 3\left(\cos\frac{7\pi}{6} + \mathrm{i}\sin\frac{7\pi}{6}\right),\ z_2 = 5(\cos\theta + \mathrm{i}\sin\theta),\ z_1 + z_2 = c + 0\mathrm{i} \Rightarrow 3\sin\frac{7\pi}{6} + 5\sin\theta = 0$ (M1) $\Rightarrow \sin\theta = \frac{3}{10} \Rightarrow \left(\frac{3}{10}\right)^2 + \cos^2\theta = 1$ (M1) $\Rightarrow \cos\theta = \pm\frac{\sqrt{91}}{10}$ (M1) $\Rightarrow z_2 = 5\left(\pm\frac{\sqrt{91}}{10} + \frac{3}{10}\mathrm{i}\right)$ or $\pm\frac{\sqrt{91}}{2} + \frac{3}{2}\mathrm{i}$ (A1A1)		

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td rowspan="5">5(c)</td><td colspan="2">M1 Requires answers of form $(\pm p)+q\mathrm{i}\ \ p>0,\ q>0$ or $(\pm p)-q\mathrm{i}\ \ p>0,\ q>0$ from (b) which could be inexact.
Allow if still have trig expressions, e.g., $\pm\sqrt{25-\left(3\sin\frac{7\pi}{6}\right)^2}-3\sin\frac{7\pi}{6}\mathrm{i}$
<u>Ignore relative positions</u> and complex number/axis labelling. May use points/lines.
More than 3 complex numbers indicated is M0.</td><td rowspan="2">M1</td></tr>
<tr><td>For $(\pm p)+q\mathrm{i}$: Three complex numbers:
1 in each of quadrant 1, Q2 and Q3 (not on axes). e.g.,

</td><td>For $(\pm p)-q\mathrm{i}$: Three complex numbers:
2 in Q3 and one in Q4 (not on axes).
e.g.,
</td></tr>
<tr><td colspan="2">A1 Requires correct answers from (b) but condone inexact equivalents $(\pm\text{awrt } 4.8)+1.5\mathrm{i}$ and allow if still have trig expressions, e.g.,
$\pm\sqrt{25-\left(3\sin\frac{7\pi}{6}\right)^2}-3\sin\frac{7\pi}{6}\mathrm{i}$ and ignore attempts to write z_2 in trig form
<u>AND</u> $r=3$ from (a) but and ignore miscalculation of z_1 $\left\{\text{i.e., if} \neq -\frac{3\sqrt{3}}{2}-\frac{3}{2}\mathrm{i}\right\}$ provided complex number is placed in Q3 and is closest to O</td><td rowspan="2">A1</td></tr>
<tr><td>
</td><td>Correct sketch with complex number in Q3 clearly closest of the three to the origin, <u>otherwise ignore relative positions</u>.
Condone e.g., asymmetry of z_{2a} and z_{2b}
May use points/lines.
If real and imaginary axes have been labelled the wrong way round then A0 but ignore all other labelling</td></tr>
<tr><td colspan="3" align="right">(2)</td></tr>
<tr><td></td><td colspan="2"></td><td>Total 8</td></tr>
</table>

Question Number	Scheme	Notes	Marks
6(a)	$f(x)=3x^2+kx-5 \Rightarrow \{\alpha\beta=\}-\frac{5}{3}$	Correct value for $\alpha\beta$. Accept $-1.\dot{6}$ Allow if e.g., "(a)" omitted and seen later	B1
			(1)
(b)	$\alpha+\beta=9\alpha\beta \Rightarrow -\frac{k}{3}=9\times"-\frac{5}{3}" \Rightarrow k=...$	Uses $\pm\frac{k}{3}$ for the sum of roots, sets equal to 9 times their product of roots and solves for k	M1
	$\{k=\}45$	Correct value (no equivalents) **from correct work**. Answer only is M1A1	A1
			(2)
(c)	$\{(\alpha+\beta)^3=\}\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$	Correct expansion seen. Terms may be uncollected e.g., $\alpha^3+2\alpha^2\beta+\alpha\beta^2+\alpha^2\beta+2\alpha\beta^2+\beta^3$	B1
	$\Rightarrow \alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha^2\beta-3\alpha\beta^2$ $=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ *	Achieves given answer via an intermediate step following expansion that is not just collecting terms on RHS of $(\alpha+\beta)^3=...$ No errors seen. Both sides must be seen but allow correct use of LHS=/RHS=. **Previous mark required.**	**d**B1*
	Working backwards: $(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3-3\alpha^2\beta-3\alpha\beta^2=\alpha^3+\beta^3$ B1: $(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$ or e.g., $\alpha^3+2\alpha^2\beta+\alpha\beta^2+\alpha^2\beta+2\alpha\beta^2+\beta^3$ **d**B1*: Correct proof with $-3\alpha\beta(\alpha+\beta)$ seen expanded. Both sides must be seen but allow correct use of LHS=/RHS=. **Previous mark required.**		
			(2)

Question Number	Scheme	Notes	Marks
6(d)	Note that the work for the first four marks might be seen embedded in a quadratic expression/equation. May also see use of $\left(x-\alpha^2-\beta\right)\left(x-\alpha-\beta^2\right)$ If the work **clearly** relies on using the solutions to $3x^2+45x-5=0$ $\left(\frac{-45\pm\sqrt{2085}}{6} \text{ or } 0.1103... \text{ \& } -15.1103...\right)$ then allow a max of 101010		
	$\alpha^2+\beta+\alpha+\beta^2=\alpha+\beta^2+\alpha^2+\beta$ $=\alpha+\beta+(\alpha+\beta)^2-2\alpha\beta$	Evidence of a correct algebraic expression in terms of $\alpha+\beta$ and $\alpha\beta$ only for the new sum of roots. If not seen in its entirety it could be implied by e.g. a numerical expression/value.	1st M1 (Sum)
	$=-15+(-15)^2-2\left(-\frac{5}{3}\right)$ $=-15+225+\frac{10}{3}=\frac{640}{3}$	Correct value for new sum**.** If inexact allow awrt 213 from a correct calculation. Allow if exact values are recovered later. **Must have used a correct expression and** $\alpha\beta=-\frac{5}{3}$, $\alpha+\beta=-\frac{45}{3}$ but allow slip in algebra if it is clearly recovered by e.g., an appropriate calculation	A1
	$\left(\alpha^2+\beta\right)\left(\alpha+\beta^2\right)=\alpha^3+\beta^3+\alpha\beta+(\alpha\beta)^2$ $=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)+\alpha\beta+(\alpha\beta)^2$ Allow $\alpha^2\beta^2$ for $(\alpha\beta)^2$ but $\alpha\beta^2$ must be recovered by later work	Evidence of a correct algebraic expression for the new product of roots in terms of $\alpha+\beta$ and $\alpha\beta$ only. If not seen in its entirety it could be implied by e.g. a numerical expression/value.	2nd M1 (Product)
	$=(-15)^3-3\left(-\frac{5}{3}\right)(-15)-\frac{5}{3}+\left(-\frac{5}{3}\right)^2$ $=-3375-75-\frac{5}{3}+\frac{25}{9}=-\frac{31040}{9}$	Correct value for new product. If inexact allow awrt -3450 from a correct calculation. **Must have used a correct expression and** $\alpha\beta=-\frac{5}{3}$, $\alpha+\beta=-\frac{45}{3}$ but allow slip in algebra if it is clearly recovered by e.g., an appropriate calculation	A1
	$x^2-\left(\frac{640}{3}\right)x+\left(-\frac{31040}{9}\right)$ $\{=0\}$	Applies $x^2-(\text{their } \textit{new} \text{ sum of roots})x+\text{their } \textit{new} \text{ product of roots}$ **correctly** (e.g., no missing "x") with non-zero **values** (which could be inexact). Allow without the "=0" for this mark. Not dependent. If just see e.g., $a=...,\ b=...,\ c=...$ then must see e.g., ax^2+bx+c	M1
	$9x^2-1920x-31040=0$	Correct equation as shown or an integer multiple. Could recover inexact values. Must include the "= 0". Could use e.g., z consistently for x. Requires all previous marks. If just see e.g., $a=...,\ b=...,\ c=...$ then must see e.g., $ax^2+bx+c=\underline{0}$	A1
			(6)
			Total 11

Question Number	Scheme	Notes	Marks
7	$\mathrm{f}(z) = Pz^4 - 36z^3 + Qz^2 + 192z + 68, \quad z = 3 + 5\mathrm{i}$ **Condone *x* used consistently for *z* throughout**		
(a)	$3 - 5\mathrm{i}$	Correct second root	B1
			(1)
(b)	Either $(z-3-5\mathrm{i})(z-3+5\mathrm{i}) = z^2 \pm 6z \pm m,\ m \in \mathbb{R}$ or $\alpha_1 + \alpha_2 = 6,\ \alpha_1\alpha_2 = M \Rightarrow z^2 \pm 6z \pm M,\ M \in \mathbb{R}$ For completing a correct strategy to find a 3 term quadratic factor. May either attempt to expand with correct starting point achieving $z^2 \pm 6z \pm m,\ m \in \mathbb{R}$, or attempts the sum and product of roots and reaches $z^2 \pm 6z \pm$ their (real) product		M1
	$z^2 - 6z + 34$	Correct factor. Accept answer only & "=0"	A1
			(2)
(c)	**Alternatives for (c) are shown overleaf**		
	$(z^2 - 6z + 34)(\text{"}a\text{"}z^2 + \text{"}b\text{"}z + \text{"}c\text{"}) = Pz^4 - 36z^3 + Qz^2 + 192z + 68$		
	$34c = 68 \Rightarrow c = 2$ "c" = 2 seen or implied. Allow $c = -2$ from $z^2 - 6z - 34$ but no other follow throughs.		B1 (ft on -34)
	$az^4 + (b-6a)z^3 + (c - 6b + 34a)z^2 + (34b - 6c)z + 34c$ $\Rightarrow b - 6a = -36,\ c - 6b + 34a = Q,\ -6c + 34b = 192$ $\{a = P,\ 34c = 68\}$ With $c = 2$ substituted: $\Rightarrow b - 6a = -36,\ 2 - 6b + 34a = Q,\ -12 + 34b = 192$	Expands $(\text{"}z^2 - 6z + 34\text{"})(az^2 + bz + c)$ [which could be implied] and compares coefficients for at least two of the z^3, z^2 and z terms obtaining at least 2 equations (could be implied) with real coefficients (may include the variable e.g., $34bz - 12z = 192z$). Must use at least 2 terms from the expansion per equation. Their 3TQ must have real coeffs.	M1
	e.g., $\Rightarrow 34b - 12 = 192 \Rightarrow b = 6, \quad b - 6a = -36 \Rightarrow a = 7 \Rightarrow P = 7$ Solves sufficient equations of correct form to find a real non-zero value for P (allow "a") or Q. No need to check algebra and accept a value following equations. Note that only 2 equations from comparing z^3 and z coefficients are needed to find P. It is possible to find Q first although it is not common e.g. $b = 6$ in $Q = 2 - 6b + 34\left(\frac{b}{6} + 6\right) \Rightarrow Q = 204$. **Previous mark required.**		**d**M1
	e.g., $Q = c - b + 34a = 2 - 36 + 238 = 204$ Solves sufficient equations of correct form to find real non-zero values for both P (allow "a") and Q. No need to check algebra and accept values following equations. **2 previous marks required.**		**dd**M1
	P = 7 and Q = 204 only Allow a = 7 if $P = a$ (only) seen	P = 7 (not "a") and Q = 204 No other answers. May be embedded in f(z)	A1
			(5)
(d)	"$7z^2 + 6z + 2$" $= 0$ (Allow with their other 3TQ factor - must have real coefficients) $\Rightarrow z = \frac{-6 \pm \sqrt{6^2 - 4 \times 7 \times 2}}{2 \times 7}$ or $\frac{-6 \pm \sqrt{-20}}{14}$ or $\frac{-6 \pm \sqrt{20}\,\mathrm{i}}{14}$ or $z^2 + \frac{6}{7}z + \frac{2}{7} = \left(z + \frac{3}{7}\right)^2 - \frac{9}{49} + \frac{2}{7} = 0 \Rightarrow z = ...\left\{-\frac{3}{7} \pm \frac{\sqrt{-5}}{7}\right\}$ Either uses correct formula **correctly** or completes the square - usual rules - **<u>shown</u>** for solving their other three term quadratic factor (**which must have complex roots**). Do not accept factorisation or just writing down **simplified** roots from calculator. 1 root is sufficient. If forms equations e.g., $z_1 + z_2 = -\frac{6}{7},\ z_1 z_2 = \frac{2}{7}$ (allow sign errors only) a **full** algebraic method for obtaining 1 root must be seen.		M1
	$\frac{-3 \pm \mathrm{i}\sqrt{5}}{7}$ or $-\frac{3 \pm \mathrm{i}\sqrt{5}}{7}$ or $\frac{-3}{7} \pm \frac{\sqrt{5}}{7}\mathrm{i}$	Correct simplified other roots from correct factor. Must see "i". Ignore the presence of $3 \pm 5\mathrm{i}$ if also listed. Ignore labelling. Does not require 5/5 in (c)	A1
			(2)
			Total 10

Question Number	Scheme/Notes		Marks
7(c)	If long division is not completed/equations not seen allow access to the marks if the **correct** values for P and/or Q are deduced provided there is no clearly inappropriate work. For example it is possible to deduce that $b = 6$ as well as $c = 2$ and compare z^3 coeffs. from long division. If their quadratic factor is incorrect then equations must be seen for the 1st M1		
Alt 1a Full Long Division **Use Alt 1b if they have a <u>value</u> for** c	Allow for equivalent work using e.g., a multiplication grid $$\begin{array}{r l} & Pz^2+(6P-36)z+(Q+2P-216) \\ z^2-6z+34 & \overline{)\,Pz^4-36z^3+Qz^2+192z+68} \\ & \underline{Pz^4-6Pz^3+34Pz^2} \\ & \underline{\underline{(6P-36)z^3}}+(Q-34P)z^2+192z \\ & \underline{(6P-36)z^3-6(6P-36)z^2+34(6P-36)z} \\ & (Q+2P-216)z^2+(1416-204P)z+68 \\ & \underline{(Q+2P-216)z^2-6(Q+2P-216)z+34(Q+2P-216)} \\ & (120-192P+6Q)z-68P-34Q+7412 \end{array}$$ $\Rightarrow 120-192P+6Q=0, -68P-34Q+7412=0$ B1**:** Attempts long division using a 3TQ with real coefficients and proceeds correctly to the first z^3 coefficient after subtraction (double underlined) **-** ignore subsequent errors. M1**:** Carries out sufficient long division and sets remainder = 0 (could be implied by equations if remainder not seen explicitly) to form a pair of linear simultaneous equations with real coefficients. Must have come from comparing coefficients oe for z terms and constant terms. Both equations must include P and Q.		B1 M1
	$\{32P-Q=20,\ 2P+Q=218\}$ $\Rightarrow P=\ldots$ or $Q=\ldots$	Solves the equations to find a real non-zero value for either P or Q. No requirement to check algebra and accept a value following equations. **Previous mark required.**	**d**M1
	$\Rightarrow P=\ldots$ and $Q=\ldots$	Solves the equations to find real non-zero values for both P and Q. No requirement to check algebra and accept values following equations. **2 previous marks required.**	**dd**M1
	P = 7 and Q = 204 only	P = 7 and Q = 204 No other answers. May be embedded in f(z)	A1
			(5)
Alt 1b Long Division: using a value for c	$$\begin{array}{r l} & Pz^2+(6P-36)z+2 \\ z^2-6z+34 & \overline{)\,Pz^4-36z^3+Qz^2+192z+68} \\ & \underline{Pz^4-6Pz^3+34Pz^2} \\ & (6P-36)z^3+(Q-34P)z^2+192z \\ & \underline{(6P-36)z^3-6(6P-36)z^2+34(6P-36)z} \\ & (Q+2P-216)z^2+(1416-204P)z+68 \\ & \underline{2z^2\qquad\qquad -12z+68} \\ & (2P+Q-218)z^2-(204P-1428)z \end{array}$$ $\Rightarrow 2P+Q-218=0,\ -204P+1428=0$ B1: $c=2$ implied M1: Carries out sufficient long division and sets remainder = 0 (which could be implied) to form at least an equation with real coefficients in just P. Note that a remainder may not be seen explicitly. Must have come from comparing coefficients (oe) of z^2 terms and z terms. **d**M1: Solves an equation in P to find a real non-zero value for P **or** solves two equations (one in just P and one in both P and Q) to find a real non-zero value for Q. However, it is very unlikely that Q will be seen with no answer for P given. **dd**M1: Solves two equations to find real non-zero values for both P and Q. Must have had one equation in P only and one equation in both P and Q. A1: P = 7 and Q = 204 only. May be embedded in f(z)		

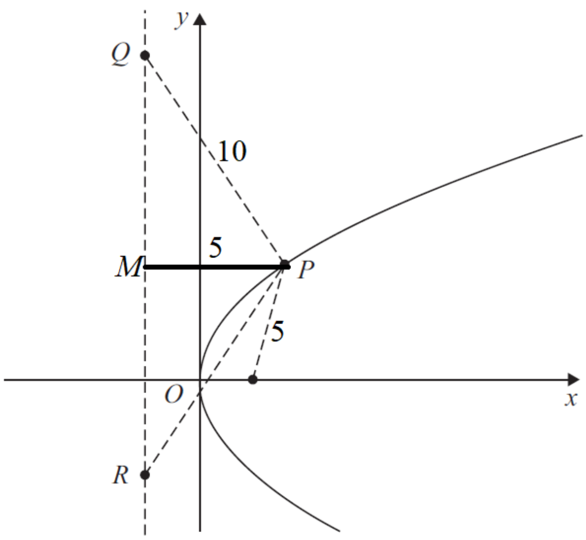
Question Number	Scheme/Notes		Marks
7(c) **Alt 2** **Substitution**	$\mathrm{f}(3\pm5\mathrm{i}) = P(3\pm5\mathrm{i})^4 - 36(3\pm5\mathrm{i})^3 + Q(3\pm5\mathrm{i})^2 + 192(3\pm5\mathrm{i}) + 68$ $= P(-644\mp960\mathrm{i}) + Q(-16\pm30\mathrm{i}) + 7772\pm600\mathrm{i}$ or $-644P - 16Q + 7772 + (\mp960P \pm 30Q \pm 600)\mathrm{i}$ Correct **six term** expression for f(3+5i) or f(3−5i). May use calculator Implied by correct equations		B1
	$\mathrm{f}(3\pm5\mathrm{i}) = 0 \Rightarrow P(-644\mp960\mathrm{i}) + Q(-16\pm30\mathrm{i}) + 7772\pm600\mathrm{i}=0$ $\Rightarrow -644P - 16Q + 7772 = 0,\ \ \mp960P \pm 30Q \pm 600 = 0$ Attempts f(3+5i) or f(3−5i) and sets equal to 0 and equates real and imaginary parts to form a pair of linear simultaneous equations with real coefficients. Both equations must have no imaginary terms and both must include *P* and *Q*. Ignore extra equations		M1
	$\{161P + 4Q = 1943,\ \ 32P - Q = 20\}$ $\Rightarrow P = \ldots$ or $Q = \ldots$	Solves the equations to find a real non-zero value for either *P* or *Q*. No requirement to check algebra and accept a value following equations. **Previous mark required.**	**d**M1
	$\Rightarrow P = \ldots$ and $Q = \ldots$	Solves the equations to find real non-zero values for both *P* and *Q*. No requirement to check algebra and accept values following equations. **2 previous marks required.**	**dd**M1
	P = 7 and *Q* = 204 only	*P* = 7 and *Q* = 204. No other answers. Maybe embedded in f(*z*)	A1
			(5)

Question Number	Scheme	Notes	Marks
8(a)	$\sum_{r=1}^{2n}(2r^2-1)=2\sum_{r=1}^{2n}r^2-\sum_{r=1}^{2n}1$ $\underline{=2\times\frac{2n}{6}(2n+1)(2(2n)+1)}-\underline{\underline{2n}}$ $\left\{=\frac{2n}{3}(2n+1)(4n+1)-2n=\frac{16n^3}{3}+4n^2-\frac{4n}{3}\right\}$	<u>M1</u>: Uses $2n$ for k in $2\times\frac{k}{6}(k+1)(2k+1)$ Must replace k with $2n$ at least once. Must be otherwise correct i.e., only allow n or $2n$ for k Note that failing to replace the first k with $2n$ leads to $\frac{n}{3}(...)$ Award M0 if it is quite clear that the wrong formula for the sum of the squares has been used.	M1
		<u>B1</u>: Correct $\sum_{r=1}^{2n}1=2n$ seen/used	B1
	$=\frac{2n}{3}(8n^2+6n-2)$ or $\frac{4n}{3}(4n^2+3n-1)$ Obtains $...n(\mathbf{3TQ}\text{ in }n)$ from a cubic with no constant. If 3TQ allow fractional coefficients. Allow implication of this mark with $\frac{4}{3}(4n^3+3n^2-n)$ or $\frac{2}{3}(8n^3+6n^2-2n)$ but not just $\frac{16n^3}{3}+4n^2-\frac{4n}{3}$. Must not have had a constant term. Can be scored if n instead of $2n$ used throughout the sum of squares formula $\left(\Rightarrow\frac{n}{3}(n^2+3n-5)\right)$ but next ddM0. Condone poor algebra.		M1
	$=\frac{4}{3}n(n+1)(4n-1)$ Allow e.g., $\frac{4n(4n-1)(n+1)}{3}$	Obtains $...n(\text{factorised 3TQ in }n)$ Apply usual quadratic rules for the factorisation. If e.g., $\frac{2n}{3}(...)\to\frac{4n}{3}(...)$ work must be on a consistent 3TQ. Factors must have all real & exact terms. **Previous 2 method marks required.**	**dd**M1
		Correct result. Allow minor recovered algebraic/bracketing slips.	A1
	Note that e.g., if $2\times\frac{2n}{6}(2n+1)(2(2n)+1)-2n$ or $\frac{16n^3}{3}+4n^2-\frac{4n}{3}$ or $\frac{2n}{3}(2n+1)(4n+1)-2n$ is immediately followed by $\frac{4}{3}n(n+1)(4n-1)$ score 11000. Allow expanding the given answer $\frac{4}{3}n(n+1)(an+b)$ and equating coefficients for the last three marks: $\frac{16n^3}{3}+4n^2-\frac{4n}{3}=\frac{4}{3}an^3+\frac{4}{3}(a+b)n^2+\frac{4}{3}bn\Rightarrow a=4,\ b=-1\Rightarrow\frac{4}{3}n(n+1)(4n-1)$ M1: Correct form for expansion of given answer and obtains a value for either a or b **dd**M1: Obtains values for both a and b A1: $\frac{4}{3}n(n+1)(4n-1)$ (allow just $a=4,\ b=-1$ if $\frac{4}{3}n(n+1)(an+b)$ seen) Summation formulae must be used. No credit for attempts using induction or e.g., setting up simultaneous equations with 2 values of n, unless there is work that can score as above.		
			(5)

Question Number	Scheme	Notes	Marks
8(b)	$\sum_{r=1}^{1} r(3r-2)^2$ or LHS $=1(3\times1-2)^2=1$ and $\frac{n^2(n+1)(9n-7)}{4}$ or RHS $=\frac{1^2\times2\times2}{4}=1$ Achieves 1 from two numerical expressions (both not just "1"). No requirement here to say "true" etc. Allow as minimum, e.g., $1\times1=\frac{4}{4}=1$. "(When n = 1,) both =1" is B0 but final A1 available.		B1
	{Assume true for n = k, then} $\sum_{r=1}^{k+1} r(3r-2)^2=\sum_{r=1}^{k} r(3r-2)^2+(k+1)(3(k+1)-2)^2$ $=\frac{k^2(k+1)(9k-7)}{4}+(k+1)(3k+1)^2$	Adds an attempt at the (k+1)th term to an attempt at the sum to k terms. Allow clear copying slips (e.g., losing the squared from the k^2) but must be a recognisable attempt at forming $\frac{k^2(k+1)(9k-7)}{4}+(k+1)(3(k+1)-2)^2$ So e.g., $\ldots+k(3k-2)^2$ is M0	M1
	$=\frac{(k+1)}{4}\left(9k^3-7k^2+4\left(9k^2+6k+1\right)\right)=\frac{(k+1)}{4}\left(9k^3+29k^2+24k+4\right)$ $\left\{\text{or } =(k+1)\left(\frac{9}{4}k^3+\frac{29}{4}k^2+6k+1\right)\right\}$ **OR** $=\frac{1}{4}\left(9k^4+38k^3+53k^2+28k+4\right)$ $\left\{\text{or } =\frac{9}{4}k^4+\frac{19}{2}k^3+\frac{53}{4}k^2+7k+1\right\}$ Reaches $\frac{(k+1)}{4}$(**4 term** cubic in k) or $(k+1)$(**4 term** cubic in k) **OR** expands to a **5 term** quartic. Must collect terms. Condone poor algebra. **Previous mark required**		**d**M1
	$=\frac{1}{4}(k+1)\left((k+1)(9k^2+20k+4)\right)$ or $\frac{1}{4}(k+1)^2(k+2)(9k+2)\Rightarrow\frac{(k+1)^2\left((k+1)+1\right)\left(9(k+1)-7\right)}{4}$ Correctly reaches the result completely in terms of k + 1 [but allow k + 2 or k + 1 + 1 for ((k + 1) + 1)] **with an intermediate step**. Condone poor notation and allow the odd recovered algebraic slip/poor bracketing provided they reach the correct $\frac{k+1}{4}$(4 term cubic in k) or $(k+1)$(4 term cubic in k) or 5 term quartic and there are no subsequent errors. Factorisation may be achieved via calculator use. Allow a final answer of $\frac{1}{4}(k+1)^2(k+2)\left(9(k+1)-7\right)$ and with brackets in any order. Meet in the middle approaches must clearly join and the final expression in k + 1 must be seen in the work		A1
	True for n = 1 and if true for n = k then it is also true for n = k + 1 so by induction it is true for (all) n Acceptable proof (i.e., must have scored the previous A) and narrative/conclusion. **Requires previous three marks and can only follow B0 if B0 was given for insufficient evidence of substitution. There must be no errors if one substitution was attempted and must have reached both sides = 1.** "Assume (true) for $n=k$" or "If true for n = k" in narrative followed by "true for $n=k+1$" is sufficient for the "if...then". Allow suitable surrogates for "true". Allow stating the result from the question paper or "P_n is true" with added reference to n. Condone $n\in\square$ but not $n\in\square$. Allow work in n rather than k throughout. There is no credit for attempts using summation formulae instead of induction unless there is work that can score via the scheme above		A1
			(5)

Question Number	Scheme	Notes	Marks
8(c)	$8\sum_{r=1}^{n} r(3r-2)^2 = 15\sum_{r=1}^{2n}(2r^2-1) \Rightarrow$ $8\times\frac{n^2(n+1)(9n-7)}{4} = 15\times\frac{4}{3}n(n+1)(4n-1)$ Attempts to form the given equation, obtaining an equation in *n* only with their answer to (a) which must be cubic but not necessarily in the right form. (Note - part (a) could have been reattempted in this part but do not allow attempts that seek to recreate an equivalent for the answer to (b) via summation formulae). Allow if 8 and 15 are swapped or if **one** is missing (but other correctly placed) provided there are no other errors. If 8 and 15 correctly placed, condone one minor copying error but must have quartic = cubic unless *n* instead of n^2 on LHS is the only error. Note that if they attempt to replace the '*n*'s in their answer to (a) with '2*n*'s then award M0 unless they have used *n* throughout part (a) in which case all marks are potentially available. If e.g., $15\times\frac{4}{3}$ is only seen evaluated it must be correct. Allow with "*a*" and "*b*" or invented values used but no further marks.		M1
	$\Rightarrow 2n(9n-7) = 20(4n-1)$ $\Rightarrow 18n^2 - 94n + 20 = 0$ $\Rightarrow 9n^2 - 47n + 10 = 0$ $\Rightarrow (9n-2)(n-5) = 0$ $\Rightarrow n = ...$ May see, e.g. $18n^3 - 94n^2 + 20n = 0$ $9n^3 - 47n^2 + 10n = 0$	Simplifies to a 3TQ or 3 term cubic with no constant (see below if 4TC or quartic) and solves to find a value for *n*. If working is shown, apply usual rules (in this case solution does not have to be a positive integer). However, if answer/s are just written down one real positive integer root must be achieved and be correct for their 3TQ/3TC with no constant. If there is no 3TQ/3TC with no constant and $n = 5$ is just written down score 100. Do not allow solutions **directly** from just a quartic e.g., $9n^4 - 38n^3 - 37n^2 + 10n = 0$ or a cubic with constant e.g., $9n^3 - 38n^2 - 37n + 10 = 0$ **unless** there is a clear **full** method to factorise (e.g., factor theorem, long division, multiplication grid). Do not allow immediate factorisation. **Previous mark required.**	**d**M1
	$\{n =\}5$ Correct answer and no other unrejected solutions e.g., $n = \frac{2}{9}, 0, -1$ (Ignore any rejected incorrect values of *n* provided quadratic oe was correct and dM1 was scored) Answer may not be labelled and accept e.g., $x = 5$		A1
			(3)
			Total 13

Question Number	Scheme	Notes	Marks
9	Let M be point where perpendicular to directrix from P meets the directrix, then $PM = 5$	States, uses or implies the horizontal distance from P to the directrix is 5. If indicated on Figure 1 or on their own diagram it must be clearly the horizontal distance from P to the directrix. Could be implied by a correct QM or QR. Do not be concerned about any preceding algebra. There is no credit for just forming expressions/equations in e.g., a and/or t until a numerical distance is found.	B1
	$QM = \sqrt{10^2 - \text{"}5\text{"}^2}$ $(= \sqrt{75} = 5\sqrt{3} \approx 8.66...)$ May see: $QM = 10\sin\frac{\pi}{3},\ 5\tan\frac{\pi}{3},\ 10\cos\frac{\pi}{6},\ \frac{5}{\tan\frac{\pi}{6}}$ $QM = 10\sin 60^\circ,\ 5\tan 60^\circ,\ 10\cos 30^\circ,\ \frac{5}{\tan 30^\circ}$	Obtains a correct numerical expression for QM (not QM^2) with their $PM = k$ where $0 < k < 10$ Pythagoras must be fully correctly applied. Implied by QR. This mark is not available for arbitrarily choosing a value for an angle but apply BOD and potentially full marks if a correct angle is used without working.	M1
	$QR = 2 \times \sqrt{10^2 - \text{"}5\text{"}^2}$ $(= 2 \times 5\sqrt{3} \approx 17.3...)$ May see: $QR = \sqrt{10^2 + 10^2 - 2 \times 10 \times 10 \times \cos\frac{2\pi}{3}}$	A correct numerical expression for QR (not QR^2) with their $PM = k$ where $0 < k < 10$ **Previous mark required.**	**d**M1
	$\{QR =\}10\sqrt{3}$	Correct answer. Allow any exact equivalent e.g., $2\sqrt{75}$, $\sqrt{300}$	A1
	Correct answer only or correct answer following no or minimal work scores 4/4 If trigonometry is used the scheme applies as above so all work must be correct for their k where $0 < k < 10$ but allow premature rounding.		
			(4)
			Total 4



Correct angles: $\angle MPQ = \angle MPR = \frac{\pi}{3} = 60^\circ$, $\angle MQP = \angle MRP = \frac{\pi}{6} = 30^\circ$, $\angle QPR = \frac{2\pi}{3} = 120^\circ$