Question	Scheme	Marks
1.(a)	$\det \mathbf{M} = (2k+1)(k+4) - k(k+7)$	M1
	$\det \mathbf{M} = k^2 + 2k + 4 \Rightarrow b^2 - 4ac = 4 - 16$ or $\det \mathbf{M} = k^2 + 2k + 4 = (k+1)^2 + 3$ or $\dfrac{\mathrm{d}(\det \mathbf{M})}{\mathrm{d}k} = 2k + 2 = 0 \Rightarrow k = -1$	M1
	$b^2 - 4ac < 0 \Rightarrow k^2 + 2k + 4 > 0$ or $\det \mathbf{M} = (k+1)^2 + 3 \ldots 3$ or $k = -1$ at minimum so det $\mathbf{M}$ $\ldots 3$ Hence $\mathbf{M}$ is non-singular for all real values of k	A1
		(3)
(b)	$\mathbf{M}^{-1} = \dfrac{1}{k^2 + 2k + 4}\begin{pmatrix} k+4 & -k \\ -k-7 & 2k+1 \end{pmatrix}$	M1 A1
		(2)
		(Total 5 marks)

Notes

(a)

M1: Attempts the determinant of **M**. Must see evidence of the attempt at subtracting but allow e.g. minor sign slips inside the brackets. Must be seen in (a).

M1: Begins a correct strategy for attempting to establish that the determinant is non-zero, must follow a valid attempt at the determinant involving all four terms.. May find the discriminant, complete the square (usual rules) on the determinant, attempt to solve the quadratic via formula, or attempt a minimisation process. There must be an attempt at a calculation.

A1: Full and correct reasoning and conclusion. Must see consideration of the sign or non-zero oe, but accept e.g. $\ldots 3$ (condone >3) as being sufficient to show det **M** cannot be zero for the reason. Showing the roots $\left(-1 \pm \mathrm{i}\sqrt{3}\right)$ are not real is acceptable The final deduction must refer to non-singular, but no need to mention **M** and condone det**M** is non-singular as conclusion.

(b)

M1: For applying $\mathbf{M}^{-1} = \dfrac{1}{\det \mathbf{M}} \times \mathrm{adj}(\mathbf{M})$ with their determinant. At least three entries in adj(**M**) must be correct initially.

A1: Correct matrix, and isw after a correct answer.

Question	Scheme	Marks
2.	$\mathrm{f}(z)=2z^3+pz^2+qz-41$	
(a)	$(z=)5+4\mathrm{i}$	B1
		(1)
(b)	$z=5\pm4\mathrm{i}\Rightarrow(z-(5+4\mathrm{i}))(z-(5-4\mathrm{i}))=...$ Or e.g. Sum of roots = 10, Product of roots = 41	M1
	$z^2-10z+41$	A1
	$\mathrm{f}(z)=(z^2-10z+41)(2z+...)$	M1
	$\Rightarrow z=\frac{1}{2},(5\pm4\mathrm{i})$	A1
		(4)
(c)	$\mathrm{f}(z)=(z^2-10z+41)(2z-1)=\ldots$	M1
	$p=-21,\ q=92$	A1
		(2)
(d)	Area = $\frac{1}{2}\times8\times\left(5-\text{"}\frac{1}{2}\text{"}\right)$	M1
	= 18	A1ft
		(2)
		(Total 9 marks)

Notes Mark (b) and (c) together

(a)

B1: Correct complex number

(b)

M1: Correct strategy to find the quadratic factor using the conjugate pair.

A1: Correct quadratic factor.

M1: Attempts to find the linear factor. Look for $2z+k$ where k is number (or allow if k is in terms of p as long as a value of p is also found at some stage). May arise from attempts at long division.

A1: Correct real root (condone if labelled x). The complex roots do not have to be stated. Must be seen in (b) (or (c) if done together).

(c)

M1: Multiplies out to obtain values for p and q. May have been found as part of a long division process in (b).

A1: Correct values. May be seen embedded in the cubic.

(d)

M1: For $\frac{1}{2}\times8\times\left|5-\text{their"}\frac{1}{2}\text{"}\right|$ where their real root is non-zero.

A1ft: Correct area (follow through their non-zero real root).

Alt 1 **(b)**	Product of complex roots = 41, Product of all roots = $\frac{\pm 41}{2}$	**M1**
	Product of complex roots = 41, Product of all roots = $\frac{41}{2}$	**A1**
	$z=\frac{\text{Product of roots of f}(z)}{\text{Product of complex roots}}=\frac{41}{2}\div 41$	**M1**
	$\Rightarrow z=\frac{1}{2},(5\pm 4\text{i})$	**A1**
		(4)
(c)	$\frac{p}{2}=-\sum\alpha_i \ \ \frac{q}{2}=\sum\alpha_i\alpha_j \Rightarrow p=...,q=...$	**M1**
	$p=-21,\ q=92$	**A1**
		(2)

Notes

Alt 1 (b)
M1: Identifies the product of roots of f(z) up to sign error, and the product of complex roots.
A1: Correct products seen or implied.
M1: Attempts to find the third root of f(z).
A1: Correct real root. The complex roots do not have to be stated. Must be seen in (b) (or (c) if done together)
(c)
M1: Applies sum and pair sum properties, or multiplies out to obtain values for p and q.
A1: Correct values. May be seen embedded in the cubic.

Alt 2 **(b)**	$\text{f}(5\pm 4\text{i})=-230\pm 472\text{i}+p(9\pm 40\text{i})+q(5\pm 4\text{i})-41=0$ $\Rightarrow -271+9p+5q=0, 472+40p+4q=0$	**M1** **A1**
	$\Rightarrow p=...,q=...\Rightarrow \text{f}(z)=2z^3+"p"z^2+"q"z-41\Rightarrow z=...$	**M1**
	$\Rightarrow z=\frac{1}{2},(5\pm 4\text{i})$	**A1**
		(4)
(c)	$\Rightarrow p=...,q=...$	**M1**
	$p=-21,\ q=92$	**A1**
		(2)

Notes Alt (b)+(c) together

Alt 2 (b)
M1: Attempts factor theorem with one of the complex roots **and** equates real and imaginary terms to produce simultaneous equations.
A1: Correct equations.
M1: Uses their p and q from solving the simultaneous equations in f(z) and solves the cubic (may just see roots).
A1: Correct real root. The complex roots do not have to be stated. Must be seen in (b) (or (c) if done together)
(c)
M1: Awarded before previous M. Solves their simultaneous equations to obtain values for p and q.
A1: Correct values. May be seen embedded in the cubic.

Question	Scheme	Marks
3.(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{t^2}$	B1
	$y - \frac{c}{t} = -\frac{1}{t^2}(x - ct)$	M1
	$\Rightarrow x = ..., y = ...$	M1
	$A(2ct, 0)$ **and** $B\left(0, \frac{2c}{t}\right)$	A1
		(4)
(b)	$\frac{1}{2} \times 2ct \times \frac{2c}{t} = 90 \Rightarrow c = ...$	M1
	$c^2 = 45 \Rightarrow c = 3\sqrt{5}$	A1cso
		(2)
		(Total 6 marks)

Notes

(a)

B1: Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. May be implied at the point of substitution into the equation if only found in terms of x and/or y initially. Allow if just stated, no working needed (may have been memorised). You may see attempts where this is derived but they must get to or imply the correct derivative in terms of t.

M1: Correctly forms the equation of the tangent. If no working for the gradient is shown accept $m = \pm\frac{1}{t^2}$ for this mark. If using $y = mx + c$ they must proceed at least as far as finding c.

M1: Uses their "tangent" (which must be a straight line equation) to find the non-zero coordinates of A and B. Both must be attempted.

A1: Both correct. Accept A, $x = 2ct$, B, $y = \frac{2c}{t}$ or list as separate coordinate as long as it is clear, but just $\left(2ct, \frac{2c}{t}\right)$ is A0. If coordinates are labelled the wrong way award A0 but allow both marks in (b).

(b)

M1: Uses their coordinates and the "90" correctly to form and solve an equation for c.

A1cso: Correct value for c (must be simplified surd so $3\sqrt{5}$). Must have come from correct coordinates. A0 if the negative root is also given (and not rejected).

Question	Scheme	Marks
4.(a)	Stretch – SF 3 parallel to the y-axis	**M1** **A1**
		(2)
(b)	$\mathbf{B}=\begin{pmatrix}-\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & -\frac{\sqrt{3}}{2}\end{pmatrix}$	**B1**
		(1)
(c)	$\mathbf{C}=\mathbf{BA}=\begin{pmatrix}-\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & -\frac{\sqrt{3}}{2}\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix}$	**M1**
	$\mathbf{C}=\begin{pmatrix}-\frac{\sqrt{3}}{2} & \frac{3}{2}\\ -\frac{1}{2} & -\frac{3\sqrt{3}}{2}\end{pmatrix}$	**A1**
		(2)
(d)	$\det\mathbf{C}=-\frac{\sqrt{3}}{2}\times-\frac{3\sqrt{3}}{2}-\frac{3}{2}\left(-\frac{1}{2}\right)=3$ So area of $H¢$ is $5\times\det\mathbf{C}=\ldots$	**M1**
	$=15$	**A1**
		(2)
		(Total 7 marks)

Notes

(a)

M1: Identifies one correct aspect, EITHER **stretch** (or allow scaling)
OR scale factor 3 **and** correct direction indicated (need not be precise).

A1: Fully correct description with both aspects correct. Must mention stretch or scaling, but be tolerant with the description of direction and scale factor as long as both are clear. Accept e.g. parallel to the y-axis, y direction, in y axis, or vertically for direction, but not "about y" (reflection implied). Accept e.g. scale factor 3, by 3, ×3 or three times for the scale factor. Ignore references to "about origin" or additional references to stretch of factor 1 parallel to the x-axis.

Some examples:

- "stretched by 3 in direction of y axis".
- "stretch the y for scale factor of 3."

  Both the above score M1A1. Stretch stated, direction and scale factor 3 both indicated.
- "enlargement scale factor 3 for y-axis."
- "A enlarge by three times paralleled to y-axis."

  Both score M1A0 Indicates direction and scale factor but does not mention stretch or scaling.
- "the y-axis will be enlarged by three times, whereas the x-axis stay the same."

  M1A0 Indicates direction and scale factor but does not mention stretch or scaling, the reference to the x-axis is ignored as not incorrect.

(b)
B1: Correct matrix. Must be exact (trig terms evaluated) and seen in (b).
(c)
M1: Attempts to multiply matrices the right way around. Implied by 3 correct entries if no product shown.
A1: Correct matrix. Must be exact.
(d)
M1: Attempts determinant of **C** (or deduces area scale factor is 3) and multiplies by 5. Implied by a correct answer if no incorrect working is seen.
A1: Cao. Allowed if scored from a **C** arising from multiplication the wrong way round in (c) or an incorrect **B** that has determinant ± 1

Question	Scheme	Marks
5.(a)	$2x^2 - 3x + 7 = 0$	
	$\alpha + \beta = \frac{3}{2}, \quad \alpha\beta = \frac{7}{2}$	B1
		(1)
(b)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1
	$= \left(\frac{3}{2}\right)^2 - 2\left(\frac{7}{2}\right) = -\frac{19}{4}$	A1
		(2)
(c)	$\text{Sum} = \alpha - \frac{1}{\beta^2} + \beta - \frac{1}{\alpha^2} = \alpha + \beta - \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{3}{2} - \frac{-\frac{19}{4}}{\left(\frac{7}{2}\right)^2} = \frac{185}{98}$	M1A1
	$\text{Prod} = \left(\alpha - \frac{1}{\beta^2}\right)\left(\beta - \frac{1}{\alpha^2}\right) = \alpha\beta - \frac{\alpha + \beta}{\alpha\beta} + \frac{1}{\alpha^2\beta^2} = \frac{7}{2} - \frac{3}{7} + \frac{4}{49} = \frac{309}{98}$	M1A1
	$x^2 - \frac{185}{98}x + \frac{309}{98} (= 0)$	M1
	$98x^2 - 185x + 309 = 0$	A1
		(6)
		(Total 9 marks)

Notes

(a)
B1: Both values correct.
(b)
M1: Attempts to use a correct identity to find the sum of square of roots.
A1: Correct value. Note do not allow recovery from $\alpha + \beta = -\frac{3}{2}$ for this mark.

(c)
M1: Attempts sum for the new roots using their values from (a) and (b). They must be substituting into a correct identity for this mark. If substitution not seen allow for any value appearing after a suitable combined identity is seen.
A1: Correct value.
M1: Attempts product for the new roots using their values from (a). Must be substituting into an expression of the correct form, but allow if a sign slip occurs when expanding. If substitution not seen allow for any value appearing after a suitable expanded identity is seen.
A1: Correct value.
M1: Applies x^2 – (their sum)x + their prod (= 0). May be implied by suitable values for p, q and r stated if no quadratic seen.
A1: Allow any integer multiple. Must include the "= 0", and must be an equation, not just values for p, q and r.
Note: Answers from solving the quadratic will gain no credit for (a) and (b) and only score in (c) if the method marks as described are earned.

Question	Scheme	Marks
6.(i)	$f(x)=x-4-\cos\left(5\sqrt{x}\right) \qquad x>0$	
(a)	$f\left(2.5\right)=-1.44...,\ f\left(3.5\right)=0.497...$	**M1**
	Sign change (negative, positive) and f(x) is **continuous** therefore (a **root**) α is between x = 2.5 and x = 3.5	**A1**
		(2)
(b)	E.g. $\dfrac{\alpha-2.5}{\lvert f\left(2.5\right)\rvert}=\dfrac{3.5-\alpha}{f\left(3.5\right)}\Rightarrow\alpha=...$ or $\dfrac{\alpha-2.5}{0-f\left(2.5\right)}=\dfrac{3.5-2.5}{f\left(3.5\right)-f\left(2.5\right)}\Rightarrow\alpha=...$	**M1**
	$\alpha=$ awrt 3.24	**A1**
		(2)
(ii)	$g(x)=\dfrac{1}{10}x^2-\dfrac{1}{2x^2}+x-11 \qquad x>0$	
(a)	$g'(x)=\dfrac{1}{5}x+\dfrac{1}{x^3}+1$	**M1**
		A1
		(2)
(b)	$x_1=6-\dfrac{g\left(6\right)}{g'\left(6\right)}=6-\dfrac{-1.41388...}{2.20462...}$	**M1**
	= 6.641	**A1cao**
		(2)
		(Total 8 marks)

Notes

(i)(a)

M1: Attempts both f(2.5) and f(3.5) with at least one correct in either radians or degrees. Note that in degrees f(2.5) = −2.49… and f(3.5) = −1.4867…

A1: Both f (2.5) = awrt −1 and f(3.5) = awrt 0.5, sign change (accept f(2.5)f(3.5) <0), continuous and conclusion all given but be forgiving with exact language. Use of degrees will be A0 as there is no change in sign.

(b)

M1: Uses a correct interpolation method to find a value for α. There are other alternative versions but look for a correct full process. E.g. may attempt the equation of the line through the two end points, then substitute y = 0 to find x. Allow if using degrees so long as a correct interpolation statement is clear.

A1: Correct value, accept awrt 3.24.

(ii)(a)

M1: $x^n\to x^{n-1}$ in at least two of the first 3 terms.

A1: All correct simplified or unsimplified.

(b)

M1: Correct application of Newton-Raphson. If no expression is seen, the method may be implied by a correct answer. (Look for the process rather than labelling if they write f but use g.)

A1cao: Correct value. Must be to 3d.p.. ISW if they try a second application of N-R.

Note: If correct answers for (b) appear after an incorrect derivative then please send to review.

Question	Scheme	Marks
7.(a)	$x=\frac{1}{3}t^2, y=\frac{2}{3}t \Rightarrow \frac{dy}{dx}=\frac{2}{3}\div\frac{2}{3}t=\frac{1}{t}$ or $y^2=\frac{4}{3}x \Rightarrow 2y\frac{dy}{dx}=\frac{4}{3} \Rightarrow \frac{dy}{dx}=\frac{2}{3y}=\frac{1}{t}$ or $y^2=\frac{4}{3}x \Rightarrow y=\frac{2}{\sqrt{3}}\sqrt{x} \Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{3}\sqrt{x}}=\frac{1}{t}$	B1
	$y-\frac{2}{3}t=-t\left(x-\frac{1}{3}t^2\right)$	M1
	$3tx+3y=t^3+2t$ *	A1*
		(3)
(b)	$t=9 \Rightarrow 27x+3y=747$	B1
	$y^2=\frac{4}{3}x \Rightarrow x=\frac{3y^2}{4} \Rightarrow 3y+3\times 9\times\frac{3y^2}{4}=729+18$ or $y^2=\frac{4}{3}x \Rightarrow \frac{1}{9}(747-27x)^2=\frac{4}{3}x \Rightarrow 729x^2-40350x+558009=0$	M1
	$27y^2+4y-996=0 \Rightarrow y=\ldots$ **or** $729x^2-40350x+558009=0 \Rightarrow x=\ldots$	M1
	$y=-\frac{166}{27},\ x=\frac{6889}{243}$	A1
		(4)
(b) ALT	$t=9 \Rightarrow 27x+3y=747$	B1
	$9x+y=249 \Rightarrow 3t^2+\frac{2}{3}t=249$	M1
	$9t^2+2t-747=0 \Rightarrow t=\ldots\left(-\frac{83}{9}\right)$	M1
	$y=-\frac{166}{27},\ x=\frac{6889}{243}$	A1
		(Total 7 marks)

Notes

(a)

B1: Correct $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of t from a calculus method. Must have seen a derivative used.

M1: Correct straight line method for the normal – must be using $\frac{-1}{\text{their } m_T}$ (or other correct approach). If using $y=mx+c$ they must proceed at least as far as finding c.

A1: cso – must have seen the evidence of use of calculus.

(b) + Alt (b)

B1: Correct equation for the normal at $t=9$

M1: Solves normal and equation of C simultaneously to obtain a quadratic equation in x or y or substitutes the parametric form to obtain a quadratic in t.

M1: Solves 3TQ in y to obtain a value (other than 6) or in x to obtain a value (other than 27) or in t to obtain a value (other than 9)

A1: Both coordinates correct and no incorrect ones (but ignore (27,6))		
Question	**Scheme**	**Marks**
8.(a)	$\sum_{r=1}^{n} r(2r^2-3r-1) = \sum_{r=1}^{n}(2r^3-3r^2-r)$ $=2\times\frac{1}{4}n^2(n+1)^2-3\times\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)$	**M1** **A1**
	$\frac{1}{2}n^2(n+1)^2-\frac{1}{2}n(n+1)(2n+1)-\frac{1}{2}n(n+1)$ $=\frac{1}{2}n(n+1)[n(n+1)-(2n+1)-1]$	**M1**
	$=\frac{1}{2}n(n+1)[n^2-n-2]=\frac{1}{2}n(n+1)(n+1)(n-2)$ $=\frac{1}{2}n(n+1)^2(n-2)$*	**A1***
		(4)
(b)	$\sum_{r=n}^{2n} r(2r^2-3r-1)=\sum_{r=1}^{2n} r(2r^2-3r-1)-\sum_{r=1}^{n-1} r(2r^2-3r-1)$ $=\frac{1}{2}(2n)(2n+1)^2(2n-2)-\frac{1}{2}(n-1)(n)^2(n-3)$	**M1**
	$=\frac{1}{2}n(n-1)\left[4(2n+1)^2-n(n-3)\right]$	**M1**
	$=\frac{1}{2}n(n-1)\left[15n^2+19n+4\right]$	**A1**
	$=\frac{1}{2}n(n-1)(15n+4)(n+1)$	**A1**
		(4)
		(Total 8 marks)

Notes

(a) Note – attempts at induction score no marks.

M1: Expands the bracket and attempt to use at least one of the standard formulae correctly.

A1: Fully correct expression

M1: Attempts to factorise out at least $n(n+1)$ - both terms must have been common factors in the terms of their expression. If expanded to a quartic, there must be a clear attempt at factorisation in stages, directly to the given answer will be M0A0. (Note if they try to find roots there needs to be evidence that −1 is a repeated root before going direct to the given answer from these.)

A1*: cso Must have achieved a suitable correct intermediate stage with a quadratic in their working.

(b)

M1: Applies f(2n) – f(k) where k is n – 1 or n with the formula from (a) or allow from restarts using the standard formulae.

dM1: Attempts to factorise out $n(n-1)$ - which must be factors of their expression, so use of f(2n) −f(n) will score dM0. Accept for this mark if they expand from a correct expression and achieve the correct answer.

A1: Correct quadratic factor. May be implied if expansion to a quartic achieves the correct answer without intermediate factorisation shown.

A1: Correct expression

Question	Scheme	Marks
9.(a)	$\frac{3z-1}{2}=\frac{\lambda+5\text{i}}{\lambda-4\text{i}}\times\frac{\lambda+4\text{i}}{\lambda+4\text{i}}$	**M1**
	$=\frac{\lambda^2+9\lambda\text{i}-20}{\lambda^2+16}$	**M1**
	$\frac{3z-1}{2}=\frac{\lambda^2+9\lambda\text{i}-20}{\lambda^2+16}\Rightarrow z=\frac{2\left(\frac{\lambda^2+9\lambda\text{i}-20}{\lambda^2+16}\right)+1}{3}$	**ddM1**
	$=\frac{\lambda^2-8}{\lambda^2+16}+\frac{6\lambda}{\lambda^2+16}\text{i}$	**A1**
		(4)
(a) **Way 2**	$\frac{3z-1}{2}=\frac{\lambda+5\text{i}}{\lambda-4\text{i}}\Rightarrow 3z=\frac{2\lambda+10\text{i}}{\lambda-4\text{i}}+1=\frac{3\lambda+6\text{i}}{\lambda-4\text{i}}$ $\Rightarrow 3z=\frac{3\lambda+6\text{i}}{\lambda-4\text{i}}\times\frac{\lambda+4\text{i}}{\lambda+4\text{i}}$ or $z=\frac{\lambda+2\text{i}}{\lambda-4\text{i}}\times\frac{\lambda+4\text{i}}{\lambda+4\text{i}}$	**M1**
	$3z=\frac{3\lambda^2+18\lambda\text{i}-24}{\lambda^2+16}$ or $z=\frac{\lambda^2+6\lambda\text{i}-8}{\lambda^2+16}$	**M1**
	$3z=\frac{3\lambda^2+18\lambda\text{i}-24}{\lambda^2+16}\Rightarrow z=\ldots$	**ddM1**
	$=\frac{\lambda^2-8}{\lambda^2+16}+\frac{6\lambda}{\lambda^2+16}\text{i}$	**A1**
(a) **Way 3**	$\frac{3z-1}{2}=\frac{\lambda+5\text{i}}{\lambda-4\text{i}}\Rightarrow(3x+3y\text{i}-1)(\lambda-4\text{i})=2\lambda+10\text{i}$ $\Rightarrow 3\lambda x-\lambda+12y+(4+3\lambda y-12x)\text{i}=2\lambda+10\text{i}$	**M1**
	$\Rightarrow 3\lambda x+12y=3\lambda,\ 3\lambda y-12x=6$	**M1**
	$\Rightarrow x=\ldots, y=\ldots$	**ddM1**
	$z=\frac{\lambda^2-8}{\lambda^2+16}+\frac{6\lambda}{\lambda^2+16}\text{i}$	**A1**
(b)	$\arg z=\frac{\pi}{4}\Rightarrow \text{Re}\,z=\text{Im}\,z\ (>0)\Rightarrow\lambda^2-6\lambda-8=0\Rightarrow\lambda=\ldots$ or $\arg z=\frac{\pi}{4}\Rightarrow\frac{6\lambda}{\lambda^2-8}=\tan\frac{\pi}{4}=1\Rightarrow\lambda^2-6\lambda-8=0\Rightarrow\lambda=\ldots$	**M1**
	(Also need $\text{Re}(z),\text{Im}(z)>0$, so $\lambda>0$) $\lambda=3+\sqrt{17}$	**A1**

		(2)
		(Total 6 marks)

Notes

(a)

M1: Multiplies rhs by $\frac{\lambda+4\text{i}}{\lambda+4\text{i}}$

M1: Applies $\text{i}^2=-1$ in both numerator and denominator and obtains a real number in the denominator.

ddM1: Rearranges to $z = \ldots$

A1: Correct and in the required form, but accept $\frac{\lambda^2-8+6\lambda\text{i}}{\lambda^2+16}$. Need not be fully simplified. Accept e.g $\frac{3\lambda^2-24}{3\lambda^2+48}+\frac{18\lambda}{3\lambda^2+48}\text{i}$

(a) Way 2

M1: Rearranges to $3z = \ldots$ (or $z = \ldots$) and multiplies numerator and denominator by the complex conjugate of their denominator.

M1: Applies $\text{i}^2=-1$ in both numerator and denominator and obtains a real number in the denominator.

ddM1: Rearranges to $z = \ldots$ if not already done so. If rearranged to z initially **M1ddM1** will be scored together.

A1: As per main scheme.

There may be variations on the rearrangement, but the key steps will remain the same.

(a) Way 3

M1: Cross multiplies, applies $z = x + \text{i}y$, expands and applies $\text{i}^2=-1$ to achieve Cartesian form terms.

M1: Equates real and imaginary parts to form two equations with real coefficients.

ddM1: Solves the equations simultaneously to find x and y in terms of λ.

A1: As per main scheme.

(b)

M1: Sets the imaginary part of z equal to their real part of z, or divides these and sets equal to 1, and forms and solves the resulting quadratic in λ. (Need not be real roots for the M.)

Watch for answers to (a) with a negative imaginary component that do not consider the minus sign, as these should score M0 as they have not set real and imaginary parts equal.

A1: Correct exact answer only. The negative solution must have been rejected. Allow if both numerators were correct in (a) if there was a slip in the denominator only (e.g. λ^2+4 or $\lambda+16$) or if they were only out by a positive scale factor (e.g. lost the 3).

Question	Scheme	Marks
10.(i)	$n=1 \Rightarrow 3^{n-1}\begin{pmatrix}2n+3 & -n\\ 4n & 3-2n\end{pmatrix}=3^0\begin{pmatrix}2(1)+3 & -1\\ 4(1) & 3-2(1)\end{pmatrix}=\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}=\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}^1$	**B1**
	Assume true for $n=k$ so that $\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}^k=3^{k-1}\begin{pmatrix}2k+3 & -k\\ 4k & 3-2k\end{pmatrix}$	
	$\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}^{k+1}=3^{k-1}\begin{pmatrix}2k+3 & -k\\ 4k & 3-2k\end{pmatrix}\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}$ or $\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}^{k+1}=3^{k-1}\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}\begin{pmatrix}2k+3 & -k\\ 4k & 3-2k\end{pmatrix}$	**M1**
	$=3^{k-1}\begin{pmatrix}10k+15-4k & -2k-3-k\\ 20k+12-8k & -4k+3-2k\end{pmatrix}$ or $3^{k-1}\begin{pmatrix}10k+15-4k & -5k-3+2k\\ 8k+12+4k & -4k+3-2k\end{pmatrix}$ or $3^{k-1}\begin{pmatrix}6k+15 & -3k-3\\ 12k+12 & -6k+3\end{pmatrix}$	**A1**
	$\left[=3^k\begin{pmatrix}2k+5 & -k-1\\ 4k+4 & -2k+1\end{pmatrix}\right]=3^k\begin{pmatrix}2(k+1)+3 & -(k+1)\\ 4(k+1) & 3-2(k+1)\end{pmatrix}$	**A1**
	If the result is true for $n=k$ then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n.	**A1cso**
		(5)
(ii)	$f(1)=8^3+6=518=74\times7$ (so true for $n=1$)	**B1**
	Assume true for $n=k$ so that $8^{2k+1}+6^{2k-1}$ is divisible by 7	
	$f(k+1)=8^{2k+3}+6^{2k+1}$	**M1**
	$=64\times\left(8^{2k+1}+6^{2k-1}\right)+\ldots$ or $36\times\left(8^{2k+1}+6^{2k-1}\right)+\ldots$	**dM1**
	$=64\times\left(8^{2k+1}+6^{2k-1}\right)-28\times6^{2k-1}$ or $36\times\left(8^{2k+1}+6^{2k-1}\right)+28\times8^{2k+1}$	**A1**
	So if the result is true for $n=k$ then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n.	**A1cso**
		(5)
(ii) Alt	$f(1)=8^3+6=518=74\times7$ (so true for $n=1$)	**B1**
	Assume true for $n=k$ so that $8^{2k+1}+6^{2k-1}$ is divisible by 7	
	$f(k+1)-Mf(k)=8^{2k+3}+6^{2k+1}-M\left(8^{2k+1}+6^{2k-1}\right)$	**M1**
	$=(64-M)\left(8^{2k+1}+6^{2k-1}\right)+\ldots$ or $(36-M)\left(8^{2k+1}+6^{2k-1}\right)+\ldots$	**dM1**
	$=(64-M)\left(8^{2k+1}+6^{2k-1}\right)-28\times6^{2k-1}$ or $(36-M)\left(8^{2k+1}+6^{2k-1}\right)+28\times8^{2k+1}$	**A1**
	$\Rightarrow f(k+1)-Mf(k)$ divisible by 7 $\Rightarrow f(k+1)$ divisible by 7. So if the result is true for $n=k$ then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n.	**A1cso**
		(Total 10 marks)

Notes
(i) **B1:** Shows the result is true for $n = 1$. The LHS may just be stated, for the RHS accept as a minimum either one unsimplified term or the 3^0 seen. (Conclusion not needed here if both sides have been found correctly.) **M1:** Attempts $\begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}^{k+1}$ either way round using the result for $n = k$. **A1:** Correct unsimplified matrix. The coefficients inside may be simplified but the common factor 3 not taken out directly. **A1:** Achieves this result with no errors, via $3^{k-1}\begin{pmatrix} 6k+15 & -3k-3 \\ 12k+12 & -6k+3 \end{pmatrix}$ or $3^k\begin{pmatrix} 2k+5 & -k-1 \\ 4k+4 & -2k+1 \end{pmatrix}$ (oe with simplified linear terms). **A1cso:** Suitable conclusion following fully correct work. Must include in some form the points "true for n =1", "true for $n = k$ implies true for $n = k + 1$" and conclude true for all n in the conclusion. Depends on the preceding MAA marks and at least stating the correct matrix for n = 1in the initial base case check (So B0M1A1A1A1 is possible). (ii) **B1:** Shows the result is true for $n = 1$, Must express as a multiple of 7 or clearly show the factor. **M1:** Attempts $f(k + 1)$ **dM1:** Attempts to express $f(k + 1)$ in terms of $f(k)$. Note they may let $f(k) = 7m$ where m is an integer and use this in the working. **A1:** Correct expression for $f(k + 1)$ in terms of $f(k)$ (or m) **A1cso:** Suitable conclusion following fully correct work. Must include in some form the points "true for n =1", "true for $n = k$ implies true for $n = k + 1$" and conclude true for all n in the conclusion. Depends on the preceding MdMA marks and finding at least $f(1) = 518$ (So B0M1dM1A1A1 is possible if all that is missing is showing the factor 7 in f(1)). (ii) Alt **B1:** Shows the result is true for $n = 1$. Must express as a multiple of 7 or clearly show the factor. **M1:** Attempt $f(k+1) - Mf(k)$ for any integer M. If $M = 0$ this is the main scheme. $M = 1$ may be seen frequently, but other value are possible. **dM1:** Attempts to express $f(k+1) - Mf(k)$ in terms of $f(k)$ or otherwise show a common factor of 7. **A1:** Correct expression for $f(k+1) - Mf(k)$ in terms of $f(k)$ or with clear common factor of 7 shown. Note if $M = 1$ is used, the expression becomes $63\times8^{2k+1}+35\times6^{2k-1}=7\left(9\times8^{2k+1}+5\times6^{2k-1}\right)$ which is fine for dM1A1. **A1cso:** Refers to divisibility of $f(k+1)$ and makes suitable conclusion following fully correct work. Must include in some form the points "true for n =1", "true for $n = k$ implies true for $n = k + 1$" and conclude true for all n in the working. Depends on the preceding MdMA marks and finding at least $f(1) = 518$ (So B0M1dM1A1A1 is possible if all that is missing is showing the factor 7 in f(1)).