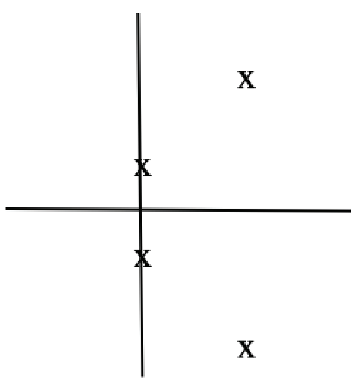
Question Number	Scheme	Notes	Marks
1	$\sum_{r=1}^{n} r^2(r+2) = \sum_{r=1}^{n} r^3 + 2\sum_{r=1}^{n} r^2$ or $\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} 2r^2$	Correct split with 2 summations. Could be implied by correct work. Condone missing or incorrect summation limits.	B1
	$= \frac{1}{4}n^2(n+1)^2 + 2\times\frac{1}{6}n(n+1)(2n+1)$	Attempts to use both standard results and obtains an expression of the form $pn^2(n+1)^2 + qn(n+1)(2n+1)$ $p, q \neq 0$ Could be implied by immediate expansion	M1
	$= \frac{1}{12}n(n+1)\left[3n(n+1)+4(2n+1)\right]$ $= \frac{1}{12}n(n+1)(3n^2+11n+4)$	**d**M1: Attempts factorisation to obtain $\frac{1}{12}n(n+1)(an^2+bn+c)$ $a,b,c \neq 0$. Condone poor algebra. Could follow cubic or quartic. Allow a consistent $a = ...$, $b = ...$, $c = ...$ if quadratic never seen simplified **Requires previous M mark.** A1: Correct expression or $a=3,\ b=11,\ c=4$ Allow e.g., $\frac{1}{12}n(n+1)$ written as $\frac{n}{12}(n+1)$	**d**M1 A1
	Note: $n(n+1)(3n^2+11n+4) = 3n^4+14n^3+15n^2+4n$		**Total 4**

Question Number	Scheme	Notes	Marks
2	$2x^4-8x^3+29x^2-12x+39=0,\quad x=2+3\text{i}$ Condone work in e.g., z throughout		
(a)	2–3i	Correct conjugate	B1
			(1)
(b)	$\big(x-(2-3\text{i})\big)\big(x-(2+3\text{i})\big)$ or $(x-2+3\text{i})(x-2-3\text{i})$ $=\ldots\quad\{x^2-4x+13\}$ or sum $=4$, product $=13$ $\Rightarrow x^2\pm4x\pm13$ or $x^2\pm13x\pm4$ or $x^2-(2+3\text{i}+2-3\text{i})x+(2+3\text{i})(2-3\text{i})$ $\Rightarrow\ldots\quad\{x^2-4x+13\}$	Attempts to multiply the two correct factors to obtain a 3 term quadratic with real coefficients. Could use $(x-2)^2=(\pm3\text{i})^2$ or $x^2-2ax+a^2+b^2$ with $a=2,\ b=\pm3$ Or uses the correct sum and product of the roots to obtain an expression of the form shown (must be some minimal working – but if just a quadratic is given the next 2 marks are available) or $x^2-(\alpha+\beta)x+\alpha\beta$ to obtain a 3 term quadratic with real coefficients.	M1
	$2x^4-8x^3+29x^2-12x+39\Rightarrow(x^2-4x+13)(2x^2+3)$	Uses their 2 or 3 term quadratic factor with real coefficients to obtain a second 2 or 3 term quadratic of the form $2x^2+\ldots$ by long division, equating coefficients or inspection. Ignore any remainder from long division. Can follow M0	M1
	$2x^2+3(=0)\Rightarrow$ $x=\pm\dfrac{\sqrt{6}}{2}\text{i}$ or $\pm\text{i}\sqrt{\dfrac{3}{2}}$ or $\pm\dfrac{\sqrt{3}}{\sqrt{2}}\text{i}$ or $\sqrt{1.5}\,\text{i}$ $\sqrt{1.5\text{i}}$ is M0 1.2247...i is M1 A0	**d**M1: Solves their second quadratic factor = 0. If 2 term must get one correct non-zero root. (Usual rules if 3TQ and one correct root if no working) Could be inexact. **Requires previous method mark.** A1: Both correct exact roots with "i" **Requires all previous marks.**	**d**M1 A1
	Solving by calculator, sometimes followed by attempts to reconstruct factors. e.g., $\text{f}(x)=(x^2-4x+13)\left(x^2+\dfrac{3}{2}\right)$ is first M1 only and working for the 3TQ must be seen		**(4)**
(c)		Allow ft on their **answers to (b)** if they are of the form $\pm k\text{i}$ or $\pm k\sqrt{-1},\ k\neq0$ regardless of how they were obtained 1st B1: One of the two pairs of roots in correct positions 2nd B1: Both pairs of roots in correct positions and correct relative to each other for their k Allow any suitable indication of the roots such as vectors. Ignore all labelling and scaling but each pair should be reasonably symmetric in x-axis for any marks (for each pair -distance of one to x-axis not less than $\frac{1}{2}$ of the other)	B1 B1 (ft on (b))
			(2)
			Total 7

Question Number	Scheme	Notes	Marks
3(a)	$y=9x^{-1}\Rightarrow\dfrac{dy}{dx}=-9x^{-2}\quad\left\{=-\dfrac{9}{(3t)^2}\right\}$ or $xy=9\Rightarrow x\dfrac{dy}{dx}+y=0\Rightarrow\dfrac{dy}{dx}=-\dfrac{y}{x}\quad\left\{=-\dfrac{\frac{3}{t}}{3t}\right\}$ or $x=3t,\ y=3t^{-1}\Rightarrow\dfrac{dx}{dt}=3,\ \dfrac{dy}{dt}=-3t^{-2}\Rightarrow\dfrac{dy}{dx}=\dfrac{-3t^{-2}}{3}$	Any correct expression for $\dfrac{dy}{dx}$ but allow e.g., $\dfrac{dx}{dy}=-9y^{-2}$ Calculus must be seen so there is no credit for just a statement e.g., $m_T=-\dfrac{1}{t^2}$	B1
	e.g., $m_N=\dfrac{(3t)^2}{9}$ or $\dfrac{3t}{\frac{3}{t}}$ or $\dfrac{3}{3t^{-2}}\quad\left\{=t^2\right\}$	Uses the perpendicular gradient rule to obtain the gradient of the normal in terms of t correct for their m_T Implied by correct use of $-\frac{dx}{dy}$	M1
	$y-\dfrac{3}{t}=t^2(x-3t)$ or $\dfrac{3}{t}=t^2(3t)+c\Rightarrow c=\ldots$ $\left\{c=\dfrac{3}{t}-3t^3\right\}$	Applies straight line method correctly with their normal (changed) gradient in terms of t. If using $y=mx+c$ coordinates must be correctly placed and $c=\ldots$ reached	M1
	$ty-t^3x=3-3t^4$ Intermediate step not required. Allow recovery from a slip.	Correct equation or f(t). Must be seen in (a). Accept equivalents for f(t) e.g., $3(1-t^4),\ -3(t^4-1)$	A1
	Allow work with $xy=c^2$ but the final mark requires use of $c^2=9$ No calculus scores a maximum of 0111 if m_T is stated and 0011 if m_N is stated		(4)
(b)	$xy=9,\ 2y-8x=3-3\times16$ e.g.,$\Rightarrow y=4x-\dfrac{45}{2}$ or $x=\dfrac{45}{8}+\dfrac{y}{4}$ $\Rightarrow x\left(4x-\dfrac{45}{2}\right)=9$ or $y\left(\dfrac{45}{8}+\dfrac{y}{4}\right)=9$	Uses $t=2$ in their $ty-t^3x=\mathrm{f}(t)\neq0$ and the equation of H to obtain an unsimplified three term quadratic equation in x or y (no variables in denominators). Only allow $\mathrm{f}(t)=\frac{9}{t}$ if stated first	M1
	$8x^2-45x-18=0$ or $2y^2+45y-72=0$ $\{\Rightarrow(8x+3)(x-6)=0$ or $(2y-3)(y+24)=0\}$ $\Rightarrow x=\ldots\ \left\{-\frac{3}{8},\ 6\right\}$ or $y=\ldots\ \left\{\frac{3}{2},\ -24\right\}$	Solves their 3TQ to find a value for x or y – apply usual rules. One root correct if no working. Can award for P provided it has come from quadratic. **Requires previous method mark.**	**d**M1
	$\left(-\dfrac{3}{8},-24\right)$ or $(-0.375,-24)$	Correct exact coordinates in simplest form from correct work. Allow $x=\ldots$, $y=\ldots$ Ignore $\left(6,\ \frac{3}{2}\right)$ but A0 for any other point shown or incorrect x or y value.	A1
	Solving in terms of t: M1: $\Rightarrow$Unsimplified 3TQ e.g., $t^2x^2+\left(\frac{3}{t}-3t^3\right)x-9=0$ M1 M1: Solves e.g, $x=\dfrac{-\frac{3}{t}+3t^3\pm\sqrt{\left(\frac{3}{t}-3t^3\right)^2+36t^2}}{2t^2}\left\{\Rightarrow\left(-\frac{3}{t^3},\ -3t^3\right)\right\}$ A1 :$t=2\Rightarrow\left(-\frac{3}{8},\ -24\right)$		(3)
	Correct final answer with no incorrect work is 111 provided f(t) was correct		**Total 7**

Question Number	Scheme	Notes	Marks
4	$\mathbf{A}=\begin{pmatrix}-3 & 8\\ -3 & k\end{pmatrix}$ $\quad\mathbf{B}=\begin{pmatrix}a & -4\\ 2 & 3\end{pmatrix}$ $\quad\mathbf{BC}=\begin{pmatrix}2 & 5 & 1\\ 1 & 4 & 2\end{pmatrix}$		
(i)	$\det\mathbf{A}=-3k-8(-3)\ \{=-3k+24\}$ Could be implied	Attempts det **A** and obtains $\pm3k\pm8(\pm3)$ or $\pm3k\pm24$	M1
	$-3k+24=3$ or $-3k+24=-3$ $\Rightarrow k=\dots$ May see $(-3k+24)^2=+\mathbf{9}\Rightarrow 9k^2-144k+567=0\Rightarrow\dots$	Equates their det **A** of form $ak+b\ \ a,b\neq0$ to 3 or –3 or equivalent work and solves for k (usual rules if quadratic and must use +9)	M1
	$k=7,\ k=9$ 1st A1: Either correct value of k from correct work. Allow e.g., $\frac{-21}{-3}$ or $\frac{-27}{3}$ 2nd A1: Both correct values of k from correct work. 7 and 9 only. No extra		A1 A1
			(4)
(ii)	$\det\mathbf{B}=1\times3a-(-4)\times2\ \{=3a+8\}$	Correct unsimplified expression for det **B**. Could be implied	B1
	$\mathbf{B}^{-1}=\dfrac{1}{\text{"}3a+8\text{"}}\begin{pmatrix}3 & 4\\ -2 & a\end{pmatrix}$	Correct $\mathbf{B}^{-1}$ with their det **B**. Adj(**B**) to be correct but allow elements to have their det **B** as denominators if incorporated.	M1
	$\mathbf{C}=\mathbf{B}^{-1}\mathbf{BC}=\dfrac{1}{3a+8}\begin{pmatrix}3 & 4\\ -2 & a\end{pmatrix}\begin{pmatrix}2 & 5 & 1\\ 1 & 4 & 2\end{pmatrix}=\dots$ Access to this mark is allowed if there is no determinant or if $\mathbf{B}^{-1}=$ det **B** × Adj(**B**) used	Multiplies **BC** by their $\mathbf{B}^{-1}$ (changed – and not just by incorporation of their determinant) **the correct way round.** Expect four correct elements for their matrices if the method is unclear. The incorrect order scores M0 even if the correct result is obtained.	M1
	$\mathbf{C}=\dfrac{1}{3a+8}\begin{pmatrix}10 & 31 & 11\\ a-4 & 4a-10 & 2a-2\end{pmatrix}$ Ignore any reference to inapplicable values of a $\left(a\neq-\frac{8}{3}\right)$	Correct **C** or equivalent with like terms collected and single fractions if necessary. e.g., $\begin{pmatrix}\dfrac{10}{3a+8} & \dfrac{31}{3a+8} & \dfrac{11}{3a+8}\\ \dfrac{a-4}{3a+8} & \dfrac{2(2a-5)}{3a+8} & \dfrac{2(a-1)}{3a+8}\end{pmatrix}$	A1
			(4)
Alt **Sim. equations**	$\begin{pmatrix}a & -4\\ 2 & 3\end{pmatrix}\begin{pmatrix}p & q & r\\ s & t & u\end{pmatrix}=\begin{pmatrix}2 & 5 & 1\\ 1 & 4 & 2\end{pmatrix}\Rightarrow\begin{matrix}ap-4s=2 & aq-4t=5 & ar-4u=1\\ 2p+3s=1 & 2q+3t=4 & 2r+3u=2\end{matrix}$ Multiplies in the correct order to obtain at least **three** correct equations		B1
	$(3a+8)p=10\quad(3a+8)q=31\quad(3a+8)r=11$ $p=\dfrac{10}{3a+8}\quad q=\dfrac{31}{3a+8}\quad r=\dfrac{11}{3a+8}$ $s=\dfrac{1}{3}\left(1-\dfrac{20}{3a+8}\right)\quad t=\dfrac{1}{3}\left(4-\dfrac{62}{3a+8}\right)\quad u=\dfrac{1}{3}\left(2-\dfrac{22}{3a+8}\right)$ $\Rightarrow\begin{pmatrix}\dfrac{10}{3a+8} & \dfrac{31}{3a+8} & \dfrac{11}{3a+8}\\ \dfrac{a-4}{3a+8} & \dfrac{4a-10}{3a+8} & \dfrac{2a-2}{3a+8}\end{pmatrix}$ $s=\dfrac{a-4}{3a+8}\quad t=\dfrac{4a-10}{3a+8}\quad u=\dfrac{2a-2}{3a+8}$ M1: Solves their equations to find expressions in terms of a for three elements M1: Finds expressions in terms of a for all six elements A1: Correct matrix – like terms collected and single fractions		M1 M1 A1
			Total 8

Question Number	Scheme	Notes	Marks
5	Solutions that rely entirely on solving the equation are generally unlikely to score but there may be attempts which include some of the work below which can receive appropriate credit.		
(a)	$\alpha+\beta=6 \quad \alpha\beta=3$	Correct sum and product. Could be implied. Allow $\frac{6}{1}$ and $\frac{3}{1}$	B1
	$(\alpha^2+1)(\beta^2+1)=\alpha^2\beta^2+\alpha^2+\beta^2+1$	Multiplies $(\alpha^2+1)(\beta^2+1)$ to obtain 3 or 4 terms with 3 correct. Do not condone $\alpha\beta^2$ for $(\alpha\beta)^2$ unless implied later	M1
	$=\alpha^2\beta^2+(\alpha+\beta)^2-2\alpha\beta+1$	Uses $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	M1
	$\{=3^2+6^2-2\times3+1\}$ $=40$	Correct answer from correct work. Use of e.g., $\alpha+\beta=-6$ is A0	A1
			(4)
(b)	Allow use of their $(\alpha^2+1)(\beta^2+1)$ which could be from (a) or a first or reattempt in (b). Numerator must be correct		
	$\frac{\alpha}{(\alpha^2+1)}+\frac{\beta}{(\beta^2+1)}=\frac{\alpha(\beta^2+1)+\beta(\alpha^2+1)}{"(\alpha^2+1)(\beta^2+1)"}$	Any correct expression with their $(\alpha^2+1)(\beta^2+1)$ for the new sum as a single fraction (or two fractions both with the common denominator)	B1
	$=\frac{\alpha\beta(\beta+\alpha)+(\alpha+\beta)}{"(\alpha^2+1)(\beta^2+1)"}=\frac{"3"\times"6"+"6"}{"40"}=\dots$	Uses a correct expression with their $(\alpha^2+1)(\beta^2+1)$ for the new sum to obtain a correct numerical expression with their denominator, $\alpha+\beta$ & $\alpha\beta$ and achieves a value.	M1
	$\frac{\alpha\beta}{"(\alpha^2+1)(\beta^2+1)"}=\frac{"3"}{"40"}$	Uses a correct expression with their $(\alpha^2+1)(\beta^2+1)$ for the new product to obtain a correct value with their denominator and $\alpha\beta$	M1
	new sum $=\frac{24}{40}\left\{=\frac{3}{5}\right\}$ **or** new product $=\frac{3}{40}$	One value for new sum or new product correct. Any equivalent fractions. Not ft. **Requires appropriate previous M mark.**	A1
	$x^2-\frac{24}{40}x+\frac{3}{40}\quad\{=0\}$	Correctly uses $x^2-(\text{sum of roots})x+(\text{product of roots})$ or equivalent work with their new sum and product. Condone use of a different variable. Allow appropriate values for p, q and r	M1
	$40x^2-24x+3=0$	Any correct equation with integer coefficients and "= 0". Condone use of a different variable. Allow e.g., $p=40$, $q=-24$, $r=3$. Requires all marks.	A1
			(6)
	Note that although $(\alpha^2+1)(\beta^2+1)$ may be attempted or reattempted in (b) there is no credit for work in (a) that is only seen in (b)		**Total 10**

Question Number	Scheme	Notes	Marks
6(a)	$\lvert z_1+z_2\rvert\left\{=\lvert 3+2\mathrm{i}+2+3\mathrm{i}\rvert=\lvert 5+5\mathrm{i}\rvert\right\}=\sqrt{5^2+5^2}$	Attempts the sum (allow one slip) and uses Pythagoras correctly	M1
	$\sqrt{50}$ or $5\sqrt{2}$	Either correct exact answer	A1
	Answer only is no marks but working can be minimal e.g., $\lvert 5+5\mathrm{i}\rvert=5\sqrt{2}$		**(2)**
(b)	$\frac{z_2z_3}{z_1}=\frac{(2+3\mathrm{i})(a+b\mathrm{i})}{(3+2\mathrm{i})}=\frac{(2+3\mathrm{i})(a+b\mathrm{i})}{(3+2\mathrm{i})}\times\frac{(3-2\mathrm{i})}{(3-2\mathrm{i})}$ or $\frac{z_2}{z_1}=\frac{2+3\mathrm{i}}{3+2\mathrm{i}}\times\frac{3-2\mathrm{i}}{3-2\mathrm{i}}$ or $\frac{z_3}{z_1}=\frac{a+b\mathrm{i}}{3+2\mathrm{i}}\times\frac{3-2\mathrm{i}}{3-2\mathrm{i}}$	Substitutes complex numbers and correct multiplier to rationalise the denominator seen or implied. See note below Could use $\times\frac{-3+2\mathrm{i}}{-3+2\mathrm{i}}$	M1
	$(3+2\mathrm{i})(3-2\mathrm{i})=13$	13 <u>obtained from</u> $(3+2\mathrm{i})(3-2\mathrm{i})$ Could be implied.	B1
	$\frac{z_2z_3}{z_1}=\frac{12a-5b}{13}+\frac{5a+12b}{13}\mathrm{i}$ or $\frac{1}{13}(12a-5b)+\frac{\mathrm{i}}{13}(5a+12b)$ or $\frac{12}{13}a-\frac{5}{13}b+\mathrm{i}\left(\frac{5}{13}a+\frac{12}{13}b\right)$ etc. Condone $\frac{(12a-5b)+(5a+12b)\mathrm{i}}{13}$	**d**M1: Attempts to simplify the numerator and collects terms to obtain $pa+qb+ra\mathrm{i}+sb\mathrm{i}$ with at least three of p, q, r and s non-zero. **Requires previous M mark**. A1: Correct answer in any form with a single "i". Correct bracketing where needed. Allow $x=\ldots, y=\ldots$	**d**M1 A1
	Note: The following marks are accessible if complex numbers are substituted in the wrong places: z_2 as denominator max 1010, z_3 as denominator max 1000		**(4)**
(c)	$\frac{12a-5b}{13}=\frac{4}{13},\ \frac{5a+12b}{13}=\frac{58}{13}\ \Rightarrow\ a=\ldots,\ b=\ldots$	Equates their x to $\frac{4}{13}$ and their y to $\frac{58}{13}$ to obtain 2 linear equations in both a and b and solves to obtain values for both a and b.	M1
	No need to check values but must be some working between equations and values. "$\frac{12a-5b}{13}=\frac{4}{13},\ \frac{5a+12b}{13}=\frac{58}{13}\quad 12a-5b=4,\ 5a+12b=58\quad a=2,\ b=4$" is M0A0 Values can immediately follow if equations are produced with coefficients of a or b of the same magnitude		
	$a=2$ and $b=4$	Correct values for a and b from correct equations with working.	A1
	SC: Allow access to both marks for the exact $a=-\frac{242}{169}$ and $b=\frac{716}{169}$ from using $w=\frac{z_1z_3}{z_2}=\frac{12a+5b}{13}+\frac{12b-5a}{13}\mathrm{i}$ There are no marks in (c) if z_3 was used as the denominator in (b) [leads to a = b = 0]		**(2)**
(d)	$\arctan\left(\frac{\frac{58}{13}}{\frac{4}{13}}\right)\{=1.5019\ldots \text{ or } 86.05\ldots^\circ\}$ or $\arctan\left(\frac{\frac{4}{13}}{\frac{58}{13}}\right)\{=0.068856\ldots \text{ or } 3.945\ldots^\circ\}$	Either correct arctan or $\tan^{-1}$ seen or implied by a correct 2sf value (awrt 1.5, 86, 0.069/0.068, 3.9) Could use equivalent trig. Note: $\tan\frac{58}{4}=-2.634$ or 0.258	M1
	1.502	1.502 only (**not** awrt) Mark final answer if 1.502 is followed by e.g., $\frac{\pi}{2}-1.502=0.06880$	A1
			(2)
			Total 10

Question Number	Scheme	Notes	Marks
7(a)	$\mathrm{f}(x)=x^{\frac{3}{2}}+x-3$ $\mathrm{f}(1)=1+1-3=-1 \quad \mathrm{f}(2)=\sqrt{8}+2-3=1.828...$	Calculates values for both f(1) and f(2) with one correct. Allow e.g., $\mathrm{f}(2)=2\sqrt{2}-1$ or awrt 2	M1
	f is **continuous** and **changes sign**, so **root** or α in [1, 2]. Correct interval [1, 2] if given. Sign change can be implied by "negative, positive", "f(1) < 0, f(2) > 0" or "f(1)f(2) < 0"	**Correct values** and sight of **continuous**, **sign change** and e.g., **root/shown/QED/true/proven/✓**	A1
			(2)
(b) **Work may be seen in a table**	$\mathrm{f}(1.5)=1.5^{\frac{3}{2}}+1.5-3 \quad \{=...0.3371...\}$	Obtains a <u>numerical expression or value</u> for f (1.5)	M1
	$\mathrm{f}(1.25)=1.25^{\frac{3}{2}}+1.25-3=... \quad \{-0.3524...\}$	Obtains a <u>value</u> for f(1.25). **Requires previous M mark.**	**d**M1
	$\Rightarrow$ root/α/x/it's in/on/$\in$ [1.25, 1.5] or "**in** [1.25, 1.5]" or $1.25 \leqslant$ root/α/$x \leqslant 1.5$	Correct values (awrt 0.3 and –0.3 or –0.4) and suitable conclusion. Allow "between $\frac{5}{4}$ and $\frac{3}{2}$ **inclusive**"	A1
	Do not accept [1.5, 1.25]. Just "f(1.25) = ... **followed by** f(1.5) = ... so..." is 100 if no evidence of interval bisection. There are no marks if it is a clear attempt at interpolation.		**(3)**
(c)(i)	$\mathrm{f}'(x)=\frac{3}{2}x^{\frac{1}{2}}+1$	Correct differentiation. Any correct equivalent e.g., $1.5\sqrt{x}+1$	B1
(ii)	$\alpha \approx 1.375-\dfrac{1.375^{\frac{3}{2}}+1.375-3}{\text{"}\frac{3}{2}\times 1.375^{\frac{1}{2}}+1\text{"}}=...$ $\left\{=1.375-\dfrac{-0.01266958256...}{2.75890591...}=1.375+0.004592248875... =1.379592249...\right\}$ $\left\{\text{exact values}: \dfrac{11}{8}-\dfrac{11\sqrt{22}-52}{32}\div\dfrac{8+3\sqrt{22}}{8}\right\}$	Correctly applies the Newton-Raphson formula with 1.375 & their $\mathrm{f}'(x)$ **and** obtains a value. Some working must be seen unless approx. root is seen correct to 6 d.p. accuracy (1.379592) or better. Allow "$...=1.375-\dfrac{\mathrm{f}(1.375)}{\mathrm{f}'(1.375)}$" followed by value but formula must be **fully** substituted if just followed by value unless "x_0" defined	M1
	awrt 1.380 or "1.38" (Ignore further iterations)	No clearly incorrect work.	A1
	NB Actual root is 1.379589808. Answer only is no marks.		**(3)**
(d)	e.g., $\dfrac{\alpha-1.25}{1.5-\alpha}=\dfrac{0.3524575141...}{0.3371173071...}$ or e.g., $\dfrac{1.5-\alpha}{0.337...}=\dfrac{1.5-1.25}{0.337...+0.352...}$	Forms an equation in e.g., α with their f(1.25) and f(1.5) allowing for sign errors only but must be using differences. Allow use of "f(1.25)" and "f(1.5)"- could recover sign error	M1
	$\alpha=1.377780737...=1.378$	**d**M1: Solves $\Rightarrow$ value **Requires previous M mark.** A1: awrt 1.378	**d**M1 A1
	May use a formula. Allow work in, e.g., x for all marks. No working required for 2nd M		**(3)**
Alt (Equation of line methods)	or $y-(-0.3524...[\text{or } 0.3371...])=\dfrac{0.3371...-(-0.3524...)}{1.5-1.25}(x-1.25[\text{or} 1.5])$ or $-0.3524...[\text{or } 0.3371...]=\dfrac{0.3371...-(-0.3524...)}{1.5-1.25}(1.25[\text{or} 1.5])+c \Rightarrow c=...$ A full method to determine the equation of the line using their f(1.25) and f(1.5) allowing for sign errors only (but allow subsequent errors finding c if $y=mx+c$ used)		M1
	$\{\Rightarrow y=2.758...x-3.800...\}$ $\alpha=1.377780737...=1.378$	**d**M1: Puts $y=0$ and solves $\Rightarrow$ value **Requires previous M mark.** A1: awrt 1.378	**d**M1 A1 **(3)**
	May use a formula. Allow work in, e.g., x for all marks. No working required for 2nd M		**Total 11**

Question Number	Scheme	Notes	Marks
8	$y^2 = 8x \quad P\left(2p^2, 4p\right) \quad Q\left(\dfrac{2}{p^2}, \dfrac{-4}{p}\right)$		
	Each part is marked separately. For example there is no credit in (c) for work seen in (b) unless that work is referred to in (c)		
(a) **Subs. both x and y into** $y^2 = 8x$	LHS or $y^2 \left\{=\left(\dfrac{-4}{p}\right)^2\right\} = \dfrac{16}{p^2}$ RHS or $8x\left\{=8\times\dfrac{2}{p^2}\right\} = \dfrac{16}{p^2}$ So Q lies on the parabola* Allow e.g., $\left(\dfrac{-4}{p}\right)^2 = 8\left(\dfrac{2}{p^2}\right) \Rightarrow \dfrac{16}{p^2} = \dfrac{16}{p^2} \Rightarrow$ true	Substitutes both coordinates of Q into the parabola equation, obtains e.g., $\dfrac{16}{p^2}$ twice and makes minimal conclusion - e.g., shown/QED/true/proven/✓ Sight of just "$y^2 = 8x$" is insufficient but allow "$y_Q{}^2 = 8x_Q$"	B1*
			(1)
Alt **Subs. x or y to find y or x**	$x = \dfrac{2}{p^2} \Rightarrow y^2 = 8\times\dfrac{2}{p^2}$ or $\dfrac{16}{p^2} \Rightarrow y = \dfrac{-4}{p}$ or $\pm\dfrac{4}{p}$ or $y = \dfrac{-4}{p} \Rightarrow \dfrac{16}{p^2} = 8x \Rightarrow x = \dfrac{2}{p^2}$ So Q lies on the parabola*	Substitutes one coordinate of Q into the parabola equation to correctly find the other coordinate and makes minimal conclusion - e.g., - e.g., shown/QED/true/proven/✓ Sight of just "$y^2 = 8x$" is insufficient but allow "$y_Q{}^2 = 8x_Q$"	B1*
			(1)
8(b)	Focus is (2, 0) or $x = 2, y = 0$ Could be seen on a diagram	Correct focus seen or used. Condone (0, 2) if $x = 2, y = 0$ used but award final A0	B1
	gradient of $PQ = \dfrac{4p + \frac{4}{p}}{2p^2 - \frac{2}{p^2}}$ or $\dfrac{-\frac{4}{p} - 4p}{\frac{2}{p^2} - 2p^2}$ $\left\{= \dfrac{4p^3 + 4p}{2p^4 - 2} = \dfrac{2p^3 + 2p}{p^4 - 1} = \dfrac{2p\left(p^2+1\right)}{p^4 - 1} = \dfrac{2p}{p^2 - 1}\right\}$	Attempts the gradient of PQ condoning one term of incorrect sign. Allow this mark is they subsequently attempt to convert it to a normal gradient. Note that m may be obtained from $4p = 2mp^2 + c,\ -\frac{4}{p} = \frac{2m}{p^2} + c \Rightarrow m = ...$	M1
	e.g., $y - 4p = \dfrac{4p + \frac{4}{p}}{2p^2 - \frac{2}{p^2}}\left(x - 2p^2\right)$	Any correct equation for PQ. May use Q. Allow this mark to be implied if their equation would have been correct but errors were made **simplifying** a correct gradient.	A1
	If $y = mx + c$ is used, one of the following expressions oe for c must be reached following correct gradient seen: $c = 4p - 2p^2\left(\text{gradient}\right)$ or $c = \dfrac{-4}{p} - \dfrac{2}{p^2}\left(\text{gradient}\right)$		
	Examples with fully simplified gradient (see overleaf for a fuller list): $x = 2 \Rightarrow y - 4p = \dfrac{2p}{p^2-1}\left(2 - 2p^2\right) \Rightarrow y = \dfrac{4p - 4p^3 + 4p^3 - 4p}{p^2 - 1} = 0$ or $y - 4p = \dfrac{2p}{p^2-1}\left(2 - 2p^2\right) \Rightarrow y - 4p = -4p \Rightarrow y = 0$ $y = 0 \Rightarrow -4p = \dfrac{2p}{p^2-1}\left(x - 2p^2\right) \Rightarrow x = \dfrac{-4p^3 + 4p + 4p^3}{2p} = 2$ $(2,0) \Rightarrow -4p = \dfrac{2p}{p^2-1}\left(2 - 2p^2\right) \Rightarrow -4p = -4p$	$\Rightarrow$ So PQ passes through the focus*	A1*

<table>
<tr><td></td><td colspan="2">Substitutes $x = 2$ and shows $y = 0$ or vice versa or substitutes both values and shows that the equation is true. Must have minimal conclusion e.g., shown/QED/true/proven/✓ and no incorrect work. Condone no conclusion if the mark in (a) was withheld for this reason only. The examples indicate the minimum level of algebra acceptable. With the exception of using (2, 0) with a fully simplified gradient, <u>look for substitution into the line followed by a further step which shows an expression that clearly leads to 0, 2 or e.g., $-4p$ or "1=1" followed by a minimal conclusion</u></td><td></td></tr>
<tr><td></td><td colspan="2">Work in "a" can only access the accuracy marks when $a = 2$ is substituted</td><td>(4)</td></tr>
<tr><td rowspan="3">Alt 1

Grad PF = Grad QF</td><td>Focus is (2, 0) or $x = 2, y = 0$
Could be seen on a diagram</td><td>Correct focus seen or used.
Condone (0, 2) if $x = 2, y = 0$ used but award final A0</td><td>B1</td></tr>
<tr><td>gradient $PF = \dfrac{4p}{2p^2-2}$ or $\dfrac{-4p}{2-2p^2}$
and gradient $QF = \dfrac{\frac{4}{p}}{2-\frac{2}{p^2}}$ or $\dfrac{-\frac{4}{p}}{\frac{2}{p^2}-2}$</td><td>M1: Obtains expressions for both gradients condoning one term of incorrect sign in either or both expressions
A1: Both correct expressions oe</td><td>M1
A1</td></tr>
<tr><td>Grad $QF = \dfrac{4p}{2p^2-2}$ = Grad PF
So PQ passes through the focus*</td><td>Shows that the gradients are the same plus minimal conclusion e.g., shown/QED/true/proven/✓ with no incorrect work. Condone no conclusion if penalised in (a).</td><td>A1*</td></tr>
<tr><td></td><td colspan="2">Note: A variation is to show grad PF or grad QF = grad PQ – marked as Alt</td><td>(4)</td></tr>
<tr><td></td><td colspan="2">Alt 2 Follows (similar triangles)</td><td></td></tr>
</table>

<table>
<tr><td colspan="2">8(b) <u>Examples</u> of minimum amount of algebra required with different expressions for gradient:</td></tr>
<tr><td colspan="2">$$y-4p=\frac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}\left(x-2p^2\right)$$</td></tr>
<tr><td>$x=2,\ y=\ldots$</td><td>$$x=2\Rightarrow y-4p=\frac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}\left(2-2p^2\right)\Rightarrow y=\frac{8p+\frac{8}{p}-8p^3-8p+8p^3-\frac{8}{p}}{2p^2-\frac{2}{p^2}}=0$$</td></tr>
<tr><td>$y=0,\ x=\ldots$</td><td>$$y=0\Rightarrow -4p=\frac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}\left(x-2p^2\right)\Rightarrow x=\frac{-8p^3+\frac{8}{p}+8p^3+8p}{4p+\frac{4}{p}}=2$$</td></tr>
<tr><td>$(2, 0)\Rightarrow$</td><td>$$(2,0)\Rightarrow -4p=\frac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}\left(2-2p^2\right)\Rightarrow -4p=\frac{8p+\frac{8}{p}-8p^3-8p}{2p^2-\frac{2}{p^2}}\Rightarrow -4p=-4p$$</td></tr>
<tr><td colspan="2">$$y-4p=\frac{4p^3+4p}{2p^4-2}\left(x-2p^2\right)$$</td></tr>
<tr><td>$x=2,\ y=\ldots$</td><td>$$x=2\Rightarrow y-4p=\frac{4p^3+4p}{2p^4-2}\left(2-2p^2\right)\Rightarrow y=\frac{8p^3+8p-8p^5-8p^3+8p^5-8p}{2p^4-2}=0$$
or $$y-4p=\frac{4p^3+4p}{2p^4-2}\left(2-2p^2\right)\Rightarrow y=\frac{-4p^3-4p+4p^3+4p}{p^2+1}=0$$</td></tr>
<tr><td>$y=0,\ x=\ldots$</td><td>$$y=0\Rightarrow -4p=\frac{4p^3+4p}{2p^4-2}\left(x-2p^2\right)\Rightarrow x=\frac{-8p^5+8p+8p^5+8p^3}{4p^3+4p}=2$$</td></tr>
</table>

$(2, 0) \Rightarrow$	$(2,0) \Rightarrow -4p = \frac{4p^3+4p}{2p^4-2}(2-2p^2) \Rightarrow -4p = \frac{8p^3+8p-8p^5-8p^3}{2p^4-2} \Rightarrow -4p = -4p$		
	$y-4p = \frac{2p}{p^2-1}(x-2p^2)$		
$x=2,\ y=\ldots$	$x=2 \Rightarrow y-4p = \frac{2p}{p^2-1}(2-2p^2) \Rightarrow y = \frac{4p-4p^3+4p^3-4p}{p^2-1} = 0$ or $y-4p = \frac{2p}{p^2-1}(2-2p^2) \Rightarrow y-4p = -4p \Rightarrow y = 0$		
$y=0,\ x=\ldots$	$y=0 \Rightarrow -4p = \frac{2p}{p^2-1}(x-2p^2) \Rightarrow x = \frac{-4p^3+4p+4p^3}{2p} = 2$		
$(2, 0) \Rightarrow$	$(2,0) \Rightarrow -4p = \frac{2p}{p^2-1}(2-2p^2) \Rightarrow -4p = -4p$		
Note that this not an exhaustive list (for example there are all the corresponding $y = mx + c$ approaches or those using Q) and the precise choice of algebra will vary widely but with the exception of the last example above this mark requires substitution into the line followed by a further step which shows an expression that clearly leads to 0, 2 or e.g., $-4p$ or "1=1" followed by a minimal conclusion (unless B0 was given in (a) for that reason).			
8(b) cont.	$y^2 = 8x \quad P(2p^2, 4p) \quad Q\left(\frac{2}{p^2}, \frac{-4}{p}\right) \quad X\left(2, \frac{-4}{p}\right) \quad Y\left(2p^2, \frac{-4}{p}\right)$		
Alt 2 **Similar triangles**	Focus is (2, 0) or $x = 2, y = 0$ Could be seen on a diagram	Correct focus seen or used. Condone (0, 2) if $x = 2, y = 0$ used but award final A0	B1
	$\frac{XF}{XP} = \frac{\frac{4}{p}}{4p+\frac{4}{p}} \quad \frac{QX}{QY} = \frac{2-\frac{2}{p^2}}{2p^2-\frac{2}{p^2}}$	M1: Obtains expressions for two ratios condoning one term of incorrect sign in either or both expressions A1: Both correct expressions oe	M1 A1
	$\frac{XF}{XP} = \frac{1}{p^2+1} = \frac{QX}{QY}$ So Δs XFQ & YPQ are similar $\Rightarrow PQ$ passes through the focus*	Shows that the ratios are the same, makes reference to similarity plus minimal conclusion e.g., shown/QED/true/proven/✓ with no incorrect work. Condone no conclusion if penalised in (a).	A1*
			(4)
8 cont.	$y^2 = 8x \quad P(2p^2, 4p) \quad Q\left(\frac{2}{p^2}, \frac{-4}{p}\right)$		
(c)	$y = \sqrt{8}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{8}x^{-\frac{1}{2}}$ or $2y\frac{dy}{dx} = 8 \Rightarrow \frac{dy}{dx} = \frac{8}{2y}$ or $x = at^2,\ y = 2at \Rightarrow \frac{dx}{dt} = 2at,\ \frac{dy}{dt} = 2at \Rightarrow \frac{dy}{dx} = \frac{2a}{2at}$ (They may use p for t and/or $a = 2$)	Achieves an expression of the correct form (sign/coefficient errors only) for the gradient. There is no requirement for calculus so they may use e.g., $m_T = \frac{1}{t}$. Must go beyond $\frac{dx}{dy}$	M1
	m_T at $P = \frac{1}{2}\sqrt{8}\frac{1}{\sqrt{2p^2}}$ or $\frac{8}{2\times 4p}$ or $\frac{2\times 2}{2\times 2\times p}$ $\left\{=\frac{1}{p}\right\}$ or m_T at $Q = -\frac{1}{2}\sqrt{8}\frac{1}{\sqrt{\frac{2}{p^2}}}$ or $\frac{8}{2\times\left(\frac{-4}{p}\right)}$ or $x = \frac{2}{p^2},\ y = \frac{-4}{p} \Rightarrow \frac{dx}{dp} = \frac{-4}{p^3},\ \frac{dy}{dp} = \frac{4}{p^2} \Rightarrow \frac{dy}{dx} = -p$ Any correct unsimplified expression for the tangent gradient in terms of p at either point. If awarding for Q via explicit differentiation must choose the negative root		A1

	Eqn of tgt at P: $y-4p=\frac{1}{p}\left(x-2p^2\right)$ oe or Eqn of tgt at Q: $y+\frac{4}{p}=-p\left(x-\frac{2}{p^2}\right)$ oe	M1: Correct straight line method for either point with their tangent gradient in terms of p (but allow if "a" also present) Coordinates correctly placed. If $y=mx+c$ is used must reach $c=$ ... following correctly placed coordinates A1: Any correct unsimplified equation for either tangent	M1 A1
	Note: $y=mx+c$: At P, $y=\frac{1}{p}x+2p$ At Q, $y=-px-\frac{2}{p}$		
	$\frac{1}{p}\left(x-2p^2\right)+4p=-p\left(x-\frac{2}{p^2}\right)-\frac{4}{p}\Rightarrow x=...$ or $\frac{1}{p}x+2p=-px-\frac{2}{p}\Rightarrow x=...$ Eliminates y from their tangent equations and solves for x (See note below if eliminate x). Gradients must be different and no clear evidence of conversion of any line to a normal. Condone poor algebra.		M1
	$x\left(\frac{1}{p}+p\right)=-\frac{2}{p}-2p\Rightarrow x=\frac{-\left(2p^2+2\right)}{\left(p^2+1\right)}=-2$	$x=-2$ only	A1
	$y=\frac{1}{p}\left(-2-2p^2\right)+4p,\quad -p\left(-2-\frac{2}{p^2}\right)-\frac{4}{p},\quad \frac{1}{p}(-2)+2p,\quad -p(-2)-\frac{2}{p}$ **d**M1: Substitutes their x (a constant or function of p) into one of their two tangent equations to obtain an expression for y. **Requires previous M mark**.		**d**M1
	e.g., $y=2p-\frac{2}{p},\quad 2\left(p-\frac{1}{p}\right),\quad \frac{2}{p}\left(p^2-1\right),\quad \frac{2p^2-2}{p},\quad \frac{2(p+1)(p-1)}{p}$ A1: Correct y in simplest form – two terms which could be factorised in any correct way and/or written as a single fraction. Note there is no requirement for coordinate notation.		A1
	Note it is obviously possible to eliminate x. In this case, award the last 4 marks in this order: M1: Eliminates x and solves for y A1: Any correct y in simplest form **d**M1: Substitutes their y (a constant or function of p) into one of their two tangent equations to obtain an expression for x. **Requires previous M mark.** A1: $x=-2$		**(8)**
	Working which involves "a" where a is never replaced by 2 can score the Ms		**Total 13**

Question Number	Scheme	Notes	Marks
9	$\mathrm{f}(n)=4^n+6n-10 \qquad n\in\mathbb{Z} \qquad n\geqslant 2$		

General guidance:
Apply the way that best fits the overall approach.
Condone work in e.g., n instead of k.
Attempts with no induction e.g., not using f(k) in an equation with f(k +1) score a max of 11000.
Using e.g., f(k + 2) – f(k + 1) requires a clear indication of assuming f(k + 1) is true to access the last three marks.
Alternative explanations are unlikely to access the last three marks unless there is a fully convincing justification of divisibility, e.g., $\mathrm{f}(k+1)-\mathrm{f}(k)=3\times4^k+6$ followed by "Since 3×4^k is a multiple of both 3 and 4 and hence 12, $3\times4^k+6$ is divisible by 18" is not a sound argument. Attempts that involve further induction on different expressions must be complete methods to access the last 3 marks.
<u>Allow use of –18 but if any different multiples of 18 are involved e.g., 36, the first A1 requires "36 is a multiple of/divisible by (but not "factor of") 18" oe for each case</u>
B1: Any correct numerical expression that is not just "18" is sufficient for this mark
e.g., 16 + 12 –10, 28 – 10, 4^2 + 2. Starting with e.g., f(3) scores a max of 01110.
Ignore an extra evaluation of $\mathrm{f}(1)$ but a comment on $\mathrm{f}(1)$'s divisibility is final A0 since $n\geqslant 2$
Final A1: There must be evidence that true for $n=k \Rightarrow$ true for $n=k+1$ but it could be minimal and be scored in a conclusion or a narrative or via both. So if e.g., "Assume true for $n=k$ ..." is seen in the work followed by "true for $n=k+1$" in a conclusion this is sufficient.
Condone "for all $n\in\mathbb{Z}$", "all $n\in\mathbb{Z}$ $n>2$", "all $\mathbb{Z}>$ (or $\geqslant$) 2" but **not** $n\in\mathbb{R}$

Question Number	Scheme	Notes	Marks
Way 1 $\mathrm{f}(k+1)-\mathrm{f}(k)$	$\mathrm{f}(2)=4^2+6\times2-10=18$	Obtains f(2) = 18 with substitution	B1
	$\mathrm{f}(k+1)=4^{k+1}+6(k+1)-10$	Attempts $\mathrm{f}(k+1)$	M1
	$\mathrm{f}(k+1)-\mathrm{f}(k)=4^{k+1}+6(k+1)-10-(4^k+6k-10)$ $=4^{k+1}-4^k+6=3\times4^k+6$ $=3(4^k+6k-10)-18k+36$	Attempts $\mathrm{f}(k+1)-\mathrm{f}(k)$, uses $4^{k+1}=4\times4^k$ & obtains $p\mathrm{f}(k)+\mathrm{g}(k)$ with g(k) linear (allow constant $\neq 0$)	M1
	$\mathrm{f}(k+1)=4\mathrm{f}(k)+18(2-k)$ f(k) may be written in full	Correct factorised expression Allow $4\mathrm{f}(k)+18\times2-18\times k$ If f(k + 1) is not made the subject then e.g., "true for f(k + 1) – f(k)" is also required	A1
	True for n = 2, if **true for n = k then true for $n=k+1$** so **true for all $n\in\mathbb{Z}$** ($n\geqslant2$) Minimum in bold.	Full conclusion/narrative and no errors. All marks needed but allow if B0 provided this mark was only withheld for insufficient working.	A1
			(5)
Way 2 $\mathrm{f}(k+1)=\ldots$	$\mathrm{f}(2)=4^2+6\times2-10=18$	Obtains f(2) = 18 with substitution	B1
	$\mathrm{f}(k+1)=4^{k+1}+6(k+1)-10$	Attempts $\mathrm{f}(k+1)$	M1
	$=4\times4^k+6k-4$ $=4(4^k+6k-10)-18k+36$	Uses $4^{k+1}=4\times4^k$ & obtains $p\mathrm{f}(k)+\mathrm{g}(k)$ with g(k) linear (allow constant $\neq 0$)	M1
	$=4\mathrm{f}(k)+18(2-k)$ f(k) may be written in full	Correct factorised expression Allow $4\mathrm{f}(k)+18\times2-18\times k$	A1
	True for n = 2, if **true for n = k then true for $n=k+1$** so **true for all $n\in\mathbb{Z}$** ($n\geqslant2$) Minimum in bold.	Full conclusion/narrative and no errors. All marks needed but allow if B0 provided this mark was only withheld for insufficient working.	A1
			(5)

Question Number	Scheme	Notes	Marks
9 cont.	$f(n)=4^n+6n-10 \quad n\in\mathbb{Z} \quad n\geqslant 2$		
Way 3 $f(k+1)-mf(k)$	$f(2)=4^2+6\times 2-10=18$	Obtains f(2) = 18 with substitution	B1
	$f(k+1)=4^{k+1}+6(k+1)-10$	Attempts $f(k+1)$	M1
	$f(k+1)-mf(k)=4^{k+1}+6(k+1)-10-m(4^k+6k-10)$ $=(4-m)4^k+(6-6m)k-4+10m$ e.g. $m=-14\Rightarrow 18\times 4^k+90k-144$ e.g. $m=4\Rightarrow -18k+36$	Attempts $f(k+1)-mf(k)$ and uses a value of m to obtain $c\times 4^k+\ldots$ where c is a multiple of their 18 or uses m = 4	M1
	e.g., $f(k+1)=-14f(k)+18(4^k+5k-8)$ $f(k+1)=4f(k)+18(2-k)$ f(k) may be written in full	A correct factorised expression Allow $-14f(k)+18\times 4^k+18\times 5k-18\times 8$ If f(k + 1) is not made the subject then e.g., "true for f(k + 1) – mf(k)" is also required	A1
	True for *n* = 2, if **true for *n* = *k* then true for *n* = *k* + 1** so **true for all *n*** $\in\mathbb{Z}$ $(n\geqslant 2)$ Minimum in bold.	Full conclusion/narrative and no errors. All marks needed but allow if B0 provided this mark was only withheld for insufficient working.	A1
			(5)
Way 4 $f(k)=18M$	$f(2)=4^2+6\times 2-10=18$	Obtains f(2) = 18 with substitution	B1
	$f(k+1)=4^{k+1}+6(k+1)-10$	Attempts $f(k+1)$	M1
	$f(k)=18M,\quad f(k+1)=4\times 4^k+6k-4$ $=4\times 18M-18k+36$	Sets $f(k)=18M$, uses $4^{k+1}=4\times 4^k$ & obtains $pf(k)+g(k)$ with g(k) linear (allow constant $\neq 0$)	M1
	$f(k+1)=18(4M+2-k)$	A correct factorised expression Allow $18\times 4M+18\times 2-18\times k$	A1
	True for *n* = 2, if **true for *n* = *k* then true for *n* = *k* + 1** so **true for all *n*** $\in\mathbb{Z}$ $(n\geqslant 2)$ Minimum in bold.	Full conclusion/narrative and no errors. All marks needed but allow if B0 provided this mark was only withheld for insufficient working.	A1
			(5)
			PAPER TOTAL: 75