Question	Scheme	Marks
1.(a)	$\begin{pmatrix} 2 & -1 & 3 \\ -2 & 3 & 0 \end{pmatrix}\begin{pmatrix} 1 & k \\ 0 & -3 \\ 2k & 2 \end{pmatrix}=\begin{pmatrix} 2+0+6k & 2k+3+6 \\ -2+0+0 & -2k-9+0 \end{pmatrix}$	M1
	$=\begin{pmatrix} 2+6k & 2k+9 \\ -2 & -2k-9 \end{pmatrix}$	A1cao
		(2)
(b)	det $\mathbf{AB}=(2+6k)(-2k-9)-(-2)(2k+9)$	M1
	det $\mathbf{AB}=0 \Rightarrow -12k^2-54k=0 \Rightarrow k=...$	dM1
	$k=-\frac{9}{2}$	A1
		(3)
		(5 marks)

Notes:

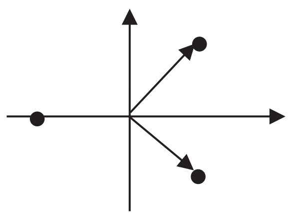
(a)
M1: Obtains a 2×2 matrix with at least two entries correct, unsimplified.
A1cao: Correct matrix with terms simplified.
(b)
M1: Attempts the determinant, be tolerant of minor slips, such as sign slips with the negatives, if the correct "*ad – bc*" form is apparent. They may give the $-(-2)(\ldots)$ as just + 2(…). Accept if seen as part of the attempt at the inverse matrix.
dM1: Expands their determinant to a **quadratic**, sets equal to zero (may be implied) and achieves a value for k via correct method (allow if a factor k is cancelled, use of formula or calculator (a correct value for their quadratic)).
A1: cso for $-\frac{9}{2}$. Accept as decimal or equivalent fractions, such as $-\frac{54}{12}$. Ignore any reference to the 0 solution.

Question	Scheme	Marks
2.	$(7r-5)^2 = 49r^2 - 70r + 25$	**B1**
	$\sum_{r=1}^{n}(7r-5)^2 = 49\sum_{r=1}^{n} r^2 - 70\sum_{r=1}^{n} r + \sum_{r=1}^{n} 25$ $= 49 \times \underline{\frac{n}{6}(n+1)(2n+1)} - 70 \times \underline{\frac{n}{2}(n+1)} + 25\times\underline{n}$	**M1** **A1ft**
	$= \frac{n}{6}\left(49(2n^2+3n+1) - 210(n+1) + 150\right)$	**M1**
	$= \frac{n}{6}\left(98n^2 - 63n - 11\right)$	**A1**
	$= \frac{n(7n+1)(14n-11)}{6}$	**A1**
		(6)
		(6 marks)

Notes:

B1: Correct expansion.
M1: Attempts the summations with at least two of the underlined formulae correct.
A1ft: Fully correct application of all three summations. Follow through on their expansion as long as there are 3 terms.
M1: Attempts to factor out at least the factor of n from their **three term** expansion – must have a common factor of n throughout to be able to score this mark which must be extracted from each term. (If the last term is +25, it is M0.) Allow if there are minor slips but the process must be correct.

Alternatively allow this mark for an attempt to expand $\frac{n}{6}(7n+1)\left(An+B\right)$ and compare coefficients with their expanded equation.
A1: Gathers terms appropriately and achieves the correct quadratic. In the alternative approach allow for $A = 14$ and $B = -11$ stated from their comparison.
A1cso: Correct answer from correct work. Any values found from the comparison approach must be substituted back in to achieve the result. Note from a correct unsimplified quadratic to correct answer, A0A1 can be awarded.

Question	Scheme		Marks
	$\mathrm{f}(z)=4z^3+pz^2-24z+108$, -3 a root.		
3(a)	$\mathrm{f}(-3)=0 \Rightarrow 4(-3)^3+p(-3)^2-24(-3)+108=0 \Rightarrow p=...$		M1
	$p=-8$		A1
			(2)
(b)	$4z^3-8z^2-24z+108=(z+3)(4z^2+...z+36)$		M1
	$=(z+3)(4z^2-20z+36)$		A1
	$4z^2-20z+36=0 \Rightarrow z=\dfrac{20\pm\sqrt{400-4\times4\times36}}{8}=...$		dM1
	Roots are $-3, \dfrac{5\pm \mathrm{i}\sqrt{11}}{2}$		A1
			(4)
(c)	e.g. Product of complex roots is $\dfrac{36}{4}=9$, so modulus is $\sqrt{"9"}$ or Modulus is $\sqrt{\left(\dfrac{5}{2}\right)^2+\left(\dfrac{\sqrt{11}}{2}\right)^2}$		M1
	Hence modulus is 3		A1
			(2)
(d)		Complex conjugate pair in correct quadrant for their roots	M1
		All three roots correctly positioned.	A1
			(2)
			(10 marks)

Notes:

Mark the question as a whole - do not be concerned part labelling.

(a)

M1: A complete method to find the value of p. Use of the factor theorem is most direct, look for setting f(−3) = 0 and solving for p. May attempt to factor out $(z+3)$ and compare coefficients, e.g.

$\mathrm{f}(z) = 4z^3 + pz^2 - 24z + 108 = (z+3)(4z^2 + bz + 36) \Rightarrow 3b + 36 = -24, 12 + b = p \Rightarrow b = .., p = ...$ or may attempt long division and set remainder equal to zero to find p or variations on these.

A1: For $p = -8$

(b)

Note: Allow marks in (b) for work seen in (a) e.g. via attempts in (a) by long division.

M1: Correct strategy to find a quadratic factor. If factorising, look for correct first and last terms. May use long division, in which case look for the correct first term and attempt to use it - may have been seen in (a).

Question instructs use of algebra so an algebraic method must be seen.

A1: Correct quadratic factor - may have been seen in (a).

dM1: Uses the quadratic formula or completing the square or calculator to find the roots of their quadratic factor (allow for attempts at a quadratic factor via long division which had non-zero remainder). If a calculator is used (no method shown), there must be at least one correct **complex** root for their equation. Factorisation is M0.

A1: Correct roots in simplest form. All three should be included at some point in the solution in (b).

(c)

M1: Any correct method to find the modulus of the complex roots. Most likely to see Pythagoras, but some may deduce from product of roots. They must have complex roots to score the marks in (c).

A1: Modulus 3 only. If -3 is also given as a modulus then score A0.

(d)

Note: Allow the marks in (d) if the i's were missing in their roots in (b) but they clearly mean the correct complex roots on their diagram.

M1: Plots the complex roots as a conjugate pair in the correct quadrants for their roots.

A1: Fully correct diagram with one root on the negative real axis, and the other as a complex pair of roughly the same length in quadrants 1 and 4.

Question	Scheme	Marks
4(a)	(i) $\mathrm{f}'(x) = \underline{Ax^{-5}} + \underline{Bx^{-\frac{9}{2}}}$ oe for at least one power	M1
	$\mathrm{f}'(x) = -\dfrac{-4x^{-5}}{8} + \dfrac{2\times -\frac{7}{2}x^{-\frac{9}{2}}}{7} = \dfrac{1}{2x^5} - \dfrac{1}{x^{\frac{9}{2}}}$ oe	A1
	(ii) Since $\mathrm{f}'(0.25) = 512 - 512 = 0$ the process cannot be carried out as it would require division by zero.	B1
	(iii) $\alpha = 0.15 - \dfrac{\mathrm{f}(0.15)}{\mathrm{f}'(0.15)} = 0.15 - \dfrac{-27.332...}{1484.137...} = ...$	M1
	$= 0.168$ to 3 d.p.	A1cso
		(5)
(b)	e.g. $\dfrac{\mathrm{f}(0.25)-\mathrm{f}(0.15)}{0.25-0.15} = \dfrac{\mathrm{f}(0.15)-0}{0.15-\alpha}$ or $\dfrac{\alpha - 0.25}{0-\mathrm{f}(0.25)} = \dfrac{\alpha - 0.15}{0-\mathrm{f}(0.15)}$ etc	M1
	$\Rightarrow \alpha = 0.15 - \dfrac{0.1\times \mathrm{f}(0.15)}{\mathrm{f}(0.25)-\mathrm{f}(0.15)} = ...$ or $\alpha = \dfrac{0.25\,\mathrm{f}(0.15) - 0.15\,\mathrm{f}(0.25)}{(\mathrm{f}(0.15)-\mathrm{f}(0.25))} = ...$ etc	M1
	$= 0.15 - \dfrac{0.1\times -27.332...}{5.571... - (-27.332)} = 0.23306... = \text{awrt}\, 0.233$ (3 d.p.)	A1
		(3)
		(8 marks)

Notes:

(a)(i)
M1: Attempts to differentiate f(x), obtaining the correct power for at least one term.
A1: Correct differentiation, need not be simplified.
(ii)
B1: Correct reason given, accept e.g. "as f'(0.25) = 0" as a minimum and isw after a correct reason is given. Just stating f'(0.25) = 0 is not sufficient, there must be an indication this is the reason why the process cannot be used but accept any indication (such as "not valid") following this.
(iii)
M1: Correct Newton-Raphson process attempted using their derivative or implied by the correct answer from use of a calculator.
A1cso: Correct answer from correct work (derivative must have been correct). Must be 3dp.
(b)
M1: Correct interpolation strategy. Accept any correct statement such as the one shown. They may use e.g. x for $\alpha - 0.15$, in which case the the method will be gained once the correct overall strategy is clear.
M1: Proceeds from a recognisable attempt at interpolation to find a value for α. Not dependent, but they must have attempted to set up a suitable equation in α. If no working is shown accept any value for the root following the suitable statement.

A1: Accept awrt 0.233 following correct working.

Question	Scheme	Marks
5(a)	$\alpha+\beta=-\frac{3}{4}$	B1
	$\alpha\beta=\frac{k}{4}$	B1
		(2)
(b)	$\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}=\frac{\alpha^3+\beta^3}{\alpha^2\beta^2}$	B1
	$=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^2};=\frac{\left(-\frac{3}{4}\right)^3-3\left(\frac{k}{4}\right)\left(-\frac{3}{4}\right)}{\left(\frac{k}{4}\right)^2}=\ldots$	M1; M1
	$=\frac{36k-27}{4k^2}=\frac{9}{k}-\frac{27}{4k^2}$	A1
		(4)
(c)	Product of roots is $\frac{\alpha\beta}{\alpha^2\beta^2}=\frac{1}{\alpha\beta}=\frac{4}{k}$	B1ft
	Equation is $x^2-\left(\frac{36k-27}{4k^2}\right)x+\frac{4}{k}=0$	M1
	$4k^2x^2-(36k-27)x+16k=0$	A1
		(3)
		(9 marks)

Notes:

(a)

B1: Correct expression for $\alpha+\beta$

B1: Correct expression for $\alpha\beta$

(b)

B1: Combines the fractions correctly.

M1: For a correct identity for the sum of cubes.

M1: Substitutes their values for $\alpha+\beta$ and $\alpha\beta$ into their equation for sum of $\frac{\alpha}{\beta^2}+\frac{\beta}{\alpha^2}$ (not dependent, so there may be a slip in the identity used for $\alpha^3+\beta^3$).

A1: Correct expression in terms of k in a simplified form - e.g. either form as shown in scheme.

(c)

B1ft: Correct product of roots in terms of k, or follow through $\frac{1}{\text{their }\alpha\beta}$ from part (a).

M1: Applies $x^2 - (\text{their sum of roots})x + \text{their product of roots}\left(=0\right)$. Allow without the "=0" for this mark.
A1: Correct equation, as shown or an integer multiple thereof. Accept equivalents for the x term (e.g. $4k^2x^2 + (27 - 36k)x + 16k = 0$. Must include the "=0".

Question	Scheme	Marks
6(a)	$a=5$	B1
		(1)
(b)	$\dfrac{dy}{dx}=-\dfrac{20}{x^2}$ or $\dfrac{dy}{dx}=-\dfrac{2\sqrt{5}}{t^2}\div 2\sqrt{5}=-\dfrac{1}{t^2}$ or $x\dfrac{dy}{dx}+y=0$ oe	B1
	Gradient of normal is $\dfrac{-1}{\text{"}-1/t^2\text{"}}=t^2$	M1
	Normal is $y-\dfrac{2\sqrt{5}}{t}=t^2\left(x-2t\sqrt{5}\right)$	M1
	$\Rightarrow ty-2\sqrt{5}=t^3x-2t^4\sqrt{5}\Rightarrow ty-t^3x-2\sqrt{5}\left(1-t^4\right)=0$ *	A1*
		(4)
(c)	$cy-c^3x-2\sqrt{5}\left(1-c^4\right)=0$ passes through $\left(-\dfrac{\sqrt{5}}{c},-4c\sqrt{5}\right)$ $\Rightarrow -4c^2\sqrt{5}+c^2\sqrt{5}-2\sqrt{5}\left(1-c^4\right)=0$	M1
	$\Rightarrow 2c^4-3c^2-2=0$ (oe)	A1
	$\Rightarrow c^2=\dfrac{3\pm\sqrt{9-4\times2\times-2}}{4}=\ldots\left(2,-\dfrac{1}{2}\right)$	dM1
	$c^2>0\Rightarrow c^2=2\Rightarrow c=\pm\sqrt{2}$	A1
		(4)
		(9 marks)

Notes:

(a)

B1: Correct value stated.

(b)

B1: Correct expression for $\dfrac{dy}{dx}$, or any equivalent correct expression including it, such as $x\dfrac{dy}{dx}+y=0$ or $\dfrac{dy}{dx}=-\dfrac{1}{t^2}$

M1: Attempts negative reciprocal gradient at the point P. Allow with a instead of 5 for this mark, so score for e.g. $m_N=\dfrac{4at^2}{20}$.

M1: Uses their normal (changed from tangent) gradient and P to find the equation of the tangent. Look for $y-\dfrac{2\sqrt{5}}{t}=\text{"}m_n\text{"}\left(x-t\sqrt{5}\right)$. If using $y=mx+c$ they must proceed as far as finding c.

A1*: Correct equation achieved from correct working with intermediate step.

(c) **M1:** Substitutes the parameter for A into the normal equation and attempts to substitute the coordinates of B to obtain an equation in one variable. Allow if there are slips during substitution. **A1:** Correct quadratic in c^2 need not be simplified. **dM1:** Solves their (at least two term) quadratic in c^2 to find a value for at least c^2 **A1:** Deduces correct values. Both required. Ignore reference to any complex roots.		
Alts		
(c)	$cy - c^3x - 2\sqrt{5}\left(1-c^4\right) = 0$ intersects H again $\Rightarrow \frac{20}{x}c - c^3x - 2\sqrt{5}\left(1-c^4\right) = 0$ or $cy - \frac{20}{y}c^3 - 2\sqrt{5}\left(1-c^4\right) = 0$ $\Rightarrow c^3x^2 + 2\sqrt{5}\left(1-c^4\right)x - 20c = 0$ or $cy^2 - 2\sqrt{5}\left(1-c^4\right)y - 20c^3 = 0$	**M1** **A1**
	$\Rightarrow \left(c^3x + 2\sqrt{5}\right)\left(x - 2c\sqrt{5}\right) = 0$ or $\left(cy - 2\sqrt{5}\right)\left(y + 2\sqrt{5}c^3\right) = 0$ $\left(x = 2c\sqrt{5} \text{ is } A \text{ so}\right)$ for B $x = -\frac{2\sqrt{5}}{c^3} = -\frac{\sqrt{5}}{c} \Rightarrow c = \ldots$ $\left(\text{or } y = \frac{2\sqrt{5}}{c} \text{ is } A \text{ so}\right)$ for B $y = -2\sqrt{5}c^3 = -4c\sqrt{5} \Rightarrow c = \ldots$	**M1**
	$\Rightarrow c^2 = 2 \Rightarrow c = \pm\sqrt{2}$	**A1**
		(4)
Notes:		
(c) **M1:** Substitutes parameters for A and equation for H into normal to obtain a quadratic in x. **A1:** Correct quadratic in x or y **M1:** Solves the quadratic in x or y, identifies correct solution and equates to the relevant coordinate of B and solves for c **A1:** Deduces correct values. Both required.		
(c)	$m_{AB} = \dfrac{\frac{2\sqrt{5}}{c} - \left(-4c\sqrt{5}\right)}{2\sqrt{5}c - \left(-\frac{\sqrt{5}}{c}\right)}$	**M1**
	$m_{AB} = \frac{2+4c^2}{2c^2+1} = 2$	**A1**
	From (b), normal at A has gradient $(t^2 =)c^2 \Rightarrow c^2 = 2$	**M1**
	$\Rightarrow c^2 = 2 \Rightarrow c = \pm\sqrt{2}$	**A1**
		(4)

Notes:
(c) **M1:** Attempts the gradient of AB. **A1:** Correct gradient need not be simplified. **M1:** Finds/deduces the gradient of the normal at A and sets equal to their gradient of AB. **A1:** Deduces correct values. Both required.

Question	Scheme	Marks
7(i)(a)	Reflection or in the line $y=-x$	M1
	Reflection in the line $y=-x$	A1
		(2)
(b)	$\begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$ or $6\times\begin{pmatrix} \pm\cos 240° & \pm\sin 240° \\ \pm\sin 240° & \pm\cos 240° \end{pmatrix}$	M1
	$\begin{pmatrix} -3 & 3\sqrt{3} \\ -3\sqrt{3} & -3 \end{pmatrix}$	A1
		(2)
(c)	$\mathbf{R}=\mathbf{QP}=\begin{pmatrix} -3 & 3\sqrt{3} \\ -3\sqrt{3} & -3 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}=\ldots$	M1
	$=\begin{pmatrix} -3\sqrt{3} & 3 \\ 3 & 3\sqrt{3} \end{pmatrix}$ **QP** correctly found	A1
		(2)
(ii)	$\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix}\begin{pmatrix} \lambda \\ 1 \end{pmatrix}=\begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}\Rightarrow\begin{pmatrix} -2\lambda+2\sqrt{3} \\ 2\lambda\sqrt{3}+2 \end{pmatrix}=\begin{pmatrix} 4\lambda \\ 4 \end{pmatrix}$	M1
	$-2\lambda+2\sqrt{3}=4\lambda$ or $2\lambda\sqrt{3}+2=4$	A1
	$\Rightarrow 6\lambda=2\sqrt{3}\Rightarrow\lambda=\ldots$ or $2\sqrt{3}\lambda=2\Rightarrow\lambda=\ldots$	dM1
	$\lambda=\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$ oe	A1
	Both $-2\lambda+2\sqrt{3}=4\lambda$ and $2\lambda\sqrt{3}+2=4$ solved leading to $\lambda=\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$	A1
		(5)
		(11 marks)

Notes:

(a)
M1: Identifies the transformation as a reflection or identifies the correct line of reflection.
A1: Fully correct description, with the equation of the line of reflection or suitable description (e.g. in the line through angle 135° with the positive x-axis). Ignore any references to a centre of reflection.

(b)
M1: Either the correct matrix for the rotation (with trig ratios evaluated) or an attempt at scaling a matrix of form shown by a factor 6 (need not evaluate ratio) – if no trig ratios seen this may be implied by the exact values in the right places. The correct answer implies the M.
A1: Correct matrix.
(c)
M1: Attempts to multiply **Q** and **P** in the correct order.
A1: QP correct
(ii)
M1: Attempts the product $\begin{pmatrix}-2 & 2\sqrt{3}\\ 2\sqrt{3} & 2\end{pmatrix}\begin{pmatrix}\lambda\\ 1\end{pmatrix}$ and sets equal to $\begin{pmatrix}4\lambda\\ 4\end{pmatrix}$. Allow for poor notation as long as the intention is clear, and it may be implied by one correct equation or follow through equation.
A1: Extracts at least one correct equation (not part of the matrix equation). May be implied by correct value for λ following correct matrix equation.
dM1: Attempts to solve the equation. May be implied by the correct value following a correct matrix equation with no extraction of separate equations.
A1: Correct value for λ from at least one equation and isw if incorrectly simplified (allow if their second equation does not concur).
A1: Correct value for λ coming from both equations, solved explicitly, or checks the value of λ from the first equation works in the second equation.

Alt (ii)	$\begin{pmatrix}-2 & 2\sqrt{3}\\ 2\sqrt{3} & 2\end{pmatrix}\begin{pmatrix}\lambda\\ 1\end{pmatrix}=\begin{pmatrix}4\lambda\\ 4\end{pmatrix}\Rightarrow\begin{pmatrix}\lambda\\ 1\end{pmatrix}=\dfrac{1}{-4-12}\begin{pmatrix}2 & -2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}\begin{pmatrix}4\lambda\\ 4\end{pmatrix}$ $\Rightarrow\begin{pmatrix}\lambda\\ 1\end{pmatrix}=-\dfrac{1}{16}\begin{pmatrix}8\lambda-8\sqrt{3}\\ -8\lambda\sqrt{3}-8\end{pmatrix}$	**M1**
	$2\lambda=\sqrt{3}-\lambda$ or $2=\lambda\sqrt{3}+1$	**A1**
	$\Rightarrow 3\lambda=\sqrt{3}\Rightarrow\lambda=...$ or $\sqrt{3}\lambda=1\Rightarrow\lambda=...$	**dM1**
	$\lambda=\dfrac{\sqrt{3}}{3}$ or $\dfrac{1}{\sqrt{3}}$	**A1**
	Both $2\lambda=\sqrt{3}-\lambda$ and $2=\lambda\sqrt{3}+1$ solved leading to $\lambda=\dfrac{\sqrt{3}}{3}$ or $\dfrac{1}{\sqrt{3}}$	**A1**
		(5)

Notes:

M1: Correct attempt at inverse, attempts the product $\begin{pmatrix}-2 & 2\sqrt{3}\\ 2\sqrt{3} & 2\end{pmatrix}^{-1}\begin{pmatrix}4\lambda\\ 4\end{pmatrix}$ and sets equal to $\begin{pmatrix}\lambda\\ 1\end{pmatrix}$.

Allow for poor notation as long as the intention is clear, and it may be implied by one correct equation or follow through equation.
A1: Extracts at least one correct equation (not part of the matrix equation). May be implied by correct value for λ following correct matrix equation.

dM1: Attempts to solve the equation. May be implied by the correct value following a correct matrix equation with no extraction of separate equations.
A1: Correct value for λ from at least one equation (allow if their second equation does not concur).
A1: Correct value for λ coming from both equations, solved explicitly, or checks the value of λ from the first equation works in the second equation.

Question	Scheme	Marks
8(a)	$y^2 = 4ax, y = k \Rightarrow P = \left(\frac{k^2}{4a}, k\right)$	B1
	Either $PS = \frac{k^2}{4a} + a = ...$ *or* $PS^2 = \left(\frac{k^2}{4a} - a\right)^2 + k^2 = ... \Rightarrow PS = ...$	M1
	$PS = \frac{k^2 + 4a^2}{4a}$ *	A1*
		(3)
(b)	Gradient of l_2 is $\frac{k}{\frac{k^2+4a^2}{4a}} = \frac{4ak}{k^2+4a^2}$ oe	B1
	$l_2 : y = \frac{4ak}{k^2+4a^2}(x+a) \Rightarrow y = \frac{4ak}{k^2+4a^2} \times (0+a) = ..$	M1
	$y\big\|_{x=0} = \frac{4a^2k}{k^2+4a^2}$ *	A1*
		(3)
(c)	Area $OSP = \frac{1}{2} \times a \times k$	B1
	Area $BPA = \frac{1}{2} \times \frac{k^2+4a^2}{4a} \times \left(k - \frac{4a^2k}{k^2+4a^2}\right) \quad \left(= \frac{k^3}{8a}\right)$	M1
	$\frac{\text{Area } BPA}{\text{Area } OSP} = \frac{\frac{k^2+4a^2}{4a} \times \left(k - \frac{4a^2k}{k^2+4a^2}\right)}{ak} = 4k^2$	M1
	$\Rightarrow k^3 + 4a^2k - 4a^2k = 16a^2k^3 \Rightarrow a = ...$	dM1
	$a = \frac{1}{4}$	A1
		(5)
		(11 marks)

Notes:

(a)
B1: Correct x coordinate at P found. May be seen on diagram.
M1: For a full method to find an expression for PS. Either use of focus-directrix property or may use Pythagoras with their coordinates.
A1*: Reaches the correct expression with a suitable intermediate step and no errors seen. If using Pythagoras the suitable step must be one with brackets expanded before factorising again.

(b)
B1: Correct expression for the gradient of l_2 given or implied by working. Need not be simplified. If using a similar triangles approach this may be scored for e.g. $\frac{k^2+4a^2}{4a} \div k = \frac{a}{y}$ or $k = \left(\frac{k^2+4a^2}{4a}\right)m$

M1: Full method to find the y intercept, e.g. by forming the equation of the line and substituting $x = 0$

May use $y - k = \frac{4ak}{k^2+4a^2}\left(x - \frac{k^2}{4a}\right) \Rightarrow y = k + \frac{-k^3}{k^2+4a^2}$

A1*: Reaches correct answer with no errors seen.
(c)
NB: If a value is chosen for k (or $k = 2a$) used, all marks are available, score for the relevant correct expressions/methods with their value.

B1: Correct area of OSP stated or implied. Note that if they go direct to ratios, the $\frac{1}{2}$ may not be seen (as it cancels with that in BPA)
M1: Correct method for the area of the triangle BPA. Allow sign slips if the method is clear (e.g. $\frac{k^2}{4a} - "-a" = \frac{k^2}{4a} - a$ if it is clear BP is meant). Allow if the negative of the area is found.
M1: Applies the ratio correctly to the problem.
dM1: Attempts to solve their equation.
A1: Correct answer. Allow if the negative of the area was found and later made positive as recovery.

Question	Scheme	Marks
9	For $n = 1$, $\sum_{r=1}^{1} \log(2r-1) = \log(2-1) = \log 1$ and $\log\left(\frac{(2\times 1)!}{2^1 1!}\right) = \log 1$ So true for $n = 1$	B1
	(Assume the result is true for $n = k$, so $\sum_{r=1}^{k} \log(2r-1) = \log\left(\frac{(2k)!}{2^k k!}\right)$ Then) $\sum_{r=1}^{k+1} \log(2r-1) = \log\left(\frac{(2k)!}{2^k k!}\right) + \log(2(k+1)-1)$	M1
	$= \log\left(\frac{(2k)!}{2^k k!} \times (2k+1)\right)$	M1
	$= \log\left(\frac{(2k+1)!}{2^k k!} \times \frac{2k+2}{2k+2}\right) = \log\left(\frac{(2k+2)!}{2^k \times 2(k+1)!}\right)$	M1
	$= \log\left(\frac{(2k+2)!}{2^{k+1}(k+1)!}\right)$	A1
	Hence result is true for $n = k+1$. As true for $n = 1$ and have shown if true for $n = k$ then it is true for $n = k+1$, so it is true for all $n \in \mathbb{N}$ by induction.	A1
		(6)
		(6 marks)

Notes:

(a)

B1: Checks the result for $n = 1$. Must see **both** sides (possibly in one line) identified as log1 or 0 but may not see much more than this.

M1: Makes the inductive assumption (may be implied) and applies it to the question by adding the $(k+1)^{\text{th}}$ term to the expression for the sum to k terms. Allow if there are minor slips (e.g. a missing factorial) if the intent is clear.

M1: Attempts to combine or split log terms appropriately. Not dependent, so may be scored if the wrong term is added in the previous M as long as it is a log term.

M1: Introduces the relevant cancelling factors to achieve the $(2k + 2)!$ term. The introduction of the factors must shown or implied in an intermediate step. Alternatively, may decompose from the k+1 statement to achieve the same intermediate expression.

A1: Achieves correct expression from correct work (or correctly shows equivalence).

A1: Completes the induction by demonstrating the result clearly, with suitable conclusion conveying "true for $n = 1$", "assumed true for $n = k$" and "shown true for $n = k + 1$", and "hence true for all n". Depends on all except the B mark, though a check for $n = 1$ must have been attempted.

NB Allow the M's and first A if n is used throughout but the steps are correct, but must have used a different variable for the final A.

Alt steps if splitting logs:	$\sum_{r=1}^{k+1} \log(2r-1) = \log\left(\frac{(2k)!}{2^k k!}\right) + \log(2k+1)$	**M1**
	$= \log(2k)! - \log\left(2^k k!\right) + \log(2k+1)$	**M1**
	$= \log(2k+1)! - \log\left(\frac{2^{k+1}(k+1)!}{2\times(k+1)}\right) = \log\left(\frac{(2k+1)!(2k+2)}{2^k (k+1)!}\right)$	**M1**