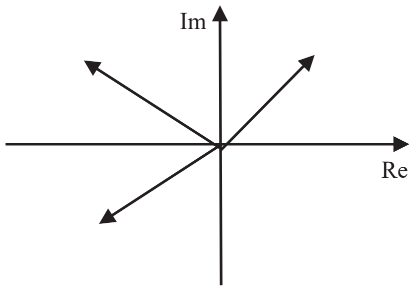
Question Number	Scheme	Notes	Marks
1(a)	$z_1 = 3+3\text{i} \qquad z_2 = p+q\text{i} \qquad p,q \in \mathbb{R}$		
	$\lvert z_1\rvert = \sqrt{3^2+3^2}$ $\lvert z_1z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert \Rightarrow \lvert z_2\rvert\sqrt{18} = 15\sqrt{2} \Rightarrow \lvert z_2\rvert = \ldots$	Attempts $\lvert z_1\rvert$ using Pythagoras and uses $\lvert z_1z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$ to find $\lvert z_2\rvert$	M1
	$\lvert z_2\rvert = 5$	Cao	A1
			(2)
ALT	$\lvert z_1z_2\rvert = 15\sqrt{2}$ $\lvert(3p-3q)+\text{i}(3p+3q)\rvert = 15\sqrt{2}$ $\sqrt{18p^2+18q^2} = 15\sqrt{2}$ $p^2+q^2 = 25$ $\lvert z_2\rvert = \sqrt{p^2+q^2} = 5$	Uses $\lvert z_1z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$ to reach $p^2+q^2 = \ldots$	M1 A1 **(2)**
(b)	$\lvert z_2\rvert = 5 \Rightarrow p^2+q^2 = 25$ $\Rightarrow (-4)^2+q^2 = 25 \Rightarrow q = \ldots$	Uses $p^2+q^2 = "5"^2$ with $p = \pm 4$ leading to a value for q.	M1
	$q = \pm 3$	Both values. Must be clear $p = 4$ has not been used	A1
			(2)
(c)	 Points to be in the correct quadrants and either with correct numbers on the axes or labelled correctly	3 + 3i plotted correctly and labelled Vectors/ lines not needed; point(s) alone are sufficient	B1
		A conjugate pair plotted correctly following through their q.	B1ft
			(2)
			Total 6

Question Number	Scheme	Notes	Marks
2	$\mathrm{f}(x)=10-2x-\dfrac{1}{2\sqrt{x}}-\dfrac{1}{x^3} \qquad x>0$		
(a)	$\mathrm{f}(0.4)=-7.21...,\ \mathrm{f}(0.5)=0.292...$	Attempts both f(0.4) and f(0.5)	M1
	Sign change (positive, negative) and $\mathrm{f}(x)$ is **continuous** therefore (a root) α is between $x=0.4$ and $x=0.5$	Both f(0.4) = awrt -7 and f(0.5) = awrt 0.3, sign change and conclusion. **Must mention continuity**. Can have $\mathrm{f}(0.4)\times\mathrm{f}(0.5)<0$ instead of "sign change"	A1
			(2)
(b)	$\mathrm{f}'(x)=-2+\dfrac{1}{4}x^{-\frac{3}{2}}+3x^{-4}$	$x^n \to x^{n-1}$ in at least 1 term other than 10	M1
		2 of the 3 terms shown correct	A1
		All correct	A1
			(3)
(c)	$x_1=0.5-\dfrac{\mathrm{f}(0.5)}{\mathrm{f}'(0.5)}=0.5-\dfrac{0.29289321...}{46.70710678...}$	Correct application of Newton-Raphson	M1
	$=0.494$	Correct value 3dp. A correct derivative must have been used	A1
			(2)
(d)	$\dfrac{4.9-\beta}{\lvert\mathrm{f}(4.9)\rvert}=\dfrac{\beta-4.8}{\mathrm{f}(4.8)}\Rightarrow\beta=...$	Uses a correct interpolation method (Signs to be correct)	M1
	$\beta=4.883$	Correct value 3dp unless penalised in (c)	A1
			(2)
ALT 1	$\beta=\dfrac{a\lvert\mathrm{f}(b)\rvert+b\lvert\mathrm{f}(a)\rvert}{\lvert\mathrm{f}(a)\rvert+\lvert\mathrm{f}(b)\rvert}$ $\beta=\dfrac{4.8\times0.0344+4.9\times0.1627}{0.0344+0.1627}=...$	Uses a correct interpolation method (Signs to be correct)	M1
	$\beta=4.883$	Correct value 3dp unless penalised in (c)	A1
			(2)
ALT 2	Gradient $=\dfrac{-0.0344-0.1627}{4.9-4.8}=-1.971$ Equation of line: $y-0.1627=-1.971(x-4.8)$ or $y=-1.971+9.6235$ Substitute $y=0$ $x=...$	Complete method for line equation followed by substitution to obtain a value for x	M1
	$\beta=4.883$	Correct value 3dp unless penalised in (c)	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
3(a)	$\mathbf{M}^{-1}=\dfrac{1}{5k-3k}\begin{pmatrix}5 & -k\\ -3 & k\end{pmatrix}$	Attempts $\mathbf{M}^{-1}=\dfrac{1}{\det\mathbf{M}}\times\text{adj}(\mathbf{M})$ Either part correct but $\text{adj}(\mathbf{M})=\mathbf{M}$ scores M0	M1
	$=\dfrac{1}{2k}\begin{pmatrix}5 & -k\\ -3 & k\end{pmatrix}$ or $\begin{pmatrix}\dfrac{5}{2k} & -\dfrac{1}{2}\\ \dfrac{-3}{2k} & \dfrac{1}{2}\end{pmatrix}$	Correct matrix $2k$ must be seen for this mark	A1
			(2)
(b)	$(\mathbf{MN})^{-1}=\mathbf{N}^{-1}\mathbf{M}^{-1}=\dfrac{1}{2k}\begin{pmatrix}k & k\\ 4 & -1\end{pmatrix}\begin{pmatrix}5 & -k\\ -3 & k\end{pmatrix}$	Applies $(\mathbf{MN})^{-1}=\mathbf{N}^{-1}\mathbf{M}^{-1}$	M1
	$=\dfrac{1}{2k}\begin{pmatrix}2k & 0\\ 23 & -5k\end{pmatrix}$ or e.g. $\begin{pmatrix}1 & 0\\ \dfrac{23}{2k} & \dfrac{-5}{2}\end{pmatrix}$	Correct matrix	A1
			(2)
ALT (b)	Find $\mathbf{N}$ (ie inverse of $\mathbf{N}^{-1}$) Find $\mathbf{MN}$ $=-\dfrac{1}{5k}\begin{pmatrix}-5k & 0\\ -23 & 2k\end{pmatrix}$ Find $(\mathbf{MN})^{-1}$	Complete method needed	M1
	$=\dfrac{1}{2k}\begin{pmatrix}2k & 0\\ 23 & -5k\end{pmatrix}$ or e.g. $\begin{pmatrix}1 & 0\\ \dfrac{23}{2k} & \dfrac{-5}{2}\end{pmatrix}$	Correct matrix	A1
			(2)
			Total 4

Question Number	Scheme	Notes	Marks
4	$f(z)=2z^4-19z^3+Az^2+Bz-156$		
(a)	$(z=)5+i$	Correct complex number	B1
			(1)
	Mark (b) and (c) together – ignore any labelling seen. Award marks in the order given for their choice of method		
(b)/(c) With (b) first	$z=5\pm i\Rightarrow(z-(5+i))(z-(5-i))=...$ Or e.g. Sum of roots = 10 Product of roots = 26	Correct strategy to find the quadratic factor using the conjugate pair	M1
	$z^2-10z+26$	Correct quadratic	A1
	$f(z)=(z^2-10z+26)(2z^2+...z+k)$	Attempts to find the other quadratic. May use inspection (apply rules for quadratic factorisation ie $"26"\lvert k\rvert=156$) or e.g. long division with quotient $2z^2+...z+...$	M1
	$\boxed{\text{NB long division gives quotient } 2z^2+z+(A-42) \text{ and remainder } (10A+B-446)z+936-26A}$		
	$2z^2+z-6$	Correct quadratic	A1
	$\Rightarrow z=\frac{3}{2},\ -2\ \ (,5\pm i)$	Correct real roots. The complex roots do not have to be stated.	A1
			(5)
	$f(z)=(z^2-10z+26)(2z^2+z-6)$ $=...$	Multiplies out both quadratics or extracts the terms needed	M1
	$A=36,\ B=86$	Correct values (can be seen in the quartic equation)	A1
			(2)
			Total 8
(b)/(c) With (c) first	$952+960i-2090-24A+10Ai+5Bi-156=0$	Substitute $(5+i)$ into the quartic (by calculator) and equate real and imag parts (can be done with $(5-i)$)	M1
	$-1294+24A+5B=0$ $-446+10A+B=0$	Correct equations	A1
	$A=36\ \ B=86$	M1 Solve simultaneously A1 One correct A1 Both correct	M1 A1A1
			(5)
	$2z^4-19z^3+36z^2+86z-156=0$ $z=...$	Solve the equation by long division, inspection or by calculator	M1
	$\Rightarrow z=\frac{3}{2},\ -2\ \ (,5\pm i)$	Correct real roots. The complex roots do not have to be stated.	A1
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
5	$2x^2 - 3x + 5 = 0$		
(a)	$\alpha + \beta = \frac{3}{2}, \quad \alpha\beta = \frac{5}{2}$	Both	B1
			(1)
(b)(i)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	Uses a correct identity	M1
	$= \left(\frac{3}{2}\right)^2 - 2\left(\frac{5}{2}\right) = -\frac{11}{4} \; (= -2.75)$	Correct value Allow to come from $\alpha + \beta = -\frac{3}{2}$	A1
(ii)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$	Reaches an identity ready for substitution	M1
	$= \left(\frac{3}{2}\right)^3 - 3\left(\frac{3}{2}\right)\left(\frac{5}{2}\right) = -\frac{63}{8} \; (= -7.875)$	Correct value	A1
			(4)
(c)	$\text{Sum} = \alpha^3 + \beta^3 - (\alpha + \beta) = -\frac{63}{8} - \frac{3}{2}\left(= -\frac{75}{8}\right)$	Attempts sum Allow eg $(\alpha^3 - \beta) + (\beta^3 - \alpha)$ followed by $(\alpha^3 + \beta^3) + (\alpha + \beta) = ...$	M1
	$\text{Prod} = (\alpha\beta)^3 - \alpha^4 - \beta^4 + \alpha\beta$ and $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$	Expands $(\alpha^3 - \beta)(\beta^3 - \alpha)$ and uses a correct identity for $\alpha^4 + \beta^4$	M1
	Alt identities: $\alpha^4 + \beta^4 =$ $(\alpha + \beta)^4 - 4\alpha\beta(\alpha^2 + \beta^2) - 6\alpha^2\beta^2$; $\alpha^4 + \beta^4 = (\alpha^3 + \beta^3)(\alpha + \beta) - \alpha\beta(\alpha^2 + \beta^2)$		
	$(\alpha\beta)^3 - \alpha^4 - \beta^4 + \alpha\beta = \left(\frac{5}{2}\right)^3 + \frac{5}{2} - \left(\left(-\frac{11}{4}\right)^2 - 2\left(\frac{5}{2}\right)^2\right) = \frac{369}{16}$		A1
	$x^2 + \frac{75}{8}x + \frac{369}{16} (= 0)$	Applies $x^2 -$ (their sum)x + their prod $(= 0)$	M1
	$16x^2 + 150x + 369 = 0$	Allow any integer multiple	A1
			(5)
			Total 10

Question Number	Scheme	Notes	Marks
6(a)	$x=9t^2, y=18t \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{18}{18t}$ or $y^2=36x \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x}=36 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{18}{y}=\frac{18}{18t}$ or $y^2=36x \Rightarrow y=6\sqrt{x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{\sqrt{x}}=\frac{3}{3t}$	Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t There must be evidence of use of calculus ($\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{t}$ with no working scores B0)	B1
	$m_T=\frac{1}{t} \Rightarrow m_N=-t$	Correct use of the perpendicular gradient rule.	M1
	$y-18t=-t\left(x-9t^2\right)$	Correct straight line method for the normal. Must use their perpendicular gradient – not dy/dx. (Any complete method – use of $y = mx + c$ requires an attempt at "c")	dM1
	$y+tx=9t^3+18t$ *	Cso All previous marks must have been earned	A1*
			(4)
(b)	$x=54, y=0 \Rightarrow 54t=9t^3+18t$ $\Rightarrow 9t^3-36t=0$	Substitutes $x = 54$ and $y = 0$ into the equation from part (a) and attempts to collect terms.	M1
	$9t^3-36t=0 \Rightarrow 9t\left(t^2-4\right)=0$ $\Rightarrow t=\pm 2 \Rightarrow y \pm 2x=9(\pm 2)^3+18(\pm 2)$	Solves to obtain at least one non zero value for t and attempts at least one normal equation	dM1
	$y=-2x+108$ **or** $y=2x-108$	One correct equation in any equivalent form	A1
	$y=-2x+108$ **and** $y=2x-108$	Both correct and in the required form	A1
			(4)
(c)	$x=-9 \Rightarrow y=18+108$ or $-18-108$	Uses $x = -9$ to find the y coordinate of A or B	M1
	Area $=\frac{1}{2}\times 252\times 18$	Fully correct strategy for the area Award M0 if their x coord of the focus is not doubled	M1
	$= 2268$	Cao	A1
			(3)
			Total 11
ALT	Last 2 marks by "shoelace" method: eg $\left\lvert \frac{1}{2}\begin{vmatrix} -9 & 9 & -9 & -9 \\ 126 & 0 & -126 & 126 \end{vmatrix} \right\rvert$ $=\left\lvert\frac{1}{2}\left(9\times-126-9\times126-\left(-9\times-126+9\times126\right)\right)\right\rvert$	Their coordinates with first and last the same ½ must be included Attempt to expand also needed	M1
	$= 2268$	Must be positive	A1

Question Number	Scheme	Notes	Marks
7(a)	$\mathbf{A}^2 = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	Correct matrix	B1
			(1)
(b)	Rotation $-60°$ (anticlockwise) about the origin	Rotation	M1
		$-60°$ (anticlockwise) (Or $60°$ clockwise or $300°$ (anticlockwise)) about (0, 0)	A1
			(2)
(c)	$n = 12$	Cao but can be embedded ie $A^{12} = I$	B1
			(1)
(d)	$\mathbf{B} = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$	Correct matrix	B1
			(1)
(e)	$\mathbf{C} = \mathbf{BA} = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	Multiplies the right way round.	M1
	$\mathbf{C} = \begin{pmatrix} -2\sqrt{3} & -2 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$	Correct matrix Accept unsimplified	A1
			(2)
(f)	$\det \mathbf{C} = -2\sqrt{3} \times -\frac{\sqrt{3}}{2} - \frac{1}{2}(-2) = 4$ So area of P is $\frac{20}{\det \mathbf{C}} = \ldots$	Attempts determinant of **C** (or deduces area scale factor is 4) and divides into 20	M1
	$= 5$	Cao Must follow a correct matrix in (e)	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
8	$\sum_{r=0}^{n}(r+1)(r+2)$		
(a)	$\sum_{r=0}^{n} r^2+3r+2 = 2+\frac{1}{6}n(n+1)(2n+1)+\frac{3}{2}n(n+1)+2n$ M1: Attempt to use at least one of the standard formulae correctly A1: For $\frac{1}{6}n(n+1)(2n+1)+\frac{3}{2}n(n+1)+(2n \text{ or } 2n+2)$ A1:Fully correct expression		M1A1A1
	$\frac{1}{6}n(n+1)(2n+1)+\frac{3}{2}n(n+1)+2n+2=(n+1)\left[\frac{1}{6}n(2n+1)+\frac{3}{2}n+2\right]$ Attempt to factorise $(n+1)$ It is a "show" question so this must be seen (in any equivalent form). If their expression does not allow for factorising $(n+1)$ score M0		M1
	$\frac{1}{3}(n+1)\left[n^2+5n+6\right]$	May obtain a cubic and extract a different factor ie $n+2$ or $n+3$	
	$\frac{1}{3}(n+1)(n+2)(n+3)$*	Cso At least one intermediate step in the working must be seen.	A1*
			(5)
(a) Way 2	$\sum_{r=0}^{n}(r+1)(r+2) = \sum_{r=1}^{n+1}r(r+1)$ $=\sum_{r=1}^{n+1}r^2+r=\frac{1}{6}(n+1)(n+2)(2(n+1)+1)+\frac{1}{2}(n+1)(n+2)$ M1: Attempt to use at least one of the standard formulae correctly with $n = n + 1$ A1: For $\frac{1}{6}(n+1)(n+2)(2(n+1)+1)$ or $\frac{1}{2}(n+1)(n+2)$ A1:Fully correct expression		M1A1A1
	$\frac{1}{6}(n+1)(n+2)(2(n+1)+1)+\frac{1}{2}(n+1)(n+2)=(n+1)\left[\frac{1}{6}(n+1)(2n+3)+\frac{1}{2}(n+2)\right]$ Attempt to factorise $(n+1)$ (see additional comments above)		M1
	$\frac{1}{3}(n+1)\left[n^2+5n+6\right]$	May obtain a cubic and extract a different factor ie $n+2$ or $n+3$	
	$\frac{1}{3}(n+1)(n+2)(n+3)$*	Cso At least one intermediate step in the working must be seen.	A1*
(b)	Upper limit = 99	Correct upper limit	B1
	$10\times11+11\times12+12\times13+...+100\times101=\sum_{r=0}^{99}(r+1)(r+2)-\sum_{r=0}^{8}(r+1)(r+2)$ Fully correct strategy for the sum using their upper limit for the first sum and upper limit 8 for the second in the result from (a). Lower limits 0 or 1		M1
	$=\frac{1}{3}(100)(101)(102)-\frac{1}{3}(9)(10)(11)$ = 343 070	Correct value	A1

			(3)
			Total 8

Question Number	Scheme	Notes	Marks
9(i)	$u_n = 5\times 2^{n-1} - n\times 2^n$		
	$n=1 \Rightarrow u_1 = 5\times 2^0 - 1\times 2 = 3$ (Shows the result is true for $n = 1$)		B1
	Assume true for $n = k$ so that $u_k = 5\times 2^{k-1} - k\times 2^k$		
	$u_{k+1} = 2\left(5\times 2^{k-1} - k\times 2^k\right) - 2^{k+1}$	Attempts u_{k+1} using the recurrence relationship	M1
	$= 5\times 2^k - k\times 2^{k+1} - 2^{k+1}$	Correct expanded expression	A1
	$= 5\times 2^k - (k+1)2^{k+1}$	Achieves this result with no errors	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted.		
			(5)
(ii)	$\mathrm{f}(n) = 5^{n+2} - 4n - 9$		
	$\mathrm{f}(1) = 125 - 4 - 9 = 112 = 16\times 7$	Shows f(1) is divisible by 16 Either of 112 or 16×7 must be seen	B1
	Assume true for $n = k$ so that $5^{k+2} - 4k - 9$ is divisible by 16		
	$\mathrm{f}(k+1) = 5^{k+3} - 4(k+1) - 9$	Attempts f(k + 1)	M1
	$= 5\times\left(5^{k+2} - 4k - 9\right) + \ldots$	Attempts to express in terms of f(k)	dM1
	$= 5\times\left(5^{k+2} - 4k - 9\right) + 16k + 32$	Correct expression for f(k + 1)	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		**(5)**
			Total 10

ii ALT 1	$f(1)=125-4-9=112=16\times7$	Shows f(1) is divisible by 16 Either of 112 or 16×7 must be seen	B1
	Assume $5^{k+2}-4k-9$ is divisible by 16		
	$f(k+1)-mf(k)=5^{k+3}-4(k+1)-9-m\left(5^{k+2}-4k-9\right)$ Attempt f(k+1) – mf(k)		M1
	$=(5-m)\left(5^{k+2}-4k-9\right)+\ldots$	Attempts to express in terms of f(k)	dM1
	$f(k+1)=5\times\left(5^{k+2}-4k-9\right)+16k+32$	Correct expression for f(k + 1)	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		
ii ALT 2	$f(1)=125-4-9=112=16\times7$	Shows f(1) is divisible by 16 Either of 112 or 16×7 must be seen	B1
	Assume $5^{k+2}-4k-9$ is divisible by 16		
	$f(k+1)-f(k)=5^{k+3}-4(k+1)-9-\left(5^{k+2}-4k-9\right)$ Attempt f(k+1) – f(k)		M1
	$f(k+1)-f(k)=5\times5^{k+2}-5^{k+2}-4k-4-9+4k+9$ $=4\times5^{k+2}-4=4\left(5^{k+2}-1\right)$ Obtains a simplified expression for the difference **and** attempts to prove $\left(5^{k+2}-1\right)$ is divisible by 4 using induction		dM1
	Correct proof for $\left(5^{k+2}-1\right)$ being divisible by 4 and states that thus as the difference is divisible by 16, $f(k+1)$ is divisible by 16		A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1 cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		
ii ALT 3	$f(1)=125-4-9=112=16\times7$	Shows f(1) is divisible by 16 Either of 112 or 16×7 must be seen	B1
	f(k) is divisible by 16 so set $f(k)=16\lambda$		
	$5^{k+2}=16\lambda+4k+9$		M1
	$f(k+1)=5^{k+3}-4(k+1)-9$ $=5\times5^{k+2}-4k-13=5(16\lambda+4k+9)-4k-13$	Expresses f(k + 1) in terms of λ and k and collects terms	dM1
	$=80\lambda+16k+32$	Correct expression May have factor of 16 taken out	A1
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.		A1cso
	The final mark depends on all except the B mark, though a check for $n = 1$ must have been attempted		