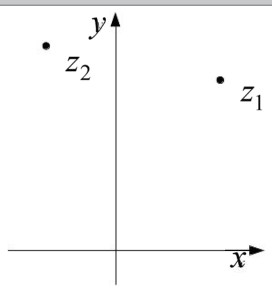
Question	Scheme	Marks
1.	$\det \mathbf{M} = 3x \times (2-x) - (4x+1) \times 7 = \ldots$	M1
	$= -3x^2 - 22x - 7$ or $3x^2 + 22x + 7$	A1
	$-3x^2 - 22x - 7 = 0 \Rightarrow (-3x-1)(x+7) = 0 \Rightarrow x = \ldots$	M1
	$-3x^2 - 22x - 7 > 0 \Rightarrow "-7" < x < "-\frac{1}{3}"$	M1
	So range is $-7 < x < -\frac{1}{3}$ or $(x \in)\left(-7, -\frac{1}{3}\right)$	A1
		(5)
		(5 marks)

Notes:

M1: Attempts to expand the determinant of **M.** Allow with + between the 2 products.
A1: Correct simplified quadratic with = or an inequality sign or neither
M1: Attempts to solve their three term quadratic, any valid means (usual rules – see front pages). Correct answers seen implies correct method. Can be awarded even if the roots are complex.
M1: Chooses the inside region for their roots, accept with strict or loose inequalities.
A1: Correct answer. Accept $x > -7 \cap x < -\frac{1}{3}$

Question	Scheme	Marks
2(a)		M1 A1
		(2)
(b) (i)	$\lvert z_1 \rvert = \sqrt{3^2+5^2} = \sqrt{34}$	B1
		(1)
(ii)	$\dfrac{z_1}{z_2} = \dfrac{3+5\text{i}}{-2+6\text{i}} \times \dfrac{-2-6\text{i}}{-2-6\text{i}} = \ldots$	M1
	$= \dfrac{-6-18\text{i}-10\text{i}+30}{40}$	A1
	$= \dfrac{3}{5} - \dfrac{7}{10}\text{i}$	A1
		(3)
(c)	$\arg\dfrac{z_1}{z_2} = \arctan\dfrac{-7/10}{3/5} = \arctan\dfrac{-7}{6} = \ldots$ but allow $\arctan\dfrac{7}{6}$ for M1	M1
	$= -0.86$ or 5.42 (awrt)	A1
		(2)
		(8 marks)

Notes:

(a)

M1: Points in correct quadrants - z_1 in quadrant 1 and z_2 in quadrant 2. Must be clearly labelled either eg z_1 or 3 + 5i or correct numbers on the axes. (Accept with vector arrows.)

A1: Correct diagram, z_1 in first quadrant further away from real axis than imaginary and z_2 in second quadrant, closer to imaginary axis but above z_1 **OR** correct nos on their axes (imag axis may include i), **but not** dashes w/o any indication of scale.

Allow M1A0 for points unlabelled but diagram otherwise correct.

(b)(i)

B1: Correct modulus. Must be evaluated to $\sqrt{34}$ Question says "without using your calculator" so decimal answers can be ignored (isw) but exact answer must be seen somewhere.

(ii)

M1: Multiplies numerator and denominator by the conjugate of their denominator.

A1: Correct unsimplified (or simplified) numerator, with the i^2 correctly dealt with, and correct denominator.

A1: Correct answer. Allow as shown, $\dfrac{6}{10} - \dfrac{7}{10}\text{i}$, or 0.6 - 0.7i.

(c)

M1: For $\arctan\left(\pm"\frac{7}{6}"\right)$ (not necessarily simplified to this) or $\tan\alpha = \pm\frac{7}{6}$ $\quad \alpha = ...$ This mark is available if answer is given in degrees. Can use $\arctan\left(\pm"\frac{6}{7}"\right)$ provided a complete method to reach the correct arg is seen.

A1: For awrt -0.86 or awrt 5.42 Must be radians.

ALT for (c):

M1: Use argz_1 – arg z_2 correctly

A1: Correct answer

Question	Scheme	Marks
3(a)	$\left(\frac{9}{2},0\right)$	B1
		(1)
(b)	$PS=9$	B1
	$x_P=-\frac{9}{2}+9=\frac{9}{2}\Rightarrow OP=\sqrt{\left(\frac{9}{2}\right)^2+\left(18\times\frac{9}{2}\right)}=\ldots$	M1
	So perimeter = $"\frac{9}{2}"+"9"+"\frac{9\sqrt{5}}{2}"$	dM1
	$=\frac{27+9\sqrt{5}}{2}$ oe	A1
		(4)
		(5 marks)

Notes:

(a)
B1: Correct coordinates**.**
(b)
B1: Deduces $PS=9$ from the focus directrix property (may be implied by seeing it embedded in an expression for the perimeter). May find coordinates of P first and then attempt Pythagoras theorem – must be correct. May be seen on the diagram. Allow even if incorrect value used later.
M1: Uses distance from directrix to find x coordinate of P and goes on to find OP by Pythagoras (with a plus sign).
dM1: Sums their **three** side lengths. Extras – including 0 – score M0. Depends on the previous M mark.
A1: Correct answer. Equivalents must be in simplified surd form.

Question	Scheme	Marks
4(a)	$4-3\text{i}$	B1
		(1)
(b)	$(x-(4+3\text{i}))(x-(4-3\text{i}))=...$	M1
	$x^2-8x+25$	A1
		(2)
(c)	E.g. Product of roots is 225, so product of real roots is $\frac{225}{25}=9$ Or $x^4+Ax^3+Bx^2+Cx+225=(x^2-8x+25)(x^2+...+9)$	M1
	Hence (as root is positive) repeated real root is 3	A1
		(2)
(d)	$(x^2-8x+25)(x^2-6x+9)$ $=x^4-6x^3+9x^2-8x^3+48x^2-72x+25x^2-150x+225$	M1
	$=x^4-14x^3+82x^2-222x+225$ Two correct middle term coefficients	A1
	So$A=-14$, $B=82$ and $C=-222$ (or accept in the quartic)	A1
		(3)
		(8 marks)

Notes:

(a)

B1: For 4 – 3i

(b)

M1: Correct strategy to find a quadratic factor. May expand as shown in scheme, or may look for sum of roots and product of roots first and then write down the factor.

A1: Correct quadratic factor. Can be written down – give M1A1 if correct, M0A0 if incorrect. Ignore "= 0" with their quadratic factor.

Alt for (b):

M1: Product of complex roots is 25, so product of real roots is $\frac{225}{25}=9$, so the (positive) real root is "3", hence quadratic factor is $(x-"3")^2$

A1: x^2-6x+9 **or** $(x-3)^2$

(c)

M1: A complete strategy to deduce the real root or its square. May consider product of roots, as in scheme, or may first attempt to factorise/long division to find the other quadratic factor – award at the point the quadratic factor with real roots is found. May have been seen in (b)

A1: Real root is 3. (No need to see rejection of the negative possibility.)

Not a "show that" so award M1A1 if correct root is written down with no working.

(d) **M1:** Attempts to expand the two quadratic factors – one of which must have a repeated root, so $\left(x^2 \pm 9\right)$ scores M0. (Alternative, may apply –(sum of roots) to find A, pair sum to find B etc – accept method for at least two constants.) **A1:** Two correct values of the three. Accept as embedded in a quartic equation. **A1:** All three correct. Accept as embedded in their quartic equation. If their answers are wrong a correct method would get M1A0A0 but w/o some working score M0

Question	Scheme	Marks
5(a)	Two of: Rotation; about O; through $60°$ $\left(\frac{\pi}{3}\right)$ (anticlockwise)	M1
	All of: Rotation about O through $60°$ $\left(\frac{\pi}{3}\right)$ (anticlockwise)	A1
		(2)
(b)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1
		(1)
(c)	$\mathbf{R} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	M1
	$= \begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ **QP** correctly found	A1
		(2)
(d)	$3\mathbf{R} = \begin{pmatrix} -\frac{3\sqrt{3}}{2} & -\frac{3}{2} \\ -\frac{3}{2} & \frac{3\sqrt{3}}{2} \end{pmatrix}$ or correctly deals with 3 as a multiple.	B1ft
	Required matrix is $(3\mathbf{R})^{-1} = \frac{1}{\left(-\frac{3\sqrt{3}}{2}\right)\left(\frac{3\sqrt{3}}{2}\right) - \left(-\frac{3}{2}\right)\left(-\frac{3}{2}\right)}\begin{pmatrix} \frac{3\sqrt{3}}{2} & \frac{3}{2} \\ \frac{3}{2} & -\frac{3\sqrt{3}}{2} \end{pmatrix} = \ldots$ Or $(\mathbf{R})^{-1} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)}\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1\sqrt{3}}{2} \end{pmatrix} = \ldots$	M1
	$(3\mathbf{R})^{-1} = \frac{1}{-9}\begin{pmatrix} \frac{3\sqrt{3}}{2} & \frac{3}{2} \\ \frac{3}{2} & -\frac{3\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{6} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{\sqrt{3}}{6} \end{pmatrix}$	A1
		(3)
		(8 marks)

Notes:
(a) **M1:** Two aspects of the type, centre of rotation and angle correct. Accept equivalent angles or angle in radians. (E.g. 300° clockwise is fine). Assume anticlockwise unless otherwise stated. **A1:** Fully correct description. Accept just 60° for the angle, but 60° clockwise is incorrect **(b)** **B1:** Correct matrix. **(c)** **M1:** Attempts to multiply **Q** and **P** in the correct order. **A1: QP** correct **(d)** **B1ft:** Multiplies all elements of their matrix by 3, or multiplies all elements of their $\mathbf{R}^{-1}$ by $\frac{1}{3}$ **M1:** Attempts the inverse of their 3**R** or **R**. This must be a complete method – ie must transpose and evaluate the determinant and use it. Alternatively, they may attempt an inverse from first principles. Award this mark if a slip is made in solving their simultaneous equations. **A1:** Correct answer. Accept alternative forms

Question	Scheme	Marks
6(a)	(i) $\alpha+\beta=-\frac{5}{A}$	B1
	(ii) $\alpha\beta=-\frac{12}{A}$	B1
		(2)
(b)	$\left(\alpha-\frac{3}{\beta}\right)+\left(\beta-\frac{3}{\alpha}\right)=(\alpha+\beta)-3\left(\frac{\alpha+\beta}{\alpha\beta}\right)=-\frac{5}{A}-3\left(\frac{-5}{A}\right)\times\frac{-A}{12}$	M1
	$-\frac{5}{A}-\frac{15}{12}=\frac{5}{4}\Rightarrow A=\ldots$	dM1
	$A=-2$	A1
		(3)
(c)	$\left(\alpha-\frac{3}{\beta}\right)\left(\beta-\frac{3}{\alpha}\right)=\alpha\beta-6+\frac{9}{\alpha\beta}=-\frac{12}{A}-6+\frac{9}{-12/A}$	M1
	$-\frac{12}{\text{"}-2\text{"}}-6-\frac{9\text{"}-2\text{"}}{12}=\frac{B}{4}\Rightarrow B=\ldots$	dM1
	$B=6$	A1
		(3)
		(8 marks)

Notes:

(a)
(i) B1: Correct expression for $\alpha+\beta$
(ii) B1: Correct expression for $\alpha\beta$
(b)
M1: Attempts the sum of roots for second equation in terms of A using results from (a). Allow slips in signs.

dM1: Equates the sum of roots to $\frac{5}{4}$ and solves for A. Depends on the previous M mark.

A1:$A=-2$
(c)
M1: Attempts the product of roots for second equation in terms of A using results from (a). Allow slips in signs. May be using their value of A or A itself

dM1: Equates the product of roots to $\frac{B}{4}$ and solves for B using their value of A. Depends on first M mark of (c).
A1: $B=6$

Question	Scheme	Marks
7(a)	$\dfrac{dy}{dx} = -\dfrac{36}{x^2}$ oe	B1
	$m_t = -\dfrac{36}{4^2} \Rightarrow m_n = \dfrac{16}{36} = \dfrac{4}{9}$	M1
	Normal is $y - 9 = \dfrac{4}{9}(x-4)$	M1
	$\Rightarrow 9y - 81 = 4x - 16 \Rightarrow 4x - 9y + 65 = 0$ *	A1*
		(4)
(b)	Normal meets H again when $4x - 9\times\dfrac{36}{x} + 65 = 0$ or $4\times\dfrac{36}{y} - 9y + 65 = 0$	M1
	$\Rightarrow 4x^2 + 65x - 324 = 0 \Rightarrow x = \ldots$ or $9y^2 - 65y - 144 = 0 \Rightarrow y = \ldots$	dM1
	$\Rightarrow Q = \left(-\dfrac{81}{4}, -\dfrac{16}{9}\right)$	A1
	At $x = -\dfrac{81}{4}$, $\dfrac{dy}{dx} = -\dfrac{36}{\left(-\dfrac{81}{4}\right)^2} = \ldots$ so tangent is $y - \left(-\dfrac{16}{9}\right) = -\dfrac{64}{729}\left(x - \left(-\dfrac{81}{4}\right)\right)$	M1
	$y = -\dfrac{64}{729}x - \dfrac{32}{9}$	A1(5)
		(9 marks)

Notes:

(a)

B1: Correct expression for $\dfrac{dy}{dx}$, or any equivalent correct expression including it, such as $x\dfrac{dy}{dx} + y = 0$ or $\dfrac{dy}{dx} = -\dfrac{1}{t^2}$

M1: Attempts negative reciprocal gradient at the point P

M1: Uses their normal (changed from tangent) gradient and $P(4,9)$ to find the equation of the normal. Look for $y - 9 = "m_n"(x-4)$ Working must be shown for their constant if $y = mx + c$ is used as this is a "show that" question.

A1*: Correct equation achieved from correct working with intermediate step.

(b)

M1: Substitutes hyperbola equation into the **given** normal to obtain an equation in one variable. Other valid means of obtaining an equation in a single variable are acceptable.

dM1: Gathers terms and solves the 3 term quadratic to find a value $\neq 4$ for x or $\neq 9$ for y. Solution by calculator allowed if correct roots (or values $\neq 4$ for x or $\neq 9$ for y) are shown

A1: Correct coordinates of intersection.

M1: Uses their x value to find the gradient at Q and then uses the intersection point with their gradient to form the equation of the line.

A1: Correct equation.

Question	Scheme		Marks
8(a)	x: 1, 2, 3, 4, 5; f(x): 0.5, **−1.2401**, −0.2885, **0.1508**, 0.5840	One correct Both correct	B1 B1
			(2)
(b)	Identify an interval where the sig changes and mention the change of sign		M1
	f is continuous on [3,4], not on [1,2] hence the root is in [3,4]		A1
			(2)
(c)	$\mathrm{f}(3.5) = -0.064... < 0$ so root in [3.5,4]		M1
	$\mathrm{f}(3.75) = 0.0435... > 0$		M1
	Hence root is in the interval [3.5, 3.75]		A1
			(3)
(d)	E.g. $\dfrac{\mathrm{f}(-0.5)-\mathrm{f}(-1)}{-0.5-(-1)} = \dfrac{\mathrm{f}(-0.5)-0}{-0.5-\beta}$ or $\dfrac{\beta-(-1)}{0-\mathrm{f}(-1)} = \dfrac{\beta-(-0.5)}{0-\mathrm{f}(-0.5)}$ etc		M1
	$\Rightarrow \beta = -0.5 - \dfrac{0.5\times \mathrm{f}(-0.5)}{\mathrm{f}(-0.5)-\mathrm{f}(-1)} = ...$ or $\beta = \dfrac{-0.5\mathrm{f}(-1)+\mathrm{f}(-0.5)}{(\mathrm{f}(-1)-\mathrm{f}(-0.5))} = ...$ etc		dM1
	$= -0.5 - \dfrac{0.5\times 0.5786...}{0.5786...-(-0.875)} = -0.6990... = -0.699$ (awrt)		A1
			(3)
			(10 marks)

Notes:

Accept open or closed intervals throughout the question where relevant and intervals described by inequalities.

(a)

B1: One correct value of the two missing.

B1: Both values correct.

(b)

M1: Identifies at least one of the intervals on which a sign change occurs – must mention sign changing.

A1: Correct interval with reason given. Accept reasons such as f not defined at $\frac{5}{3}$ in [1,2] or $x = \frac{5}{3}$ is an asymptote as reason for dismissing this interval.

(c)

M1: Evaluates f at the midpoint of their chosen interval from (b) and selects interval of length 0.5 in which the root lies. This mark can be awarded if the interval was incorrect (even if no change of sign in that interval)

M1: Evaluates f at the midpoint of their interval of length 0.5, and considers the signs or chooses the "correct" interval of length 0.25. There must have been a change of sign in their initial interval for this mark to be awarded.

A1: Correct interval selected with all values correct to at least 1 s.f. rounded or truncated. No extra intervals included.

(d)

M1: Correct interpolation strategy. Accept any correct statement such as the one shown. Sign errors imply an incorrect formula unless they follow a correct general statement.

dM1: Rearranges to find β and evaluates.

A1: Accept awrt -0.699 following correct working.

Question	Scheme	Marks
9(a)	For $n=1$, $\sum_{r=1}^{1} r^3 = 1$ and $\frac{1}{4}(1^2)(1+1)^2 = \frac{1}{4}\times 1\times 4 = 1$ So true for $n = 1$	B1
	(Assume the result is true for $n=k$, so $\sum_{r=1}^{k} r^3 = \frac{1}{4}k^2(k+1)^2$) Then $\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$	M1
	$= \frac{1}{4}(k+1)^2\left[k^2+4(k+1)\right] = \frac{1}{4}(k+1)^2\left[k^2+4k+4\right]$	M1
	$= \frac{1}{4}(k+1)^2(k+2)^2$	A1
	$\left[= \frac{1}{4}(k+1)^2((k+1)+1)^2\right]$ Hence result is true for $n = k+1$. As **true for $n = 1$** and have shown **if true for $n = k$ then it is true for $n = k+1$, so it is true for all** $n \in \mathbb{N}$ by induction.	A1
		(5)
(b)	$\sum_{r=1}^{n} r(r+1)(r-1) = \sum_{r=1}^{n} r^3 - r$	B1
	$= \frac{1}{4}n^2(n+1)^2 - \frac{1}{2}n(n+1)$	M1
	(Please note the mark above is incorrectly labelled as A1 on e-PEN)	
	$= \frac{1}{4}n(n+1)\left[n^2+n-2\right] = \frac{1}{4}n(n+1)(n+...)(n+...)$	M1
	$= \frac{1}{4}n(n+1)(n-1)(n+2)$	A1
		(4)
(c)	$\sum_{r=n}^{2n} r^2 = \frac{1}{6}(2n)(2n+1)(2(2n)+1) - \frac{1}{6}(n-1)(n)(2(n-1)+1)$	M1 A1
	$3\sum_{r=1}^{n} r(r+1)(r-1) = 17\sum_{r=n}^{2n} r^2$ $\Rightarrow \frac{3}{4}n(n+1)(n-1)(n+2) = \frac{17}{6}n\left(2(8n^2+6n+1)-(2n^2-3n+1)\right)$ $\Rightarrow 18(n+1)(n-1)(n+2) = 68\left(14n^2+15n+1\right) = 68(14n+1)(n+1)$	dM1

	$\Rightarrow 18(n-1)(n+2)=68(14n+1)$ $\Rightarrow 18n^2-934n-104=0 \Rightarrow n=...$	**ddM1**
	$n = 52$	**A1**
		(5)
		(14 marks)

Notes:

(a)

B1: Checks the result for $n = 1$. Should see a clear substitution into both sides, accept minimum of seeing $\frac{1}{4}\times1\times4$, or $\frac{1}{4}\times1\times2^2$, or $\frac{1}{4}\times1\times(1+1)^2=1$ for right hand side.

M1:(Makes or assumes the inductive assumption, and) adds $(k + 1)^3$ to the result for $n = k$

M1: Attempts to take at least $(k+1)^2$ as a factor out of the expression. Allow if an expansion to a quartic is followed by the factorised expression.

A1: Reaches the correct expression for $n = k + 1$from correct working with sufficient working seen, so expect at least seeing the quadratic before a factorised form.. Need not see the "k+1" explicitly for this mark.

A1: Completes the induction by demonstrating the result clearly, with suitable conclusion conveying "true for $n = 1$", "assumed true for $n = k$" and "shown true for $n = k + 1$", and "hence true for all n". All these statements (or equivalents) must be seen in their conclusion (not simply scattered through the work). Depends on all except the B mark, though a check for $n = 1$ must have been attempted.

(b)

B1: Correct expansion.

M1: Applies the standard formula for $\sum r$ and the result from (a) to their sum.

If the expansion is given as $\sum r^3 - r^2$, allow the use of $\sum r^2$ instead of $\sum r$

M1: Takes out the common factors n and $(n+1)$ and attempts to simplify to required form OR factorises their quartic.

A1: Correct answer. (Ignore A, B and C listed explicitly.) Correct answer can be obtained from a cubic or a quartic. Award M1A1 in either of these cases.

(c)

M1: Attempts to apply $\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n-1} r^2$ with the standard result for $\sum r^2$ Accept with n instead of $n-1$ in second expression.

A1: Correct expression for the RHS seen, no need to be simplified.

dM1: Applies the summations to the equation in the question and cancels/factorises out the factor n. Depends on the first M mark of (c)

ddM1: Simplifies the quadratic factor of the right hand side and cancels/factors out the n+1 and solves the resulting quadratic. Note M1M1 is implied by sights of the correct roots $-\frac{1}{9}, 52, 0, -1$ of the quartic. Depends on both previous M marks in (c)

A1: Correct answer. Must reject other roots. The correct answer obtained from a quartic or cubic equation solved by calculator gains all relevant marks.