# Pearson Edexcel A Level Mathematics 9MA0

## Unit Test 8 Differentiation

**Time allowed:** 50 **minutes**

| Question | Points | Score |
|---|---|---|
| 1 | 6 | |
| 2 | 8 | |
| 3 | 6 | |
| 4 | 5 | |
| 5 | 5 | |
| 6 | 8 | |
| 7 | 6 | |
| 8 | 2 | |
| 9 | 4 | |
| Total: | 50 | |

**School:** www.CasperYC.club

**Name:**

**Teacher:**

**How I can achieve better:**

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1. (a) Given that $f(x) = \sin(x)$, show that [4]

$$f'(x) = \lim_{h \to 0} \left[ \left( \frac{\cos(h) - 1}{h} \right) \sin(x) + \frac{\sin(h)}{h} \cos(x) \right]$$

(b) Hence prove that $f'(x) = \cos(x)$. [2]

Total: 6

2. A toy soldier is connected to a parachute.

   The soldier is thrown into the air from ground level.

   The height, in metres, of the soldier above the ground can be modelled by the equation

$$H = \frac{4t^{\frac{2}{3}}}{t^2 + 1} \qquad 0 \leq t \leq 6s$$

   where $H$ is height of the soldier above the ground and $t$ is the time since the soldier was thrown.

   (a) Show that [4]

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{8(1 - 2t^2)}{3\sqrt[3]{t}(t^2 + 1)^2}$$

   (b) Using the differentiated function, explain whether the soldier was increasing or decreasing in height after 2 seconds. [2]

   (c) Find the exact time when the soldier reaches a maximum height. [2]

Total: 8

3. A curve has the equation [6]

$$y = \ln(3x) - e^{-2x}.$$

Show that the equation of the tangent at the point with an $x$-coordinate of 1 is

$$y = \left(\frac{e^2 + 2}{e^2}\right)x - \left(\frac{e^2 + 3}{e^2}\right) + \ln(3).$$

4. Given that $x = \sec(4y)$, find

   (a) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $y$. [2]

   (b) Show that [3]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{x\sqrt{x^2 - 1}}$$

   where $k$ is a constant which should be found.

Total: 5

5. A curve $C$ has equation $4^x = 2xy$ for $x > 0$. [5]

   Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point $C$ with coordinates $(2, 4)$.

6. A curve has parametric equations

$$x = \cos(2t), \quad \text{and} \quad y = \sin(t), \qquad -\pi \leq t \leq \pi.$$

(a) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $t$. [3]

Leave your answer as a single trigonometric ratio.

(b) Find an equation of the normal to the curve at the point $A$ where $t = -\dfrac{5\pi}{6}$. [5]

Total: 8

7. The curve $C$ has equation $y = x^3 + 6x^2 - 12x + 6$.

   (a) Show that $C$ is concave on the interval $[-5, -3]$. [3]

   (b) Find the coordinates of the point of inflection. [3]

Total: 6

8. In a rainforest, the area covered by trees, $F$, has been measured every year since 1990. [2]

   It was found that the rate of loss of trees is proportional to the remaining area covered by trees.

   Write down a differential equation relating $F$ to $t$, where $t$ is the numbers of years since 1990.

9. The volume of a sphere $V$ cm$^3$ is related to its radius $r$ cm by the formula $V = \frac{4}{3}\pi r^3$. [4]

   The surface area of the sphere is also related to the radius by the formula $S = 4\pi r^2$.

   Given that the rate of decrease in surface area, in cm$^2$s$^{-1}$, is $\frac{\mathrm{d}S}{\mathrm{d}t} = -12$, find the rate of decrease of volume $\frac{\mathrm{d}V}{\mathrm{d}t}$.