Pearson Edexcel

A Level Mathematics 9MA0

Unit Test

9 Numerical Methods

Time allowed: 50 **minutes**

School:

Name:

Teacher:

Question	Points	Score
1	7	
2	5	
3	7	
4	10	
5	10	
6	11	
Total:	50	

1. $f(x) = x^4 - 8x^2 + 2$.

   (a) Show that the equation $f(x) = 0$ can be written as [2]

$$x = \sqrt{ax^4 + b}, x > 0$$

   where a and b are constants to be found.

   (b) Let $x_0 = 1.5$. Use the iteration formula [2]

$$x_{n+1} = \sqrt{ax_n^4 + b}$$

   together with your values of a and b from part (a), to find, to 4 decimal places, the values of x_1, x_2, x_3 and x_4.

   A root of $f(x) = 0$ is α.

   (c) By choosing a suitable interval, prove that $\alpha = -2.782$ to 3 decimal places. [3]

Total: 7

2.

$$g(x) = 3\sin\left(\frac{x}{6}\right)^3 - \frac{1}{10}x - 1, -40 < x < 20,$$

   x is in radians.

   (a) Show that the equation $g(x) = 0$ can be written as [3]

$$x = 6\left(\sqrt[3]{\arcsin\left(\frac{1}{3} + \frac{1}{30}x\right)}\right)$$

   (b) Using the formula [2]

$$x_{n+1} = 6\left(\sqrt[3]{\arcsin\left(\frac{1}{3} + \frac{1}{30}x_n\right)}\right),$$

   with $x_0 = 4$, find to 3 decimal places, the values of x_1, x_2 and x_3.

Total: 5

3. $f(x) = 2 - 3\sin^3(x) - \cos(x)$, where x is in radians.

   (a) Show that $f(x) = 0$ has a root α between $x = 1.9$ and $x = 2.0$. [2]

(b) Using $x_0 = 1.95$ as a first approximation, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places. [5]

Total: 7

4. $g(x) = \frac{2}{x-1} - e^x$

(a) By drawing an appropriate sketch, show that there is only one solution to the equation $g(x) = 0$. [2]

(b) Show that the equation $g(x) = 0$ may be written in the form $x = 2e^{-x} + 1$ [2]

(c) Let $x_0 = 1.5$. [2]

Use the iterative formula to find to 4 decimal places the values of x_1, x_2, x_3 and x_4.

(d) Using $x_0 = 1.5$ as a first approximation, apply the Newton-Raphson procedure once to $g(x)$ to find a second approximation to α, giving your answer to 4 decimal places. [4]

Total: 10

5.

$$h(t) = 40\ln(t+1) + \sin\left(\frac{t}{5}\right) - \frac{1}{4}t^2, \quad t \geq 0$$

The graph $y = h(t)$ models the height of a rocket t seconds after launch.

(a) Show that the rocket returns to the ground between 19.3 and 19.4 seconds after launch. [2]

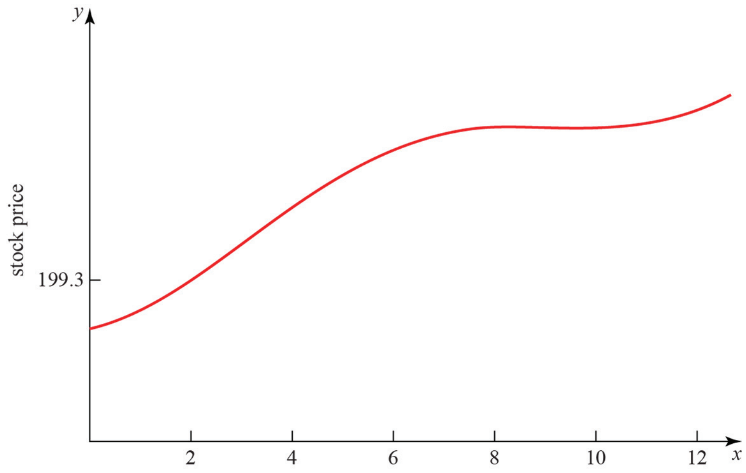
(b) Using $t_0 = 19.35$ as a first approximation to α, apply the Newton-Raphson procedure once to $h(t)$ to find a second approximation to α, giving your answer to 3 decimal places. [5]

(c) By considering the change of sign of $h(t)$ over an appropriate interval, determine if your answer to part (b) is correct to 3 decimal places. [3]

Total: 10

6.

$$p(t) = \frac{1}{10}\ln(t+1) - \cos\left(\frac{t}{2}\right) + \frac{1}{10}t^{\frac{3}{2}} + 199.3, \qquad 0 \leq t \leq 12$$



(a) Above is a graph of the price of a stock during a 12-hour trading window. The equation of the curve is given above. Show that the price reaches a local maximum in the interval $8.5 < t < 8.6$. [5]

(b) Above shows that the price reaches a local minimum between 9 and 11 hours after trading begins. Using the Newton-Raphson procedure once and taking $t_0 = 9.9$ as a first approximation, find a second approximation of when the price reaches a local minimum. [6]

Total: 11