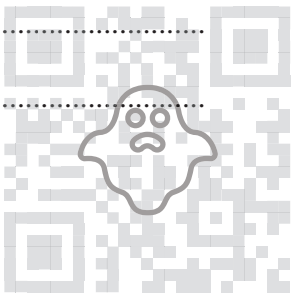
**1** Solve the equation $2^{3x-1} = 5(3^{1-x})$. Give your answer in the form $\frac{\ln a}{\ln b}$ where $a$ and $b$ are integers. [4]



**2** The polynomial $2x^3 - x^2 + a$, where $a$ is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 3)$ is a factor of $\mathrm{p}(x)$.

**(a)** Find the value of $a$. [2]

**(b)** When $a$ has this value, solve the inequality $\mathrm{p}(x) < 0$. [4]

**3** The equation of a curve is $y = \sin x \sin 2x$. The curve has a stationary point in the interval $0 < x < \frac{1}{2}\pi$.

Find the $x$-coordinate of this point, giving your answer correct to 3 significant figures. [6]

**4** **(a)** Express $4\cos x - \sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of $R$ and give $\alpha$ correct to 2 decimal places. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**(b)** Hence solve the equation $4\cos 2x - \sin 2x = 3$ for $0° < x < 180°$. [5]

.....

.....

.....

.....

.....

.....

.....

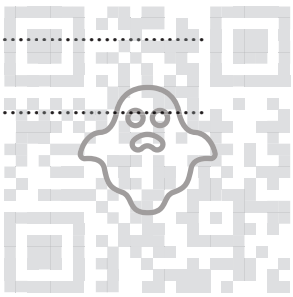
.....

.....

.....

.....

.....



**5** **(a)** Solve the equation $z^2 - 6iz - 12 = 0$, giving the answers in the form $x + iy$, where $x$ and $y$ are real and exact. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**(b)** On a sketch of an Argand diagram with origin $O$, show points $A$ and $B$ representing the roots of the equation in part **(a)**. [1]

**(c)** Find the exact modulus and argument of each root. [3]

**(d)** Hence show that the triangle $OAB$ is equilateral. [1]

**6** Relative to the origin $O$, the points $A$, $B$ and $C$ have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}.$$

**(a)** Using a scalar product, find the cosine of angle $BAC$. [4]

**(b)** Hence find the area of triangle $ABC$. Give your answer in a simplified exact form. [4]

**7** The variables $x$ and $\theta$ satisfy the differential equation

$$x \sin^2 \theta \frac{\mathrm{d}x}{\mathrm{d}\theta} = \tan^2 \theta - 2 \cot \theta,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 2$ when $\theta = \frac{1}{4}\pi$.

**(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}\theta}(\cot^2 \theta) = -\frac{2 \cot \theta}{\sin^2 \theta}$.

(You may assume without proof that the derivative of $\cot \theta$ with respect to $\theta$ is $-\operatorname{cosec}^2 \theta$.) [1]

.....

.....

.....

.....

.....

.....

.....

.....

**(b)** Solve the differential equation and find the value of $x$ when $\theta = \frac{1}{6}\pi$. [7]

.....

.....

.....

.....

.....

.....

.....

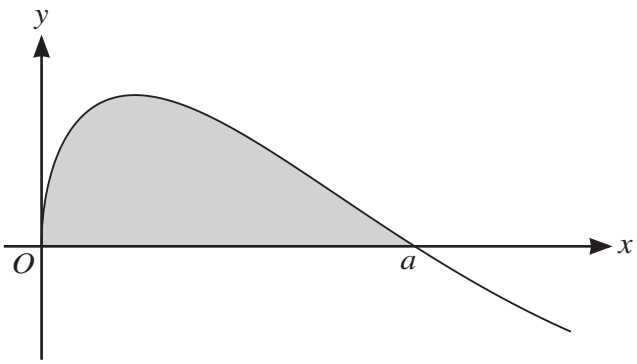
.....

.....

.....

.....

**8**




The diagram shows part of the curve $y = \sin\sqrt{x}$. This part of the curve intersects the $x$-axis at the point where $x = a$.

**(a)** State the exact value of $a$. [1]

.....

.....

.....

**(b)** Using the substitution $u = \sqrt{x}$, find the exact area of the shaded region in the first quadrant bounded by this part of the curve and the $x$-axis. [7]

.....

.....

.....

.....

.....

.....

.....

.....

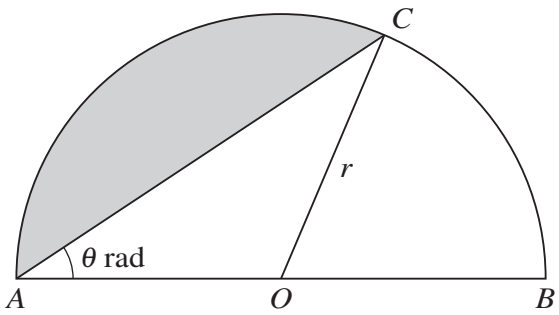
.....

.....

.....

.....

**9**




The diagram shows a semicircle with diameter $AB$, centre $O$ and radius $r$. The shaded region is the minor segment on the chord $AC$ and its area is one third of the area of the semicircle. The angle $CAB$ is $\theta$ radians.

**(a)** Show that $\theta = \frac{1}{3}(\pi - 1.5 \sin 2\theta)$. [4]

**(b)** Verify by calculation that $0.5 < \theta < 0.7$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**(c)** Use an iterative formula based on the equation in part **(a)** to determine $\theta$ correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

.....

.....

.....

.....

.....

.....

.....

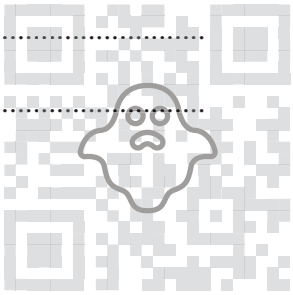
.....

.....

.....

.....

.....



**10** Let $f(x) = \dfrac{4 - x + x^2}{(1 + x)(2 + x^2)}$.

**(a)** Express $f(x)$ in partial fractions. [5]

**(b)** Find the exact value of $\int_0^4 f(x)\,dx$. Give your answer as a single logarithm. [5]