1 Solve the equation $8\sin^2\theta + 6\cos\theta + 1 = 0$ for $0° < \theta < 180°$. [3]

2 The function f is defined by $f(x) = -2x^2 - 8x - 13$ for $x < -3$.

(a) Express $f(x)$ in the form $-2(x + a)^2 + b$, where a and b are integers. [2]

(b) Find the range of f. [1]

(c) Find an expression for $f^{-1}(x)$. [3]

3 **(a)** Find the first three terms in ascending powers of x of the expansion of $(1 + 2x)^5$. [2]

(b) Find the first three terms in ascending powers of x of the expansion of $(1 - 3x)^4$. [2]

(c) Hence find the coefficient of x^2 in the expansion of $(1 + 2x)^5(1 - 3x)^4$. [2]

4 A large industrial water tank is such that, when the depth of the water in the tank is x metres, the volume $V\,\text{m}^3$ of water in the tank is given by $V = 243 - \frac{1}{3}(9 - x)^3$. Water is being pumped into the tank at a constant rate of $3.6\,\text{m}^3$ per hour.

Find the rate of increase of the depth of the water when the depth is 4 m, giving your answer in cm per minute. [5]

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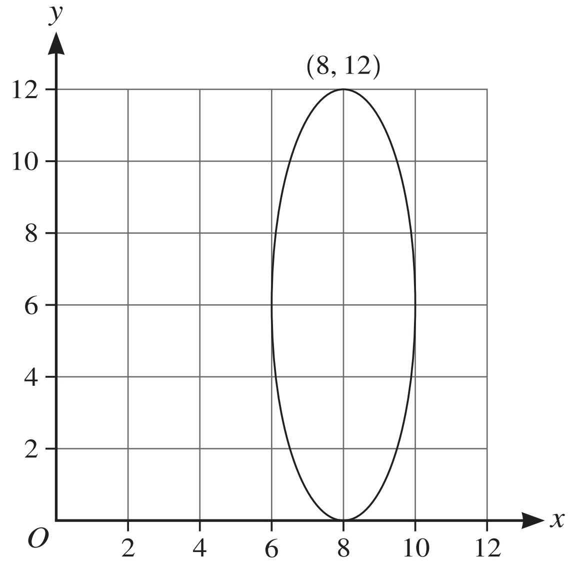

The diagram shows a curve which has a maximum point at $(8, 12)$ and a minimum point at $(8, 0)$. The curve is the result of applying a combination of two transformations to a circle. The first transformation applied is a translation of $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$. The second transformation applied is a stretch in the y-direction.

(a) State the scale factor of the stretch. [1]

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(b) State the radius of the original circle. [1]

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(c) State the coordinates of the centre of the circle after the translation has been completed but before the stretch is applied. [2]

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(d) State the coordinates of the centre of the original circle. [2]

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6 It is given that $\alpha = \cos^{-1}\left(\frac{8}{17}\right)$.

Find, without using the trigonometric functions on your calculator, the exact value of $\dfrac{1}{\sin\alpha} + \dfrac{1}{\tan\alpha}$. [5]

7 The curve $y = f(x)$ is such that $f'(x) = \dfrac{-3}{(x+2)^4}$.

(a) The tangent at a point on the curve where $x = a$ has gradient $-\frac{16}{27}$.

Find the possible values of a. [4]

(b) Find $f(x)$ given that the curve passes through the point $(-1, 5)$. [3]

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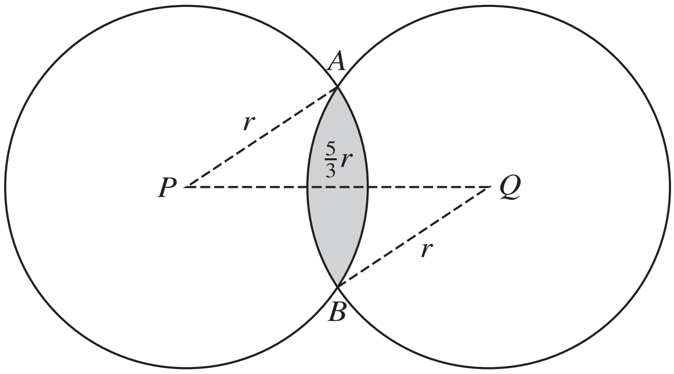

The diagram shows two identical circles intersecting at points A and B and with centres at P and Q. The radius of each circle is r and the distance PQ is $\frac{5}{3}r$.

(a) Find the perimeter of the shaded region in terms of r. [4]

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(b) Find the area of the shaded region in terms of r. [3]

9 The first term of a geometric progression is 216 and the fourth term is 64.

(a) Find the sum to infinity of the progression. [3]

The second term of the geometric progression is equal to the second term of an arithmetic progression.

The third term of the geometric progression is equal to the fifth term of the same arithmetic progression.

(b) Find the sum of the first 21 terms of the arithmetic progression. [6]

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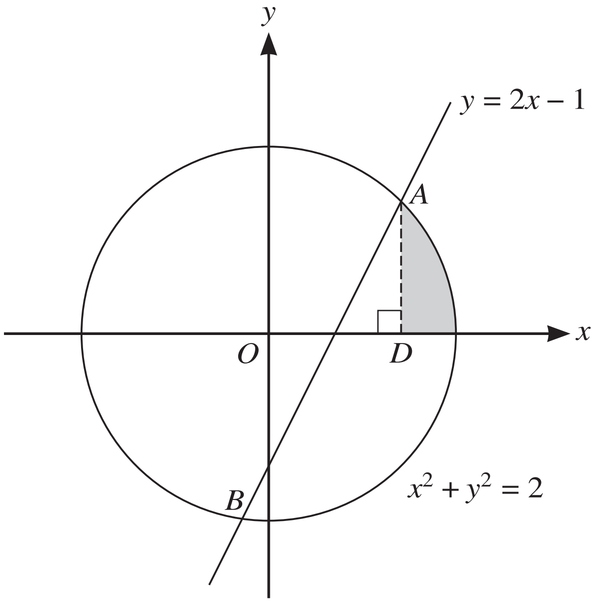

The diagram shows the circle $x^2 + y^2 = 2$ and the straight line $y = 2x - 1$ intersecting at the points A and B. The point D on the x-axis is such that AD is perpendicular to the x-axis.

(a) Find the coordinates of A. [4]

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(b) Find the volume of revolution when the shaded region is rotated through 360° about the x-axis. Give your answer in the form $\frac{\pi}{a}(b\sqrt{c} - d)$, where a, b, c and d are integers. [4]

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(c) Find an exact expression for the perimeter of the shaded region. [2]

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11 The coordinates of points A, B and C are $A(5, -2)$, $B(10, 3)$ and $C(2p, p)$, where p is a constant.

(a) Given that AC and BC are equal in length, find the value of the fraction p. [3]

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(b) It is now given instead that AC is perpendicular to BC and that p is an integer.

(i) Find the value of p. [4]

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(ii) Find the equation of the circle which passes through A, B and C, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants. [4]