1 Solve the equation $3x + 2 = \dfrac{2}{x - 1}$. [3]

2 The equation of a curve is such that $\frac{dy}{dx} = 12\left(\frac{1}{2}x - 1\right)^{-4}$. It is given that the curve passes through the point $P\,(6, 4)$.

(a) Find the equation of the tangent to the curve at P. [2]

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(b) Find the equation of the curve. [4]

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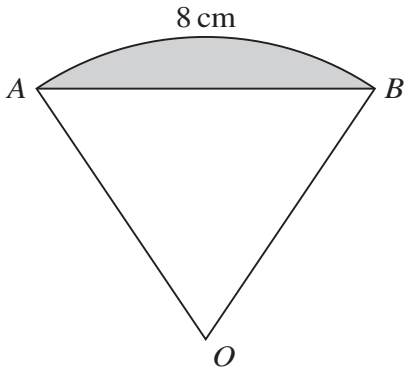
3 A curve has equation $y = ax^{\frac{1}{2}} - 2x$, where $x > 0$ and a is a constant. The curve has a stationary point at the point P, which has x-coordinate 9.

Find the y-coordinate of P. [5]

4 The coefficient of x^2 in the expansion of $\left(1+\frac{2}{p}x\right)^5+(1+px)^6$ is 70.

Find the possible values of the constant p. [6]

5



The diagram shows a sector OAB of a circle with centre O. The length of the arc AB is 8 cm. It is given that the perimeter of the sector is 20 cm.

(a) Find the perimeter of the shaded segment. [4]

(b) Find the area of the shaded segment. [2]

6 **(a)** Show that the equation

$$\frac{1}{\sin\theta + \cos\theta} + \frac{1}{\sin\theta - \cos\theta} = 1$$

may be expressed in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are constants to be found. [3]

(b) Hence solve the equation $\frac{1}{\sin\theta + \cos\theta} + \frac{1}{\sin\theta - \cos\theta} = 1$ for $0° \leqslant \theta \leqslant 360°$. [3]

7 A tool for putting fence posts into the ground is called a 'post-rammer'. The distances in millimetres that the post sinks into the ground on each impact of the post-rammer follow a geometric progression. The first three impacts cause the post to sink into the ground by 50 mm, 40 mm and 32 mm respectively.

(a) Verify that the 9th impact is the first in which the post sinks less than 10 mm into the ground. [3]

(b) Find, to the nearest millimetre, the total depth of the post in the ground after 20 impacts. [2]

(c) Find the greatest total depth in the ground which could theoretically be achieved. [2]

8 The function f is defined by $f(x) = 2 - \frac{3}{4x - p}$ for $x > \frac{p}{4}$, where p is a constant.

(a) Find $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [3]

(b) Express $f^{-1}(x)$ in the form $\frac{p}{a} - \frac{b}{cx-d}$, where a, b, c and d are integers. [4]

(c) Hence state the value of p for which $f^{-1}(x) \equiv f(x)$. [1]

9 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 4x + 9,$$
$$g(x) = 2x^2 + 4x + 12.$$

(a) Express $f(x)$ in the form $(x - a)^2 + b$. [1]

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(b) Express $g(x)$ in the form $2[(x + c)^2 + d]$. [2]

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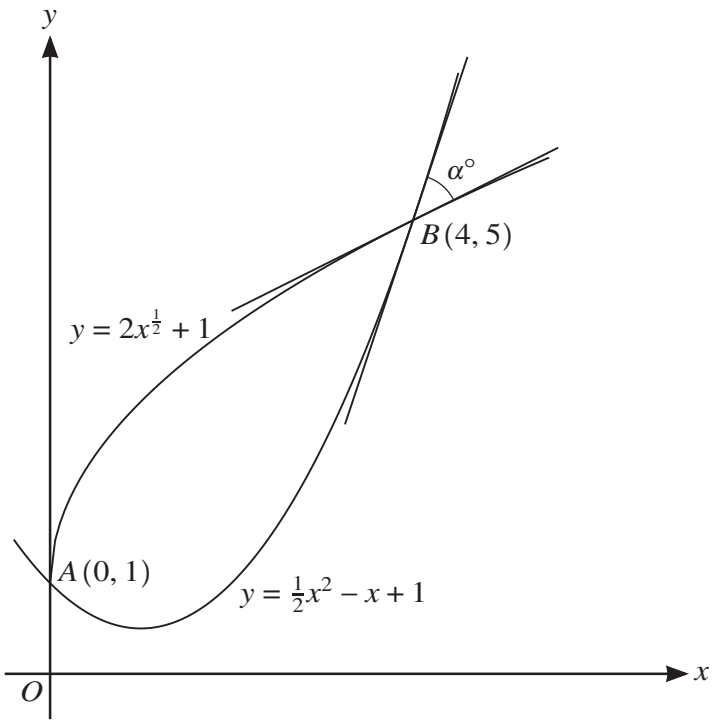
(c) Express $g(x)$ in the form $kf(x + h)$, where k and h are integers. [1]

(d) Describe fully the two transformations that have been combined to transform the graph of $y = f(x)$ to the graph of $y = g(x)$. [4]

10



Curves with equations $y = 2x^{\frac{1}{2}} + 1$ and $y = \frac{1}{2}x^2 - x + 1$ intersect at $A\,(0,\ 1)$ and $B\,(4,\ 5)$, as shown in the diagram.

(a) Find the area of the region between the two curves. [5]

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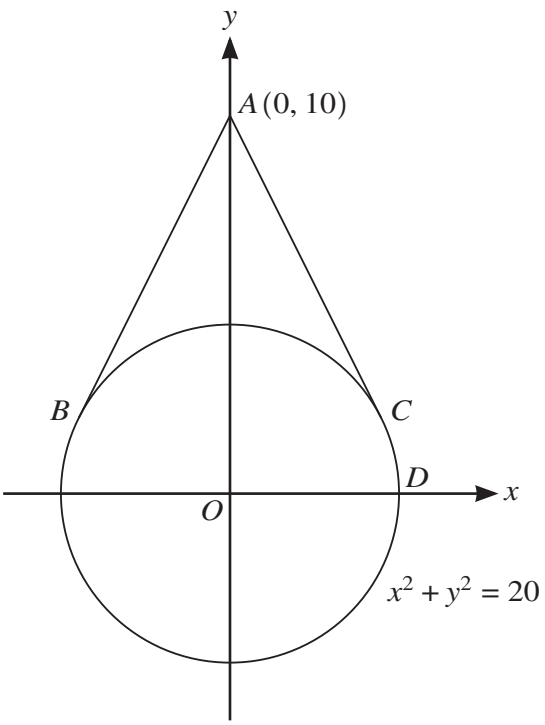
The acute angle between the two tangents at B is denoted by $\alpha°$, and the scales on the axes are the same.

(b) Find α. [5]

11



The diagram shows the circle with equation $x^2 + y^2 = 20$. Tangents touching the circle at points B and C pass through the point A (0, 10).

(a) By letting the equation of a tangent be $y = mx + 10$, find the two possible values of m. [4]

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(b) Find the coordinates of B and C. [3]

The point D is where the circle crosses the positive x-axis.

(c) Find angle BDC in degrees. [3]