

- 2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| \geq 2$ and $|z - 1 + i| \leq 1$. [4]



4 Solve the equation

$$\log_{10}(2x + 1) = 2 \log_{10}(x + 1) - 1.$$

Give your answers correct to 3 decimal places.

[6]

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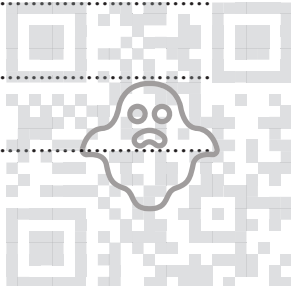
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- 5 (a) By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec} x = 1 + e^{-\frac{1}{2}x}$ has exactly two roots in the interval $0 < x < \pi$. [2]

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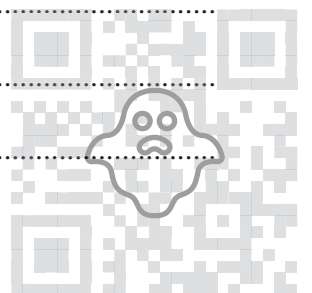
- (b) The sequence of values given by the iterative formula

$$x_{n+1} = \pi - \sin^{-1} \left(\frac{1}{e^{-\frac{1}{2}x_n} + 1} \right),$$

with initial value $x_1 = 2$, converges to one of these roots.

Use the formula to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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(b) Find the other roots of this equation. [4]

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8 The coordinates (x, y) of a general point of a curve satisfy the differential equation

$$x \frac{dy}{dx} = (1 - 2x^2)y,$$

for $x > 0$. It is given that $y = 1$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x . [6]

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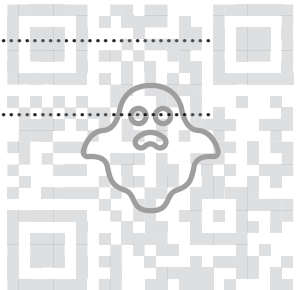
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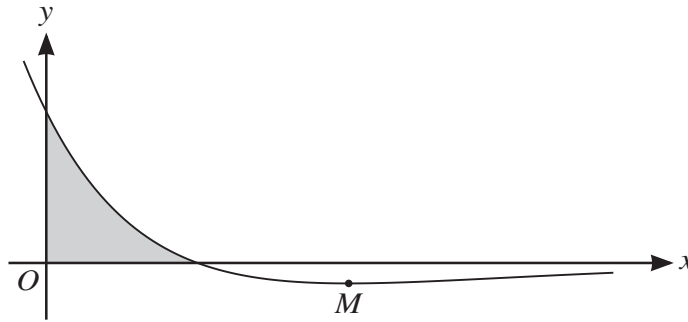
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A series of horizontal dotted lines for writing.



10



The diagram shows the curve $y = (2 - x)e^{-\frac{1}{2}x}$, and its minimum point M .

- (a) Find the exact coordinates of M . [5]

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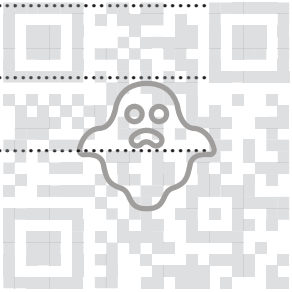
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- (b) Find the area of the shaded region bounded by the curve and the axes. Give your answer in terms of e . [5]

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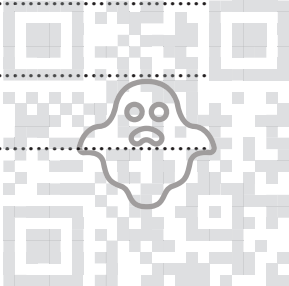
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- (b) Given instead that the acute angle between the directions of the two lines is $\cos^{-1}(\frac{1}{6})$, find the two possible values of a . [6]

Dotted lines for writing the answer.

