1 The coefficient of x^3 in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.

Find the value of the constant k. [4]

2 The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where p is positive.

Find the sum to infinity of the progression. [5]

3 The equation of a curve is $y = 2x^2 + m(2x + 1)$, where m is a constant, and the equation of a line is $y = 6x + 4$.

Show that, for all values of m, the line intersects the curve at two distinct points. [5]

4 The sum, S_n, of the first n terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The kth term in the progression is greater than 200.

Find the smallest possible value of k. [5]

5 Functions f and g are defined by

$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$
$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

(a) Find the value of fg(7). [1]

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(b) Find the values of x for which $f^{-1}(x) = g^{-1}(x)$. [5]

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6 **(a)** Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$. [4]

(b) Hence solve the equation $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

7 The point (4, 7) lies on the curve $y = f(x)$ and it is given that $f'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

(a) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second.

Find the rate of increase of the y-coordinate when $x = 4$. [3]

(b) Find the equation of the curve. [4]

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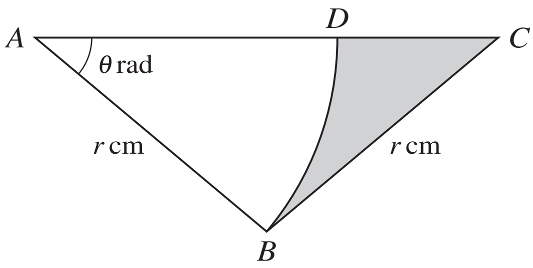

In the diagram, ABC is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector of a circle with centre A.

(a) Express the area of the shaded region in terms of r and θ. [3]

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(b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]

9 A circle has centre at the point $B(5, 1)$. The point $A(-1, -2)$ lies on the circle.

(a) Find the equation of the circle. [3]

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Point C is such that AC is a diameter of the circle. Point D has coordinates $(5, 16)$.

(b) Show that DC is a tangent to the circle. [4]

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The other tangent from D to the circle touches the circle at E.

(c) Find the coordinates of E. [2]

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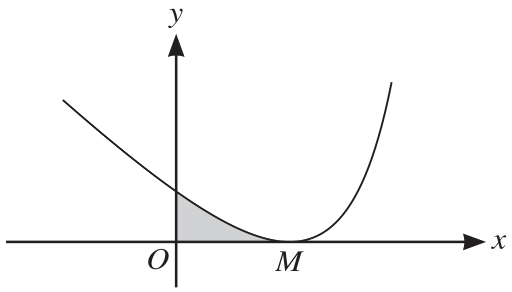

The diagram shows part of the curve $y = \dfrac{2}{(3-2x)^2} - x$ and its minimum point M, which lies on the x-axis.

(a) Find expressions for $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ and $\displaystyle\int y\,dx$. [6]

(b) Find, by calculation, the x-coordinate of M. [2]

(c) Find the area of the shaded region bounded by the curve and the coordinate axes. [2]

11 A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

(a) State the greatest and least values of y. [2]

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(b) Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]

(c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

(i) $k = -3$ [1]

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(ii) $k = 1$ [1]

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(iii) $k = 3$ [1]

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Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$f(x) = 3\cos 2x + 2,$$
$$g(x) = f(2x) + 4,$$
$$h(x) = 2f\left(x + \tfrac{1}{2}\pi\right).$$

(d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]

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(e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.