1 Find the set of values of m for which the line with equation $y = mx - 3$ and the curve with equation $y = 2x^2 + 5$ do not meet. [3]

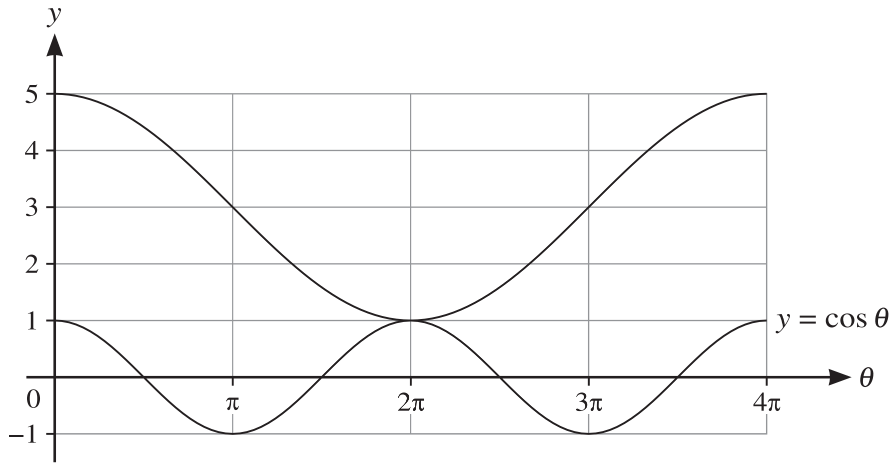
2 The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{(x-3)^2} + x$. It is given that the curve passes through the point (2, 7).

Find the equation of the curve. [4]

3 Air is being pumped into a balloon in the shape of a sphere so that its volume is increasing at a constant rate of $50\,\text{cm}^3\,\text{s}^{-1}$.

Find the rate at which the radius of the balloon is increasing when the radius is 10 cm. [3]

4



In the diagram, the lower curve has equation $y = \cos\theta$. The upper curve shows the result of applying a combination of transformations to $y = \cos\theta$.

Find, in terms of a cosine function, the equation of the upper curve. [3]

5 In the expansion of $\left(2x^2 + \frac{a}{x}\right)^6$, the coefficients of x^6 and x^3 are equal.

(a) Find the value of the non-zero constant a. [4]

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(b) Find the coefficient of x^6 in the expansion of $(1 - x^3)\left(2x^2 + \frac{a}{x}\right)^6$. [1]

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6 The equation of a curve is $y = 2 + \sqrt{25 - x^2}$.

Find the coordinates of the point on the curve at which the gradient is $\frac{4}{3}$. [5]

7 **(a)** Show that $\frac{\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1+\sin\theta} \equiv 2\tan^2\theta$. [3]

(b) Hence solve the equation $\dfrac{\sin\theta}{1-\sin\theta}-\dfrac{\sin\theta}{1+\sin\theta}=8$, for $0° < \theta < 180°$. [3]

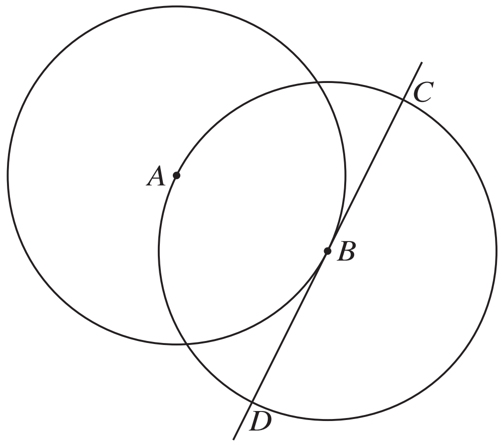
8 A geometric progression has first term a, common ratio r and sum to infinity S. A second geometric progression has first term a, common ratio R and sum to infinity $2S$.

(a) Show that $r = 2R - 1$. [3]

It is now given that the 3rd term of the first progression is equal to the 2nd term of the second progression.

(b) Express S in terms of a. [4]

9



The diagram shows a circle with centre A passing through the point B. A second circle has centre B and passes through A. The tangent at B to the first circle intersects the second circle at C and D.

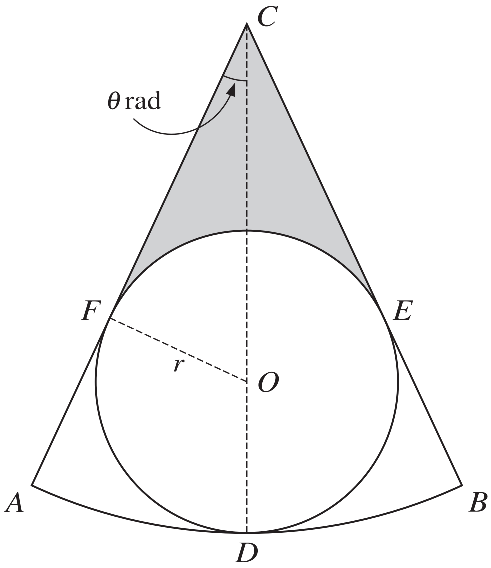
The coordinates of A are $(-1, 4)$ and the coordinates of B are $(3, 2)$.

(a) Find the equation of the tangent CBD. [2]

(b) Find an equation of the circle with centre B. [3]

(c) Find, by calculation, the x-coordinates of C and D. [3]

10



The diagram shows a sector CAB which is part of a circle with centre C. A circle with centre O and radius r lies within the sector and touches it at D, E and F, where COD is a straight line and angle ACD is θ radians.

(a) Find CD in terms of r and $\sin\theta$. [3]

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It is now given that $r = 4$ and $\theta = \frac{1}{6}\pi$.

(b) Find the perimeter of sector CAB in terms of π. [3]

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(c) Find the area of the shaded region in terms of π and $\sqrt{3}$. [4]

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11 The functions f and g are defined by

$$f(x) = x^2 + 3 \quad \text{for } x > 0,$$
$$g(x) = 2x + 1 \quad \text{for } x > -\tfrac{1}{2}.$$

(a) Find an expression for $fg(x)$. [1]

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(b) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [4]

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(c) Solve the equation $fg(x) - 3 = gf(x)$. [4]

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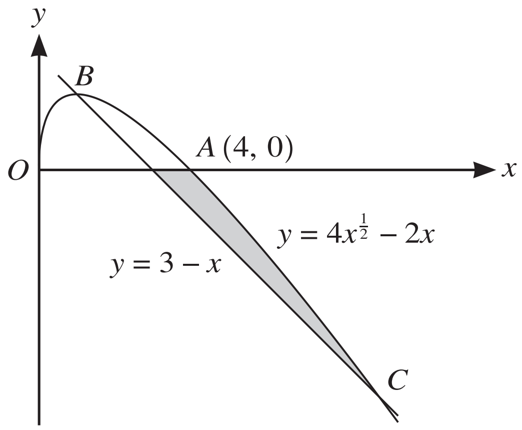

The diagram shows a curve with equation $y = 4x^{\frac{1}{2}} - 2x$ for $x \geqslant 0$, and a straight line with equation $y = 3 - x$. The curve crosses the x-axis at $A\,(4,\ 0)$ and crosses the straight line at B and C.

(a) Find, by calculation, the x-coordinates of B and C. [4]

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(b) Show that B is a stationary point on the curve. [2]

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(c) Find the area of the shaded region. [6]