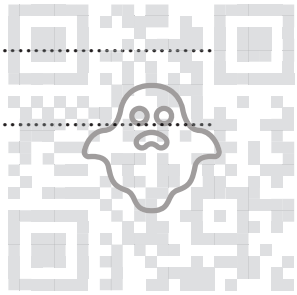
1 Solve the equation $\ln(e^{2x} + 3) = 2x + \ln 3$, giving your answer correct to 3 decimal places. [4]

2 Solve the equation $3\cos 2\theta = 3\cos\theta + 2$, for $0° \leqslant \theta \leqslant 360°$. [5]



3 The polynomial $ax^3 + x^2 + bx + 3$ is denoted by $\mathrm{p}(x)$. It is given that $\mathrm{p}(x)$ is divisible by $(2x - 1)$ and that when $\mathrm{p}(x)$ is divided by $(x + 2)$ the remainder is 5.

Find the values of a and b. [5]

4 The equation of a curve is $y = \cos^3 x\sqrt{\sin x}$. It is given that the curve has one stationary point in the interval $0 < x < \frac{1}{2}\pi$.

Find the x-coordinate of this stationary point, giving your answer correct to 3 significant figures. [6]

5 **(a)** By sketching a suitable pair of graphs, show that the equation $\ln x = 3x - x^2$ has one real root. [2]

(b) Verify by calculation that the root lies between 2 and 2.8. [2]

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(c) Use the iterative formula $x_{n+1} = \sqrt{3x_n - \ln x_n}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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6 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = xe^{y-x},$$

and $y = 0$ when $x = 0$.

(a) Solve the differential equation, obtaining an expression for y in terms of x. [7]

(b) Find the value of y when $x = 1$, giving your answer in the form $a - \ln b$, where a and b are integers. [1]

7 The equation of a curve is $x^3 + 3x^2y - y^3 = 3$.

(a) Show that $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$. [4]

(b) Find the coordinates of the points on the curve where the tangent is parallel to the x-axis. [5]

8 Let $f(x) = \dfrac{x^2 + 9x}{(3x - 1)(x^2 + 3)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find $\int_1^3 f(x)\,dx$, giving your answer in a simplified exact form. [5]

9 The lines l and m have vector equations

$$\mathbf{r} = -\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} - \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} + \mathbf{k})$$

respectively, where a and b are constants.

(a) Given that l and m intersect, show that $2b - a = 4$. [4]

(b) Given also that l and m are perpendicular, find the values of a and b. [4]

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(c) When a and b have these values, find the position vector of the point of intersection of l and m. [2]

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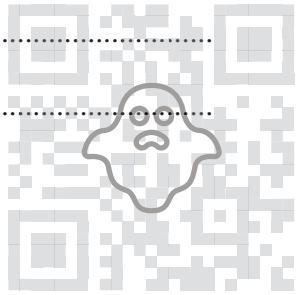

10 The complex number $-1 + \sqrt{7}\mathrm{i}$ is denoted by u. It is given that u is a root of the equation

$$2x^3 + 3x^2 + 14x + k = 0,$$

where k is a real constant.

(a) Find the value of k. [3]

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(b) Find the other two roots of the equation. [4]

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(c) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying the equation $|z - u| = 2$. [2]

(d) Determine the greatest value of $\arg z$ for points on this locus, giving your answer in radians. [2]

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