1 (a) Express $x^2 - 8x + 11$ in the form $(x + p)^2 + q$ where p and q are constants. [2]

(b) Hence find the exact solutions of the equation $x^2 - 8x + 11 = 1$. [2]

2 The thirteenth term of an arithmetic progression is 12 and the sum of the first 30 terms is -15.

Find the sum of the first 50 terms of the progression. [5]

3 The coefficient of x^4 in the expansion of $\left(2x^2 + \dfrac{k^2}{x}\right)^5$ is a. The coefficient of x^2 in the expansion of $(2kx - 1)^4$ is b.

(a) Find a and b in terms of the constant k. [3]

(b) Given that $a + b = 216$, find the possible values of k. [3]

4 **(a)** Prove the identity $\dfrac{\sin^3\theta}{\sin\theta - 1} - \dfrac{\sin^2\theta}{1 + \sin\theta} \equiv -\tan^2\theta(1 + \sin^2\theta)$. [4]

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(b) Hence solve the equation

$$\frac{\sin^3 \theta}{\sin \theta - 1} - \frac{\sin^2 \theta}{1 + \sin \theta} = \tan^2 \theta (1 - \sin^2 \theta)$$

for $0 < \theta < 2\pi$. [2]

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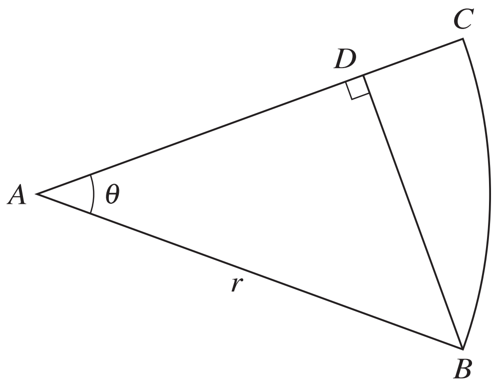

The diagram shows a sector ABC of a circle with centre A and radius r. The line BD is perpendicular to AC. Angle CAB is θ radians.

(a) Given that $\theta = \frac{1}{6}\pi$, find the exact area of BCD in terms of r. [3]

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(b) Given instead that the length of BD is $\frac{\sqrt{3}}{2}r$, find the exact perimeter of BCD in terms of r. [4]

6 The function f is defined as follows:

$$f(x) = \frac{x^2 - 4}{x^2 + 4} \quad \text{for } x > 2.$$

(a) Find an expression for $f^{-1}(x)$. [3]

(b) Show that $1 - \dfrac{8}{x^2 + 4}$ can be expressed as $\dfrac{x^2 - 4}{x^2 + 4}$ and hence state the range of f. [4]

(c) Explain why the composite function ff cannot be formed. [1]

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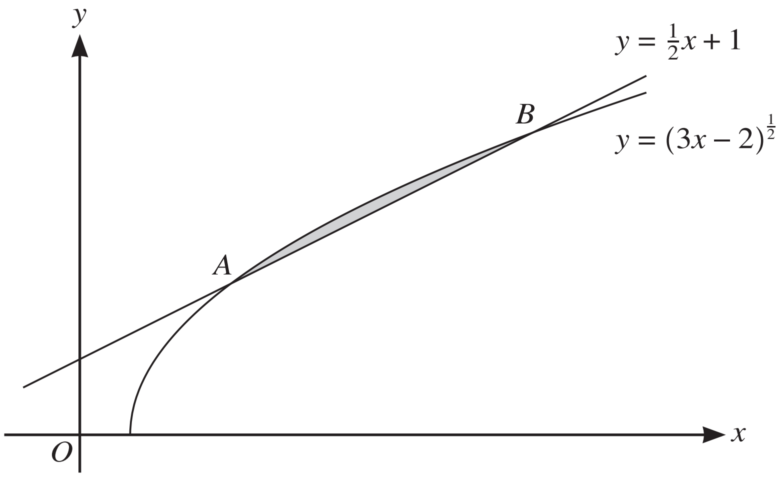

The diagram shows the curve with equation $y = (3x - 2)^{\frac{1}{2}}$ and the line $y = \frac{1}{2}x + 1$. The curve and the line intersect at points A and B.

(a) Find the coordinates of A and B. [4]

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(b) Hence find the area of the region enclosed between the curve and the line. [5]

8 **(a)** The curve $y = \sin x$ is transformed to the curve $y = 4\sin\left(\frac{1}{2}x - 30°\right)$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied. [5]

(b) Find the exact solutions of the equation $4\sin\left(\frac{1}{2}x - 30°\right) = 2\sqrt{2}$ for $0° \leqslant x \leqslant 360°$. [3]

9 The equation of a circle is $x^2 + y^2 + 6x - 2y - 26 = 0$.

(a) Find the coordinates of the centre of the circle and the radius. Hence find the coordinates of the lowest point on the circle. [4]

(b) Find the set of values of the constant k for which the line with equation $y = kx - 5$ intersects the circle at two distinct points. [6]

10 The equation of a curve is such that $\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$. The curve has a stationary point at $\left(-1, \frac{9}{2}\right)$.

(a) Determine the nature of the stationary point at $\left(-1, \frac{9}{2}\right)$. [1]

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(b) Find the equation of the curve. [5]

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(c) Show that the curve has no other stationary points. [3]

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(d) A point A is moving along the curve and the y-coordinate of A is increasing at a rate of 5 units per second.

Find the rate of increase of the x-coordinate of A at the point where $x = 1$. [3]

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