**1** Expand $(1 + 3x)^{\frac{2}{3}}$ in ascending powers of $x$, up to and including the term in $x^3$, simplifying the coefficients. [4]

**2** Solve the equation $4^x = 3 + 4^{-x}$. Give your answer correct to 3 decimal places. [5]

 

**3** The parametric equations of a curve are

$$x = t + \ln(t + 2), \qquad y = (t - 1)e^{-2t},$$

where $t > -2$.

**(a)** Express $\frac{dy}{dx}$ in terms of $t$, simplifying your answer. [5]

**(b)** Find the exact $y$-coordinate of the stationary point of the curve. [2]

**4** Let $\mathrm{f}(x) = \dfrac{15 - 6x}{(1 + 2x)(4 - x)}$.

**(a)** Express $\mathrm{f}(x)$ in partial fractions. [3]

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**(b)** Hence find $\displaystyle\int_1^2 \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in the form $\ln\left(\dfrac{a}{b}\right)$, where $a$ and $b$ are integers. [4]

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5 **(a)** By first expanding $\tan(2\theta + 2\theta)$, show that the equation $\tan 4\theta = \frac{1}{2}\tan\theta$ may be expressed as $\tan^4\theta + 2\tan^2\theta - 7 = 0$. [4]

**(b)** Hence solve the equation $\tan 4\theta = \frac{1}{2}\tan\theta$, for $0° < \theta < 180°$. [3]

6 **(a)** By sketching a suitable pair of graphs, show that the equation $\cot \frac{1}{2}x = 1 + e^{-x}$ has exactly one root in the interval $0 < x \leqslant \pi$. [2]

**(b)** Verify by calculation that this root lies between 1 and 1.5. [2]

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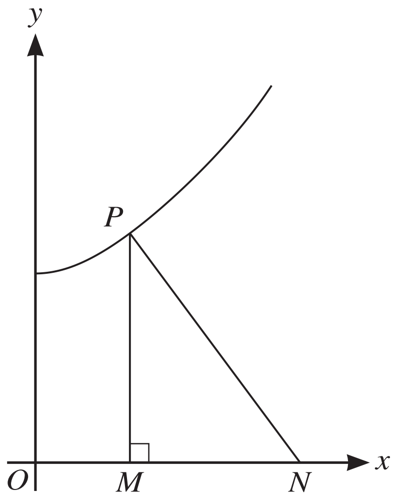
**(c)** Use the iterative formula $x_{n+1} = 2\tan^{-1}\left(\dfrac{1}{1 + e^{-x_n}}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

**7**




For the curve shown in the diagram, the normal to the curve at the point $P$ with coordinates $(x, y)$ meets the $x$-axis at $N$. The point $M$ is the foot of the perpendicular from $P$ to the $x$-axis.

The curve is such that for all values of $x$ in the interval $0 \leqslant x < \frac{1}{2}\pi$, the area of triangle $PMN$ is equal to $\tan x$.

**(a)** **(i)** Show that $\dfrac{MN}{y} = \dfrac{dy}{dx}$. [1]

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**(ii)** Hence show that $x$ and $y$ satisfy the differential equation $\frac{1}{2}y^2\dfrac{dy}{dx} = \tan x$. [2]

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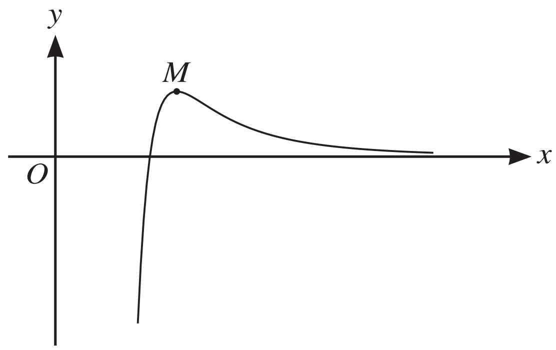
**(b)** Given that $y = 1$ when $x = 0$, solve this differential equation to find the equation of the curve, expressing $y$ in terms of $x$. [6]

**8**




The diagram shows the curve $y = \dfrac{\ln x}{x^4}$ and its maximum point $M$.

**(a)** Find the exact coordinates of $M$. [4]

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**(b)** By using integration by parts, show that for all $a > 1$, $\int_1^a \frac{\ln x}{x^4}\,dx < \frac{1}{9}$. [6]

 

**9** The quadrilateral $ABCD$ is a trapezium in which $AB$ and $DC$ are parallel. With respect to the origin $O$, the position vectors of $A$, $B$ and $C$ are given by $\overrightarrow{OA} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OB} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$.

**(a)** Given that $\overrightarrow{DC} = 3\overrightarrow{AB}$, find the position vector of $D$. [3]

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**(b)** State a vector equation for the line through $A$ and $B$. [1]

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**(c)** Find the distance between the parallel sides and hence find the area of the trapezium. [5]

**10** **(a)** Verify that $-1 + \sqrt{2}\mathrm{i}$ is a root of the equation $z^4 + 3z^2 + 2z + 12 = 0$. [3]

**(b)** Find the other roots of this equation. [7]