1 Solve the inequality $2|3x - 1| < |x + 1|$. [4]

2 Find the real root of the equation $\frac{2e^x + e^{-x}}{2 + e^x} = 3$, giving your answer correct to 3 decimal places. Your working should show clearly that the equation has only one real root. [5]

3 **(a)** Given that $\cos(x - 30°) = 2\sin(x + 30°)$, show that $\tan x = \dfrac{2 - \sqrt{3}}{1 - 2\sqrt{3}}$. [4]

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(b) Hence solve the equation

$$\cos(x - 30°) = 2\sin(x + 30°),$$

for $0° < x < 360°$. [2]

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4 **(a)** Prove that $\dfrac{1-\cos 2\theta}{1+\cos 2\theta} \equiv \tan^2\theta$. [2]

(b) Hence find the exact value of $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \frac{1-\cos 2\theta}{1+\cos 2\theta}\,\mathrm{d}\theta$. [4]

5 **(a)** Solve the equation $z^2 - 2piz - q = 0$, where p and q are real constants. [2]

In an Argand diagram with origin O, the roots of this equation are represented by the distinct points A and B.

(b) Given that A and B lie on the imaginary axis, find a relation between p and q. [2]

(c) Given instead that triangle OAB is equilateral, express q in terms of p. [3]

6 The parametric equations of a curve are

$$x = \ln(2 + 3t), \qquad y = \frac{t}{2 + 3t}.$$

(a) Show that the gradient of the curve is always positive. [5]

(b) Find the equation of the tangent to the curve at the point where it intersects the y-axis. [3]

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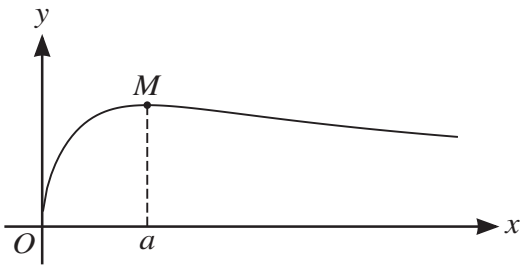

The diagram shows the curve $y = \dfrac{\tan^{-1} x}{\sqrt{x}}$ and its maximum point M where $x = a$.

(a) Show that a satisfies the equation

$$a = \tan\left(\frac{2a}{1 + a^2}\right).$$ [4]

(b) Verify by calculation that a lies between 1.3 and 1.5. [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 With respect to the origin O, the points A and B have position vectors given by $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$. The line l has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$.

(a) Find the acute angle between the directions of AB and l. [4]

(b) Find the position vector of the point P on l such that $AP = BP$. [5]

9 The equation of a curve is $y = x^{-\frac{2}{3}} \ln x$ for $x > 0$. The curve has one stationary point.

(a) Find the exact coordinates of the stationary point. [5]

(b) Show that $\int_1^8 y \, dx = 18 \ln 2 - 9$. [5]

10 The variables x and t satisfy the differential equation $\frac{dx}{dt} = x^2(1 + 2x)$, and $x = 1$ when $t = 0$.

Using partial fractions, solve the differential equation, obtaining an expression for t in terms of x. [11]