1 **(a)** Express $16x^2 - 24x + 10$ in the form $(4x + a)^2 + b$. [2]

(b) It is given that the equation $16x^2 - 24x + 10 = k$, where k is a constant, has exactly one root.

Find the value of this root. [2]

2 **(a)** The graph of $y = f(x)$ is transformed to the graph of $y = 2f(x - 1)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

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(b) The curve $y = \sin 2x - 5x$ is reflected in the y-axis and then stretched by scale factor $\frac{1}{3}$ in the x-direction.

Write down the equation of the transformed curve. [2]

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3 The equation of a curve is $y = (x - 3)\sqrt{x + 1} + 3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places.

$A\,(2,\ k)$ $\quad B\,(2.9,\ 2.8025)$ $\quad C\,(2.99,\ 2.9800)$ $\quad D\,(2.999,\ 2.9980)$ $\quad E\,(3,\ 3)$

(a) Find k, giving your answer correct to 4 decimal places. [1]

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(b) Find the gradient of AE, giving your answer correct to 4 decimal places. [1]

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The gradients of BE, CE and DE, rounded to 4 decimal places, are 1.9748, 1.9975 and 1.9997 respectively.

(c) State, giving a reason for your answer, what the values of the four gradients suggest about the gradient of the curve at the point E. [2]

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4 The coefficient of x in the expansion of $\left(4x + \frac{10}{x}\right)^3$ is p. The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{k}{x^2}\right)^5$ is q.

Given that $p = 6q$, find the possible values of k. [5]

5 The function f is defined by $f(x) = 2x^2 + 3$ for $x \geqslant 0$.

(a) Find and simplify an expression for $ff(x)$. [2]

(b) Solve the equation $ff(x) = 34x^2 + 19$. [4]

6 Points A and B have coordinates $(8, 3)$ and (p, q) respectively. The equation of the perpendicular bisector of AB is $y = -2x + 4$.

Find the values of p and q. [4]

7 The point A has coordinates $(1, 5)$ and the line l has gradient $-\frac{2}{3}$ and passes through A. A circle has centre $(5, 11)$ and radius $\sqrt{52}$.

(a) Show that l is the tangent to the circle at A. [2]

(b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also the tangent at A. [3]

8 The first, second and third terms of an arithmetic progression are a, $\frac{3}{2}a$ and b respectively, where a and b are positive constants. The first, second and third terms of a geometric progression are a, 18 and $b + 3$ respectively.

(a) Find the values of a and b. [5]

(b) Find the sum of the first 20 terms of the arithmetic progression. [3]

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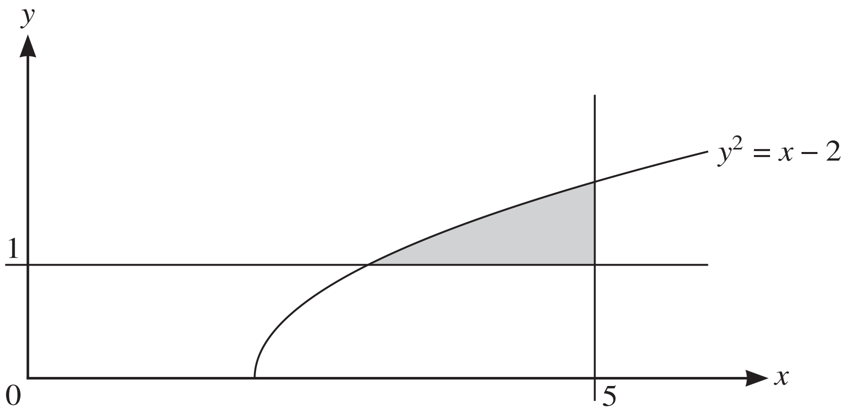

The diagram shows part of the curve with equation $y^2 = x - 2$ and the lines $x = 5$ and $y = 1$. The shaded region enclosed by the curve and the lines is rotated through 360° about the x-axis.

Find the volume obtained. [6]

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10 **(a)** Prove the identity $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} \equiv \dfrac{4\tan x}{\cos x}$. [4]

(b) Hence solve the equation $\frac{1+\sin x}{1-\sin x} - \frac{1-\sin x}{1+\sin x} = 8\tan x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [3]

11 The gradient of a curve is given by $\frac{dy}{dx} = 6(3x - 5)^3 - kx^2$, where k is a constant. The curve has a stationary point at $(2, -3.5)$.

(a) Find the value of k. [2]

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(b) Find the equation of the curve. [4]

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(c) Find $\frac{d^2y}{dx^2}$. [2]

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(d) Determine the nature of the stationary point at $(2, -3.5)$. [2]

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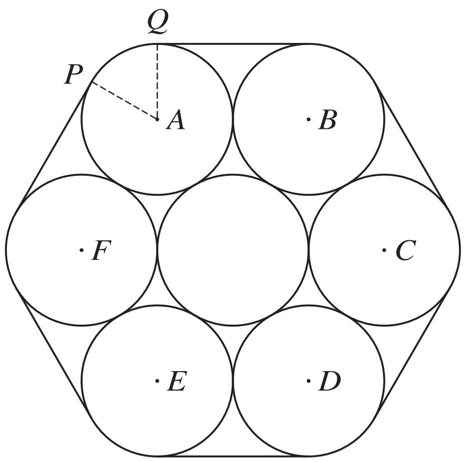

The diagram shows a cross-section of seven cylindrical pipes, each of radius 20 cm, held together by a thin rope which is wrapped tightly around the pipes. The centres of the six outer pipes are A, B, C, D, E and F. Points P and Q are situated where straight sections of the rope meet the pipe with centre A.

(a) Show that angle $PAQ = \frac{1}{3}\pi$ radians. [2]

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(b) Find the length of the rope. [4]

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(c) Find the area of the hexagon *ABCDEF*, giving your answer in terms of $\sqrt{3}$. [2]

(d) Find the area of the complete region enclosed by the rope. [3]