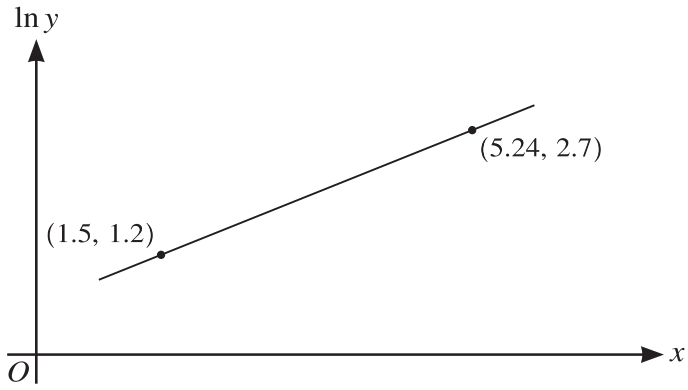
1 Find the quotient and remainder when $6x^4 + x^3 - x^2 + 5x - 6$ is divided by $2x^2 - x + 1$. [3]

2



The variables x and y satisfy the equation $y^2 = Ae^{kx}$, where A and k are constants. The graph of $\ln y$ against x is a straight line passing through the points (1.5, 1.2) and (5.24, 2.7) as shown in the diagram.

Find the values of A and k correct to 2 decimal places. [5]

3 Find the exact value of

$$\int_1^4 x^{\frac{3}{2}} \ln x \, dx.$$ [5]

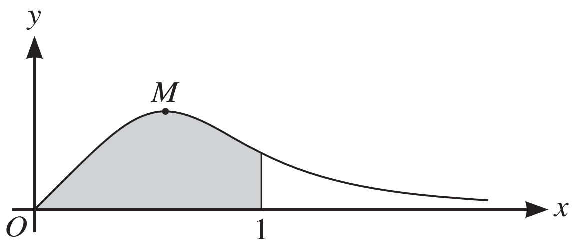
4 A curve has equation $y = \cos x \sin 2x$.

Find the x-coordinate of the stationary point in the interval $0 < x < \frac{1}{2}\pi$, giving your answer correct to 3 significant figures. [6]

5 **(a)** Express $\sqrt{2}\cos x - \sqrt{5}\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 3 decimal places. [3]

(b) Hence solve the equation $\sqrt{2}\cos 2\theta - \sqrt{5}\sin 2\theta = 1$, for $0° < \theta < 180°$. [4]

6



The diagram shows the curve $y = \dfrac{x}{1 + 3x^4}$, for $x \geqslant 0$, and its maximum point M.

(a) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [4]

(b) Using the substitution $u = \sqrt{3}x^2$, find by integration the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 1$. [5]

7 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{y-1}{(x+1)(x+3)}.$$

It is given that $y = 2$ when $x = 0$.

Solve the differential equation, obtaining an expression for y in terms of x. [9]

8 **(a)** Solve the equation $(1 + 2\mathrm{i})w + \mathrm{i}w^* = 3 + 5\mathrm{i}$. Give your answer in the form $x + \mathrm{i}y$, where x and y are real. [4]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2 - 2\mathrm{i}| \leqslant 1$ and $\arg(z - 4\mathrm{i}) \geqslant -\frac{1}{4}\pi$. [4]

(ii) Find the least value of $\operatorname{Im} z$ for points in this region, giving your answer in an exact form. [2]

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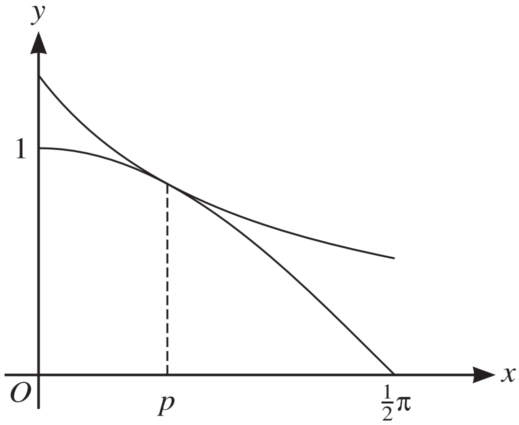

The diagram shows the curves $y = \cos x$ and $y = \dfrac{k}{1+x}$, where k is a constant, for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The curves touch at the point where $x = p$.

(a) Show that p satisfies the equation $\tan p = \dfrac{1}{1+p}$. [5]

(b) Use the iterative formula $p_{n+1} = \tan^{-1}\left(\frac{1}{1 + p_n}\right)$ to determine the value of p correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

...

(c) Hence find the value of k correct to 2 decimal places. [2]

...

10 With respect to the origin O, the points A and B have position vectors given by $\overrightarrow{OA} = 6\mathbf{i} + 2\mathbf{j}$ and $\overrightarrow{OB} = 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. The midpoint of OA is M. The point N lying on AB, between A and B, is such that $AN = 2NB$.

(a) Find a vector equation for the line through M and N. [5]

The line through M and N intersects the line through O and B at the point P.

(b) Find the position vector of P. [3]

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(c) Calculate angle OPM, giving your answer in degrees. [3]

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