1 Find the set of values of x for which $2(3^{1-2x}) < 5^x$. Give your answer in a simplified exact form. [4]

2 **(a)** Expand $(2 - 3x)^{-2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

(b) State the set of values of x for which the expansion is valid. [1]

3 Express the equation $\tan(\theta + 60°) = 2 + \tan(60° - \theta)$ as a quadratic equation in $\tan\theta$, and hence solve the equation for $0° \leqslant \theta \leqslant 180°$. [6]

4 The curve with equation $y = e^{2x}(\sin x + 3\cos x)$ has a stationary point in the interval $0 \leqslant x \leqslant \pi$.

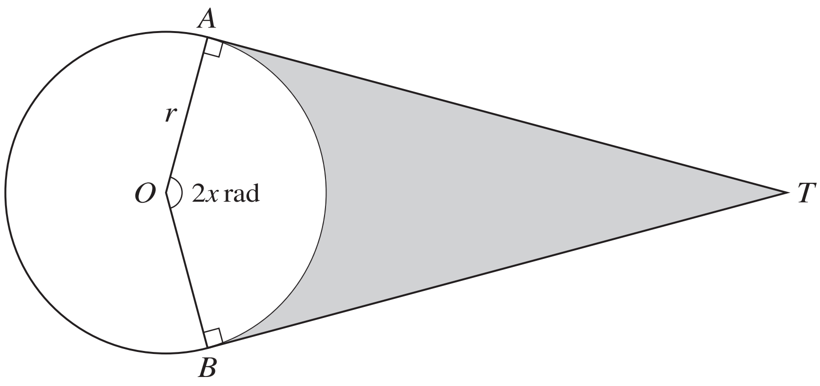
(a) Find the x-coordinate of this point, giving your answer correct to 2 decimal places. [4]

(b) Determine whether the stationary point is a maximum or a minimum. [2]

5 (a) Find the quotient and remainder when $2x^3 - x^2 + 6x + 3$ is divided by $x^2 + 3$. [3]

(b) Using your answer to part **(a)**, find the exact value of $\int_1^3 \frac{2x^3 - x^2 + 6x + 3}{x^2 + 3}\, dx$. [5]

6



The diagram shows a circle with centre O and radius r. The tangents to the circle at the points A and B meet at T, and angle AOB is $2x$ radians. The shaded region is bounded by the tangents AT and BT, and by the minor arc AB. The area of the shaded region is equal to the area of the circle.

(a) Show that x satisfies the equation $\tan x = \pi + x$. [3]

(b) This equation has one root in the interval $0 < x < \frac{1}{2}\pi$. Verify by calculation that this root lies between 1 and 1.4. [2]

(c) Use the iterative formula

$$x_{n+1} = \tan^{-1}(\pi + x_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 Let $f(x) = \dfrac{\cos x}{1 + \sin x}$.

(a) Show that $f'(x) < 0$ for all x in the interval $-\frac{1}{2}\pi < x < \frac{3}{2}\pi$. [4]

(b) Find $\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \mathrm{f}(x)\,\mathrm{d}x$. Give your answer in a simplified exact form. [4]

8 A certain curve is such that its gradient at a point (x, y) is proportional to $\dfrac{y}{x\sqrt{x}}$. The curve passes through the points with coordinates $(1, 1)$ and $(4, \mathrm{e})$.

(a) By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x. [8]

(b) Describe what happens to y as x tends to infinity. [1]

9 With respect to the origin O, the vertices of a triangle ABC have position vectors

$$\overrightarrow{OA} = 2\mathbf{i} + 5\mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = \mathbf{i} + \mathbf{j} + \mathbf{k}.$$

(a) Using a scalar product, show that angle ABC is a right angle. [3]

(b) Show that triangle ABC is isosceles. [2]

(c) Find the exact length of the perpendicular from O to the line through B and C. [4]

10 **(a)** The complex number u is defined by $u = \frac{3\mathrm{i}}{a + 2\mathrm{i}}$, where a is real.

(i) Express u in the Cartesian form $x + \mathrm{i}y$, where x and y are in terms of a. [3]

....

....

....

....

....

....

....

....

....

....

....

(ii) Find the exact value of a for which $\arg u^* = \frac{1}{3}\pi$. [3]

....

....

....

....

....

....

....

....

....

....

....

....

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2\mathrm{i}| \leqslant |z - 1 - \mathrm{i}|$ and $|z - 2 - \mathrm{i}| \leqslant 2$. [4]

(ii) Calculate the least value of $\arg z$ for points in this region. [2]

....

....

....

....

....

....

....

....

....

....

....

....

....