

4 The variables x and y satisfy the differential equation

$$(1 - \cos x) \frac{dy}{dx} = y \sin x.$$

It is given that $y = 4$ when $x = \pi$.

(a) Solve the differential equation, obtaining an expression for y in terms of x . [6]

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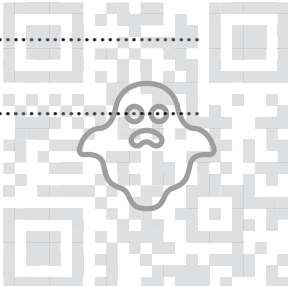
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(b) Sketch the graph of y against x for $0 < x < 2\pi$. [1]



- 5 (a) Express $\sqrt{7} \sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 2 decimal places. [3]

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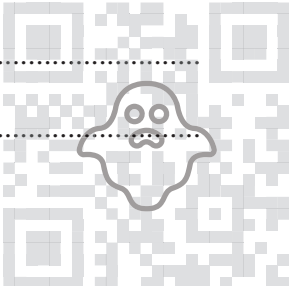
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(b) Hence solve the equation $\sqrt{7} \sin 2\theta + 2 \cos 2\theta = 1$, for $0^\circ < \theta < 180^\circ$. [5]

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(b) Hence show that $\int_a^{2a} f(x) dx = \ln 6$. [4]

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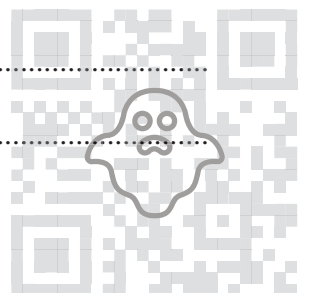
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(b) Find the acute angle between the directions of the two lines.

[3]

Dotted lines for writing the answer.



8 The complex numbers u and v are defined by $u = -4 + 2i$ and $v = 3 + i$.

(a) Find $\frac{u}{v}$ in the form $x + iy$, where x and y are real. [3]

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(b) Hence express $\frac{u}{v}$ in the form $re^{i\theta}$, where r and θ are exact. [2]

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In an Argand diagram, with origin O , the points A , B and C represent the complex numbers u , v and $2u + v$ respectively.

(c) State fully the geometrical relationship between OA and BC . [2]

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(d) Prove that angle $AOB = \frac{3}{4}\pi$. [2]

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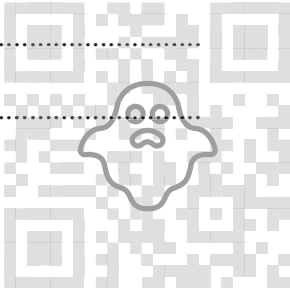
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9 Let $f(x) = \frac{e^{2x} + 1}{e^{2x} - 1}$, for $x > 0$.

(a) The equation $x = f(x)$ has one root, denoted by a .

Verify by calculation that a lies between 1 and 1.5. [2]

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(b) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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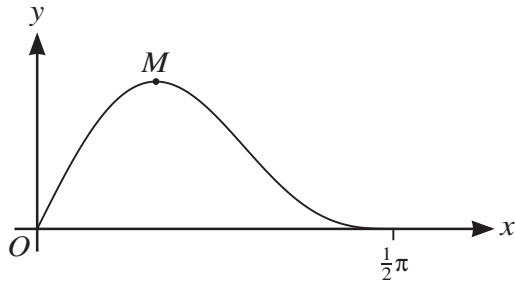
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The diagram shows the curve $y = \sin 2x \cos^2 x$ for $0 \leq x \leq \frac{1}{2}\pi$, and its maximum point M .

- (a) Using the substitution $u = \sin x$, find the exact area of the region bounded by the curve and the x -axis. [5]

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