1 Showing all necessary working, solve the equation $4x - 11x^{\frac{1}{2}} + 6 = 0$. [3]

$$\begin{matrix} 4 & -3 \\ 1 & -2 \end{matrix}$$

$(4x^{1/2} - 3)(x^{1/2} - 2) = 0$

$x^{1/2} = 3/4$ OR $x^{1/2} = 2$

$x = 9/16$ OR $x = 4$

2 A line has equation $y = x + 1$ and a curve has equation $y = x^2 + bx + 5$. Find the set of values of the constant b for which the line meets the curve. [4]

$x + 1 = x^2 + bx + 5$

$x^2 + bx - x + 4 = 0$

$\Delta = (b-1)^2 - 16 \geq 0$

$(b-1)^2 \geq 16$

$b - 1 \geq 4$ OR $b - 1 \leq -4$

$b \geq 5$ OR $b \leq -3$

3 Two points A and B have coordinates $(3a, -a)$ and $(-a, 2a)$ respectively, where a is a positive constant.

(i) Find the equation of the line through the origin parallel to AB. [2]

$$m_{AB} = \frac{2a+a}{-a-3a} = \frac{3}{-4}$$

$$y = -\tfrac{3}{4}x$$

(ii) The length of the line AB is $3\frac{1}{3}$ units. Find the value of a. [3]

$$AB^2 = 16a^2 + 9a^2 = \frac{100}{9}$$

$$25a^2 = \frac{100}{9}$$

$$a^2 = \frac{4}{9}$$

$$a = \tfrac{2}{3}$$

4 The first term of a series is 6 and the second term is 2.

(i) For the case where the series is an arithmetic progression, find the sum of the first 80 terms. [3]

AP :
$$\boxed{\begin{aligned} a &= 6 \\ d &= -4 \end{aligned}}$$

$$S_{80} = \frac{80}{2}\left[12 + 79(-4)\right]$$

$$= \boxed{\begin{array}{l} 40\times(12-79\times4) \\ \hfill -12160 \end{array}}$$

(ii) For the case where the series is a geometric progression, find the sum to infinity. [2]

GP
$$\boxed{\begin{aligned} a &= 6 \\ r &= \frac{2}{6} = \frac{1}{3} \end{aligned}}$$

$$S_\infty = \frac{6}{1-1/3} = 9$$

5 **(i)** Show that the equation

$$\frac{\cos\theta - 4}{\sin\theta} - \frac{4\sin\theta}{5\cos\theta - 2} = 0$$

may be expressed as $9\cos^2\theta - 22\cos\theta + 4 = 0$. [3]

let $\cos(\theta) = y$, $\sin^2(\theta) = 1 - y^2$

$(y-4)(5y-2) - 4\sin^2(\theta) = 0$

$5y^2 - 22y + 8 - 4(1-y^2) = 0$

$5y^2 - 22y + 8 - 4 + 4y^2 = 0$

$9y^2 - 22y + 4 = 0$

$9\cos^2(\theta) - 22\cos(\theta) + 4 = 0$

as required.

(ii) Hence solve the equation

$$\frac{\cos\theta - 4}{\sin\theta} - \frac{4\sin\theta}{5\cos\theta - 2} = 0$$

for $0° \leqslant \theta \leqslant 360°$. [3]

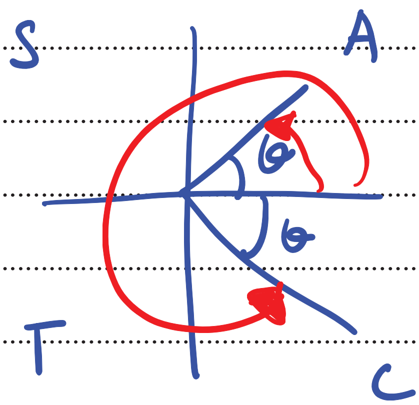
$$\cos(\theta) = \frac{22 \pm \sqrt{340}}{18}$$

$\theta =$ $\cos^{-1}\left(\frac{22-\sqrt{340}}{18}\right)$

78.59000195

360−Ans

281.4099981



$\theta = 78.6°, 281.4°$

6 A curve has a stationary point at $(3, 9\frac{1}{2})$ and has an equation for which $\frac{dy}{dx} = ax^2 + a^2x$, where a is a non-zero constant.

(i) Find the value of a. [2]

$$\frac{dy}{dx} = a \cdot 9 + a^2 \cdot 3 = 0$$

$$9 + 3a = 0$$

$$a = -3$$

(ii) Find the equation of the curve. [4]

$$y = \int -3x^2 + 9x \, dx$$

$$y = -x^3 + \frac{9}{2}x^2 + c$$

$$9\tfrac{1}{2} = -27 + \frac{81}{2} + c$$

$$c = -4$$

$$y = -x^3 + \frac{9}{2}x^2 - 4$$

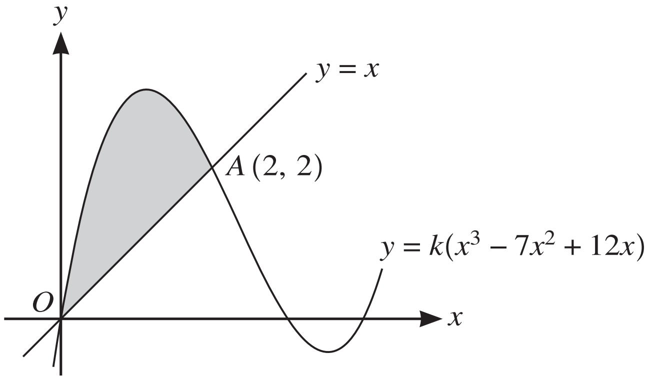
(iii) Determine, showing all necessary working, the nature of the stationary point. [2]

$$\frac{d^2y}{dx^2} = -6x + 9$$

$$= -6(3) + 9 < 0$$

maximum

7



The diagram shows part of the curve with equation $y = k(x^3 - 7x^2 + 12x)$ for some constant k. The curve intersects the line $y = x$ at the origin O and at the point $A\ (2, 2)$.

(i) Find the value of k. [1]

$2 = k(8 - 28 + 24)$

$2 = 4k$, $k = \frac{1}{2}$

(ii) Verify that the curve meets the line $y = x$ again when $x = 5$. [2]

$x = 5$, $y = \frac{1}{2}(125 - 175 + 60)$

$= 5 = x$

(iii) Find, showing all necessary working, the area of the shaded region. [5]

SHADED

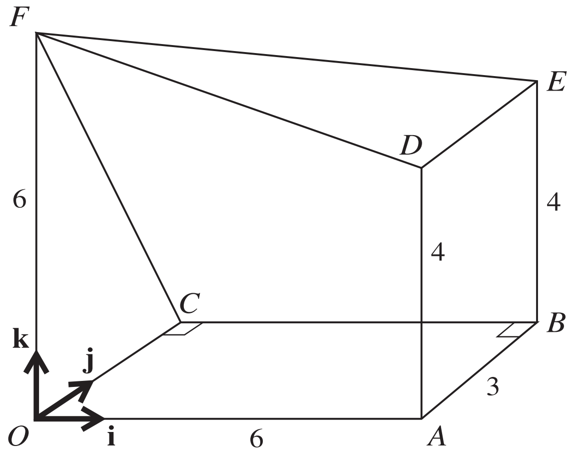
$$= \int_0^2 \tfrac{1}{2}(x^3 - 7x^2 + 12x) - x \, dx$$

$$= \left[\tfrac{1}{8}x^4 - \tfrac{7}{6}x^3 + 3x^2 - \frac{x^2}{2} \right]_0^2$$

$$= 2 - \frac{56}{6} + 12 - 2$$

$$= \boxed{\frac{8}{3}}$$

8



The diagram shows a solid figure $OABCDEF$ having a horizontal rectangular base $OABC$ with $OA = 6$ units and $AB = 3$ units. The vertical edges OF, AD and BE have lengths 6 units, 4 units and 4 units respectively. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OC and OF respectively.

(i) Find $\overrightarrow{DF}$. [1]

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(ii) Find the unit vector in the direction of $\overrightarrow{EF}$. [3]

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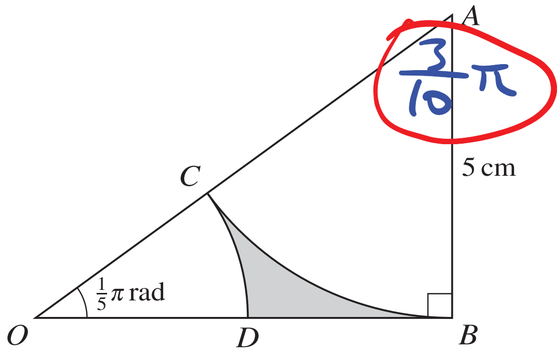
(iii) Use a scalar product to find angle EFD. [4]

9



The diagram shows a triangle OAB in which angle ABO is a right angle, angle $AOB = \frac{1}{5}\pi$ radians and $AB = 5$ cm. The arc BC is part of a circle with centre A and meets OA at C. The arc CD is part of a circle with centre O and meets OB at D. Find the area of the shaded region. [8]

$\frac{5}{OB} = \tan(\pi/5)$

$$\frac{5}{\tan(\pi \div 5)} \quad 6.881909602$$

$OB =$ $\text{Ans} \rightarrow A \quad 6.881909602$

$OC =$ $\sqrt{A^2+25}-5 \quad 3.506508084$

$\text{Ans} \rightarrow B \quad 3.506508084$

$\text{SHADED} = \frac{1}{2}\cdot 5 \cdot OB - \frac{1}{2}\cdot OC^2 \cdot \frac{\pi}{5} - \frac{1}{2}\cdot 5^2 \cdot \frac{3\pi}{10}$

$= \frac{5A}{2} - \frac{B^2\pi}{10} - \frac{75\pi}{20} \quad 1.561025225$

1.56 OR 1.57

10 A curve has equation $y = \frac{1}{2}(4x - 3)^{-1}$. The point A on the curve has coordinates $\left(1, \frac{1}{2}\right)$.

(i) (a) Find and simplify the equation of the normal through A. [5]

$$\frac{dy}{dx} = -2(4x-3)^{-2}$$

$$= -2(1)^{-2} = -2$$

NORMAL:

$$y - \tfrac{1}{2} = \tfrac{1}{2}(x-1)$$

$$y = \tfrac{1}{2}x$$

(b) Find the x-coordinate of the point where this normal meets the curve again. [3]

$\frac{1}{2}x = \frac{1}{2}(4x-3)^{-1}$

$x = \dfrac{1}{4x-3}$

$4x^2 - 3x - 1 = 0$

$4x \quad +1$
$1x \quad -1$

$(4x+1)(x-1) = 0$

$x = -\frac{1}{4}$

(ii) A point is moving along the curve in such a way that as it passes through A its x-coordinate is decreasing at the rate of 0.3 units per second. Find the rate of change of its y-coordinate at A. [2]

$\dfrac{dx}{dt} = -0.3$

$\dfrac{dy}{dt} = \boxed{\dfrac{dy}{dx} \cdot \dfrac{dx}{dt}} = -0.3\left(-2(4x-3)^{-2}\right)$

$= 0.6(1)^{-2}$

$= 0.6$

11 **(a)** The one-one function f is defined by $f(x) = (x-3)^2 - 1$ for $x < a$, where a is a constant.

(i) State the greatest possible value of a. [1]

VERTEX : $(3, -1)$

3

(ii) It is given that a takes this greatest possible value. State the range of f and find an expression for $f^{-1}(x)$. [3]

$f > -1$

$y = (x-3)^2 - 1$

$y + 1 = (x-3)^2$ $(x < 3)$

$-\sqrt{y+1} = x - 3$

$f^{-1}(x) = 3 - \sqrt{x+1}$

(b) The function g is defined by $g(x) = (x-3)^2$ for $x \geqslant 0$.

(i) Show that $gg(2x)$ can be expressed in the form $(2x-3)^4 + b(2x-3)^2 + c$, where b and c are constants to be found. [2]

$$g\left((2x-3)^2\right) = \left((2x-3)^2 - 3\right)^2$$

$$= (2x-3)^4 - 6(2x-3)^2 + 9$$

$$\boxed{b = -6, \; c = 9}$$

(ii) Hence expand $gg(2x)$ completely, simplifying your answer. [4]

$$gg(2x) = (2x)^4 + 4(2x)^3(-3) + \binom{4}{2}(2x)^2(-3)^2 + 4(2x)(-3)^3 + (-3)^4$$

$$-6(4x^2 - 12x + 9) + 9$$

$$= 16x^4 - 96x^3 + 216x^2 - 216x + 81 - 24x^2 + 72x - 45$$

$$= \boxed{16x^4 - 96x^3 + 192x^2 - 144x + 36}$$