Question	Answer	Marks	Guidance
1	Remove logarithms and state $4-3^x = e^{1.2}$, or equivalent	B1	Accept $4-3^x = 3.32(01169.....)$ 3 s.f. or better
	Use correct method to solve an equation of the form $3^x = a$, where $a > 0$.	M1	$(3^x = 0.67988..)$ Complete method to $x = \ldots$ If using $\log_3$ the subscript can be implied
	Obtain answer $x = -0.351$ only	A1	CAO must be to 3 d.p.
		3	

Question	Answer	Marks	Guidance
2	Use correct quotient rule or correct product rule	**M1**	
	Obtain correct derivative in any form	**A1**	$\frac{dy}{dx}=\frac{-2e^{-2x}\left(1-x^2\right)+2xe^{-2x}}{\left(1-x^2\right)^2}$
	Equate derivative to zero and obtain a 3 term quadratic in x	**M1**	
	Obtain a correct 3-term equation e.g. $2x^2+2x-2=0$ or $x^2+x=1$	**A1**	From correct work only
	Solve and obtain $x = 0.618$ only	**A1**	From correct work only
		5	

Question	Answer	Marks	Guidance
3	Commence division and reach partial quotient $x^2 + kx$	**M1**	
	Obtain correct quotient $x^2 + 2x - 1$	**A1**	
	Set their linear remainder equal to $2x + 3$ and solve for a or for b	**M1**	Remainder = $(a+3)x + (b-1)$
	Obtain answer $a = -1$	**A1**	
	Obtain answer $b = 4$	**A1**	
	Alternative method for question 3		
	State $x^4 + 3x^3 + ax + b = (x^2 + x - 1)(x^2 + Ax + B) + 2x + 3$ and form and solve two equations in A and B	**M1**	e.g. $3 = 1 + A$ and $0 = -1 + A + B$
	Obtain $A = 2,\ B = -1$	**A1**	
	Form and solve equations for a or b	**M1**	e.g. $a = B - A + 2, \quad b = -B + 3$
	Obtain answer $a = -1$	**A1**	
	Obtain answer $b = 4$	**A1**	
		5	

Question	Answer	Marks	Guidance
3	**Alternative method for question 3**		
	Use remainder theorem with $x = \frac{-1 \pm \sqrt{5}}{2}$	**M1**	Allow for correct use of either root in exact or decimal form.
	Obtain $-\frac{a}{2} \pm \frac{a\sqrt{5}}{2} + b = \frac{9}{2} \mp \frac{\sqrt{5}}{2}$	**A1**	Expand brackets and obtain exact equation for either root. Accept exact equivalent.
	Solve simultaneous equations for a or b	**M1**	
	Obtain answer $a = -1$ from exact working	**A1**	
	Obtain answer $b = 4$ from exact working	**A1**	
		5	

Question	Answer	Marks	Guidance
4(i)	State $R = \sqrt{7}$	**B1**	
	Use correct trig formulae to find α	**M1**	e.g. $\tan\alpha = \frac{1}{\sqrt{6}}$, $\sin\alpha = \frac{1}{\sqrt{7}}$, or $\cos\alpha = \frac{\sqrt{6}}{\sqrt{7}}$
	Obtain $\alpha = 22.208°$	**A1**	ISW
		3	

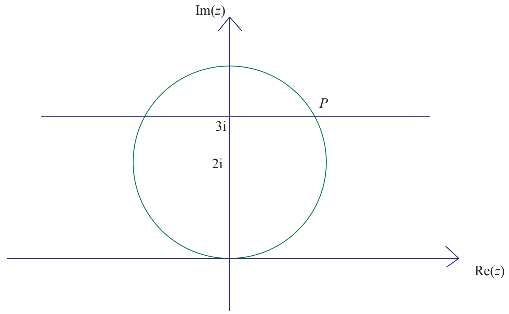
Question	Answer	Marks	Guidance
4(ii)	Evaluate $\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$ to at least 1 d.p.	**B1FT**	$49.107°$ to 3 d.p. B1 can be implied by correct answer(s) later. The FT is on *their R*
			SC: allow B1 for a correct alternative equation e.g. $3\tan^2\theta - 2\sqrt{6}\tan\theta + 1 = 0$
	Use correct method to find a value of θ in the interval	**M1**	Must get to θ
	Obtain answer, e.g. $13.4°$	**A1**	Accept correct over-specified answers. 13.449…, 54.3425…
	Obtain second answer, e.g. $54.3°$ and no extras in the given interval	**A1**	Ignore answers outside the given interval.
		4	

Question	Answer	Marks	Guidance
5	State $4xy + 2x^2 \dfrac{\mathrm{d}y}{\mathrm{d}x}$, or equivalent, as derivative of $2x^2y$	B1	
	State $y^2 + 2xy \dfrac{\mathrm{d}y}{\mathrm{d}x}$, or equivalent, as derivative of xy^2	B1	
	Equate attempted derivative of LHS to zero and set $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero (or set numerator equal to zero)	*M1	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y^2 - 4xy}{2x^2 - 2xy}$
	Reject $y = 0$	B1	Allow from $y^2 - kxy = 0$
	Obtain $y = 4x$	A1	OE from correct numerator. ISW
	Obtain an equation in y (or in x) and solve for y (or for x) in terms of a	DM1	$8x^3 - 16x^3 = a^3$ or $\dfrac{y^3}{8} - \dfrac{y^3}{4} = a^3$
	Obtain $y = -2a$	A1	With no errors seen
		7	

Question	Answer	Marks	Guidance
5	**Alternative method for question 5**		
	Rewrite as $y = \dfrac{a^3}{2x^2 - xy}$ and differentiate	**M1**	Correct use of function of a function and implicit differentiation
	Obtain correct derivative (in any form)	**A1**	$\dfrac{dy}{dx} = \dfrac{-a^3\left(4x - y - x\dfrac{dy}{dx}\right)}{\left(2x^2 - xy\right)^2}$
	set $\dfrac{dy}{dx}$ equal to zero (or set numerator equal to zero)	***M1**	
	Obtain $4x - y = 0$	**A1**	
	Confirm $2x^2 - xy \neq 0$	**B1**	$x = 0$ and $2x = y$ both give $a = 0$
	Obtain an equation in y (or in x) and solve for y (or for x)	**DM1**	$8x^3 - 16x^3 = a^3$ or $\dfrac{y^3}{8} - \dfrac{y^3}{4} = a^3$
	Obtain $y = -2a$	**A1**	With no errors seen
		7	

Question	Answer	Marks	Guidance
6	Separate variables correctly to obtain $\int \frac{1}{x+2}\,\mathrm{d}x = \int \cot\frac{1}{2}\theta\ \mathrm{d}\theta$	B1	Or equivalent integrands. Integral signs SOI
	Obtain term $\ln(x+2)$	B1	Modulus signs not needed.
	Obtain term of the form $k\ln\sin\frac{1}{2}\theta$	M1	
	Obtain term $2\ln\sin\frac{1}{2}\theta$	A1	
	Use $x = 1$, $\theta = \frac{1}{3}\pi$ to evaluate a constant, or as limits, in an expression containing $p\ln(x+2)$ and $q\ln\left(\sin\frac{1}{2}\theta\right)$	M1	Reach $C =$ an expression or a decimal value
	Obtain correct solution in any form e.g. $\ln(x+2) = 2\ln\sin\frac{1}{2}\theta + \ln 12$	A1	$\ln 12 = 2.4849$….. Accept constant to at least 3 s.f. Accept with $\ln 3 - 2\ln\frac{1}{2}$
	Remove logarithms and use correct double angle formula	M1	Need correct algebraic process. $\left(\frac{x+2}{12} = \frac{1-\cos\theta}{2}\right)$
	Obtain answer $x = 4 - 6\cos\theta$	A1	
		8	

Question	Answer	Marks	Guidance
7(a)	Substitute and obtain a correct horizontal equation in x and y in any form	**B1**	$zz^* + iz - 2z^* = 0 \Rightarrow$ $x^2 + y^2 + ix - y - 2x + 2iy = 0$ Allow if still includes brackets and/or i^2
	Use $i^2 = -1$ and equate real and imaginary parts to zero OE	***M1**	For their horizontal equation
	Obtain two correct equations e.g. $x^2 + y^2 - y - 2x = 0$ and $x + 2y = 0$	**A1**	Allow $ix + 2iy = 0$
	Solve for x or for y	**DM1**	
	Obtain answer $\frac{6}{5} - \frac{3}{5}i$ and no other	**A1**	OE, condone $\frac{1}{5}(6 - 3i)$
		5	

Question	Answer	Marks	Guidance
7(b)(i)	Show a circle with centre 2i and radius 2	**B1**	
	Show horizontal line $y = 3$ – in first and second quadrant	**B1**	
			SC: For clearly labelled axes not in the conventional directions, allow B1 for a fully 'correct' diagram.
		2	
7(b)(ii)	Carry out a complete method for finding the argument. (Not by measuring the sketch)	**M1**	$\left(z = \sqrt{3} + 3\mathrm{i}\right)$ Must show working if using 1.7 in place of $\sqrt{3}$.
	Obtain answer $\frac{1}{3}\pi$ (or $60°$)	**A1**	**SC:** Allow B2 for 60° with no working
		2	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $\dfrac{A}{2x-1}+\dfrac{Bx+C}{x^2+2}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A=4$, $B=-1$, $C=0$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
8(ii)	Integrate and obtain term $2\ln(2x-1)$	**B1FT**	The FT is on A. $\frac{1}{2}A\ln(2x-1)$
	Integrate and obtain term of the form $k\ln(x^2+2)$	***M1**	From $\dfrac{nx}{x^2+2}$
	Obtain term $-\frac{1}{2}\ln(x^2+2)$	**A1FT**	The FT is on B
	Substitute limits correctly in an integral of the form $a\ln(2x-1)+b\ln(x^2+2)$, where $ab\neq 0$	**DM1**	$2\ln 9(-2\ln 1)-\frac{1}{2}\ln 27+\frac{1}{2}\ln 3$
	Obtain answer ln 27 after full and correct exact working	**A1**	ISW
		5	

Question	Answer	Marks	Guidance
9(i)	Commence integration by parts, reaching $ax\sin\frac{1}{3}x - b\int\sin\frac{1}{3}x\,dx$	*M1	
	Obtain $3x\sin\frac{1}{3}x - 3\int\sin\frac{1}{3}x\,dx$	A1	
	Complete integration and obtain $3x\sin\frac{1}{3}x + 9\cos\frac{1}{3}x$	A1	
	Substitute limits correctly and equate result to 3 in an integral of the form $px\sin\frac{1}{3}x + q\cos\frac{1}{3}x$	DM1	$3 = 3a\sin\frac{a}{3} + 9\cos\frac{a}{3}(-0) - 9$
	Obtain $a = \dfrac{4 - 3\cos\frac{a}{3}}{\sin\frac{a}{3}}$ correctly	A1	With sufficient evidence to show how they reach the given equation
		5	
9(ii)	Calculate values at $a = 2.5$ and $a = 3$ of a relevant expression or pair of expressions.	M1	$2.5 < 2.679$ and $3 > 2.827$ If using 2.679 and 2.827 must be linked explicitly to 2.5 and 3. Solving $f(a) = 0$, $f(2.5) = 0.179$. and $f(3) = -0.173$ or if $f(a) = a\sin\frac{1}{3}a + 3\cos\frac{1}{3}a - 4 \Rightarrow f(2.5) = -0.13..,f(3) = 0.145...$
	Complete the argument correctly with correct calculated values	A1	Accept values to 1 sf. or better
		2	

Question	Answer	Marks	Guidance
9(iii)	Use the iterative process $a_{n+1} = a_{n+1}\dfrac{4-3\cos\frac{1}{3}a_n}{\sin\frac{1}{3}a_n}$ correctly at least once	M1	
	Show sufficient iterations to at least 5 d.p. to justify 2.736 to 3d.p., or show a sign change in the interval (2.7355, 2.7365)	A1	
	Obtain final answer 2.736	A1	
		3	

Question	Answer	Marks	Guidance
10(i)	Express general point of l in component form e.g. $(1+\lambda, 3-2\lambda, -2+3\lambda)$	B1	
	Substitute in equation of p and solve for λ	M1	
	Obtain final answer $\frac{5}{3}\mathbf{i}+\frac{5}{3}\mathbf{j}$ from $\lambda=\frac{2}{3}$	A1	OE Accept $1.67\mathbf{i}+1.67\mathbf{j}$ or better
		3	

Question	Answer	Marks	Guidance
10(ii)	Use correct method to evaluate a scalar product of relevant vectors e.g. (**i** – 2**j** + 3**k**).(2**i** + **j** – 3**k**)	**M1**	
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse sine or cosine of the result	**M1**	$\lvert\sin\theta\rvert = \frac{9}{14}$
	Obtain answer 40.0° or 0.698 radians	**A1**	AWRT
		3	
	Alternative method for question 10(ii)		
	Use correct method to evaluate a vector product of relevant vectors e.g. (**i** – 2**j** + 3**k**)x(2**i** + **j** – 3**k**)	**M1**	
	Using the correct process for calculating the moduli, divide the modulus of the vector product by the product of the moduli of the two vectors and evaluate the inverse sine or cosine of the result	**M1**	$\cos\theta = \frac{\sqrt{115}}{14}$
	Obtain answer 40.0° or 0.698 radians	**A1**	AWRT
		3	

Question	Answer	Marks	Guidance
10(iii)	State $a - 2b + 3c = 0$ or $2a + b - 3c = 0$	**B1**	
	Obtain two relevant equations and solve for one ratio, e.g. $a : b$	**M1**	Could use $2a + b - 3c = 0$ and $\begin{cases} a + 3b - 2c = d \\ \frac{5}{3}a + \frac{5}{3}b = d \end{cases}$ i.e. use two points on the line rather than the direction of the line. The second M1 is not scored until they solve for d.
	Obtain $a : b : c = 3 : 9 : 5$	**A1**	OE
	Substitute a, b, c and a relevant point in the plane equation and evaluate d	**M1**	Using their calculated normal and a relevant point
	Obtain answer $3x + 9y + 5z = 20$	**A1**	OE
	Alternative method for question 10(iii)		
	Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$	**M1**	
	Obtain two correct components	**A1**	
	Obtain correct answer, e.g. $3\mathbf{i} + 9\mathbf{j} + 5\mathbf{k}$	**A1**	
	Use the product and a relevant point to find d	**M1**	Using *their* calculated normal and a relevant point
	Obtain answer $3x + 9y + 5z = 20$, or equivalent	**A1**	OE

Question	Answer	Marks	Guidance
10(iii)	**Alternative method for question 10(iii)**		
	Attempt to form a 2-parameter equation with relevant vectors	**M1**	
	State a correct equation e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$	**A1**	
	State 3 equations in x, y, z, λ and μ	**A1**	
	Eliminate λ and μ	**M1**	
	Obtain answer $3x + 9y + 5z = 2$	**A1**	OE
		5	