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Question	Answer	Marks	Guidance
1(i)	$1 + 6y + 15y^2$	B1	CAO
		1	
1(ii)	$1+6(px-2x^2)+15(px-2x^2)^2$	M1	SOI. Allow $6C1 \times 1^5 (px - 2x^2)$, $6C2 \times 1^4 (px - 2x^2)^2$
	$(15p^2-12)(x^2) = 48(x^2)$	A1	1 term from each bracket and equate to 48
	<i>p</i> = 2	A1	SC: A1 $p = 4$ from $15p - 12 = 48$
		3	

Question	Answer	Marks	Guidance
2	$(y=)\left[\left(x-3\right)^2\right]\left[-2\right]$	*B1 DB1	DB1 dependent on 3 in 1st bracket
	$x-3 = (\pm)\sqrt{y+2}$ or $y-3 = (\pm)\sqrt{x+2}$	M1	Correct order of operations
	$\left(g^{-1}\left(x\right)\right) = 3 + \sqrt{x+2}$	A1	Must be in terms of x
	Domain (of g^{-1}) is (x) > -1	B1	Allow $(-1, \infty)$. Do not allow $y \ge -1$ or $g(x) \ge -1$ or $g^{-1}(x) \ge -1$
		5	

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Question	Answer	Marks	Guidance
3	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2x - 8$	B1	
	Set to zero (SOI) and solve	M1	
	(Min) $a = -2$, (Max) $b = 4/3$. – in terms of <i>a</i> and <i>b</i> .	A1 A1	Accept $a \ge -2$, $b \le \frac{4}{3}$ SC: A1 for $a > -2$, $b < \frac{4}{3}$ or for $-2 < x < \frac{4}{3}$
		4	

Question	Answer	Marks	Guidance
4(i)	Angle $CAO = \frac{\pi}{3}$	B1	
		1	
4(ii)	(Sector AOC) = $\frac{1}{2}r^2 \times their\frac{\pi}{3}$	M1	SOI
	$(\Delta ABC) = \frac{1}{2}(r)(2r)\sin\left(their\frac{\pi}{3}\right)$ or $\frac{1}{2}(2r)(r)\frac{\sqrt{3}}{2}$ or $\frac{1}{2}(r)(r)\sqrt{3}$	M1	For M1M1, <i>their</i> $\frac{\pi}{3}$ must be of the form $k\pi$ where $0 < k < \frac{1}{2}$
	$(\Delta ABC) = \frac{1}{2}(r)(2r)\sin\left(\frac{\pi}{3}\right)$ or $\frac{1}{2}(2r)(r)\frac{\sqrt{3}}{2}$ or $\frac{1}{2}(r)(r)\sqrt{3}$	A1	All correct
	$r^2\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2}r^2\left(\frac{\pi}{3}\right)$	A1	
		4	

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Question	Answer	Marks	Guidance
5(i)	$S = 28x^2, V = 8x^3$	B1B1	SOI
	$7V^{\frac{2}{3}} = 7 \times 4x^2 = S$	B1	AG, WWW
		3	
5(ii)	$\left(\frac{\mathrm{d}S}{\mathrm{d}V}\right) = \frac{14V^{-\frac{1}{3}}}{3} = \frac{14}{30}$ SOI when $V = 1000$	*M1 A1	Attempt to differentiate For M mark $\left(\frac{dS}{dV}\right)$ to be of form $kV^{-\frac{1}{3}}$
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}S}{\mathrm{d}t} \times \frac{\mathrm{d}V}{\mathrm{d}S}\right) \text{ OE used with } \frac{\mathrm{d}S}{\mathrm{d}t} = 2 \text{ and } \frac{1}{their\frac{14}{30}}$	DM1	
	$\frac{30}{7}$ or 4.29	A1	OE
	Alternative method for question 5(ii)		
	$V = \frac{S^{\frac{3}{2}}}{7\sqrt{7}} \rightarrow \left(\frac{\mathrm{d}V}{\mathrm{d}S}\right) = \frac{3}{2} \times S^{\frac{1}{2}} \times \frac{1}{7\sqrt{7}} = \frac{30}{14} \text{ SOI when } S = 700$	*M1 A1	
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}S}{\mathrm{d}t} \times \frac{\mathrm{d}V}{\mathrm{d}S}\right) \text{ OE used with } \frac{\mathrm{d}S}{\mathrm{d}t} = 2 \text{ and } \frac{1}{their \frac{14}{30}}$	DM1	
	$\frac{30}{7}$ or 4.29	A1	OE

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Question	Answer	Marks	Guidance	
5(ii)	Alternative method for question 5(ii)			
	Attempt to find either $\frac{dV}{dx}$ or $\left(\frac{dS}{dx} \text{ and } \frac{dV}{dS}\right)$ together with either $\frac{dx}{dt}$ or x	*M1		
	$\frac{\mathrm{d}V}{\mathrm{d}x} = 24x^2 \text{ or } \left(\frac{\mathrm{d}S}{\mathrm{d}x} = 56x \text{ and } \frac{\mathrm{d}V}{\mathrm{d}S} = \frac{3x}{7}\right), \ \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{140} \text{ or } x = 5 \text{ (A1)}$	A1		
	Correct method for $\frac{\mathrm{d}V}{\mathrm{d}t}$	DM1		
	$\frac{30}{7}$ or 4.29	A1	OE	
		4		

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Question	Answer	Marks	Guidance
6(i)	$3kx - 2k = x^{2} - kx + 2 \rightarrow x^{2} - 4kx + 2k + 2 (= 0)$	B1	kx terms combined correctly-implied by correct $b^2 - 4ac$
	Attempt to find $b^2 - 4ac$	M1	Form a quadratic equation in k
	1 and $-\frac{1}{2}$	A1	SOI
	$k > 1, \ k < -\frac{1}{2}$	A1	Allow $x > 1$, $x < -1/2$
		4	
6(ii)	$y = 3x - 2, y = -\frac{3}{2}x + 1$	M1	Use of <i>their k</i> values (twice) in $y = 3kx - 2k$
	$3x-2=-\frac{3}{2}x+1$ OR $y+2=2-2y$	M1	Equate <i>their</i> tangent equations OR substitute $y = 0$ into both lines
	$x = \frac{2}{3}, \rightarrow y = 0$ in one or both lines	A1	Substitute $x = \frac{2}{3}$ in one or both lines
		3	

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Question	Answer	Marks	Guidance
7(i)	$3\cos^4\theta + 4\left(1 - \cos^2\theta\right) - 3(=0)$	M1	Use $s^2 = 1 - c^2$
	$3x^{2} + 4(1-x) - 3(=0) \rightarrow 3x^{2} - 4x + 1(=0)$	A1	AG
		2	
7(ii)	Attempt to solve for <i>x</i>	M1	Expect $x = 1, 1/3$
	$\cos\theta = (\pm)1, \ (\pm)0.5774$	A1	Accept $(\pm)\left(\frac{1}{\sqrt{3}}\right)$ SOI
	$(\theta =) 0^{\circ}, 180^{\circ}, 54.7^{\circ}, 125.3^{\circ}$	A3,2,1,0	A2,1,0 if more than 4 solutions in range
		5	

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Question	Answer	Marks	Guidance
8(i)	$(2x-1)^{\frac{1}{2}} < 2 \text{ or } 3(2x-1)^{\frac{1}{2}} < 6$	M1	SOI
	2x - 1 < 4	A1	SOI
	$\frac{1}{2} < x < \frac{5}{2}$	A1 A1	Allow 2 separate statements
		4	
8(ii)	$f(x) = [3(2x-1)^{3/2} \div (\frac{3}{2}) \div (2)] [-6x] \ (+c)$	B1 B1	
	Subsitute $x = 1$, $y = -3$ into an integrated expression.	M1	Dependent on <i>c</i> being present ($c = 2$)
	$f(x) = (2x-1)^{\frac{3}{2}} - 6x + 2$	A1	
		4	

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Question	Answer	Marks	Guidance
9(i)	$\frac{5k-6}{3k} = \frac{6k-4}{5k-6} \to (5k-6)^2 = 3k(6k-4)$	M1	OR any valid relationship
	$25k^2 - 60k + 36 = 18k^2 - 12k \rightarrow 7k^2 - 48k + 36$	A1	AG
		2	
9(ii)	$k = \frac{6}{7} , 6$	B1B1	Allow 0.857(1) for $\frac{6}{7}$
	When $k = \frac{6}{7}, r = -\frac{2}{3}$	B1	Must be exact
	When $k = 6, r = \frac{4}{3}$	B1	
		4	
9(iii)	Use of $S_{\infty} = \frac{a}{1-r}$ with $r = their - \frac{2}{3}$ and $a = 3 \times their \frac{6}{7}$	M1	Provided $0 < their - 2/3 < 1$
	$\frac{18}{7} \div \left(1 + \frac{2}{3}\right) = \frac{54}{35} \text{ or } 1.54$	A1	FT if 0.857(1) has been used in part (ii).
		2	

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Question	Answer	Marks	Guidance
10(i)	$\mathbf{AX} = \begin{pmatrix} 6\\2\\3 \end{pmatrix}, \text{ and one of } \mathbf{AB} = \begin{pmatrix} 18\\6\\9 \end{pmatrix}, \mathbf{XB} = \begin{pmatrix} 12\\4\\6 \end{pmatrix}, \mathbf{BX} = \begin{pmatrix} -12\\-4\\-6 \end{pmatrix}$	B1B1	
	State $AB = 3AX$ (or $XB = 2AX$ or $AB = \frac{3}{2}XB$ etc) hence straight line OR $\frac{AXAB}{ AX AB } = 1 (\rightarrow \theta = 0)$ or $\frac{AXBX}{ AX BX } = -1 (\rightarrow \theta = 180)$ hence straight line	B1	WWW A conclusion (i.e. a straight line) is required.
		3	
10(ii)	$\mathbf{CX} = \begin{pmatrix} -3\\6\\2 \end{pmatrix}$	B1	
	CX.AX = -18 + 12 + 6	M1	
	= 0 (hence CX is perpendicular to AX)	A1	
		3	
10(iii)	$ \mathbf{CX} = \sqrt{3^2 + 6^2 + 2^2}, \mathbf{AB} = \sqrt{18^2 + 6^2 + 9^2}$ Both attempted	M1	
	Area $\triangle ABC = \frac{1}{2} \times their 21 \times their 7 = 73\frac{1}{2}$	M1A1	Accept answers which round to 73.5
		3	

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Question	Answer	Marks	Guidance
11(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -2\left(x-1\right)^{-3}$	B1	
	When $x = 2$, $m = -2 \rightarrow$ gradient of normal $= -\frac{1}{m}$	M1	<i>m</i> must come from differentiation
	Equation of normal is $y-3 = \frac{1}{2}(x-2) \rightarrow y = \frac{1}{2}x+2$	A1	AG Through (2, 3) with gradient $-\frac{1}{m}$. Simplify to AG
		3	

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Question	Answer	Marks	Guidance
11(ii)	$(\pi)\int y_1^2(dx), (\pi)\int y_2^2(dx)$	*M1	Attempt to integrate y^2 for at least one of the functions
	$ (\pi) \int \left(\frac{1}{2}x+2\right)^2 \text{ or } \left(\frac{1}{4}x^2+2x+4\right) \\ (\pi) \int \left(\left(x-1\right)^{-4}+4\left(x-1\right)^{-2}+4\right) $	A1A1	A1 for $(\frac{1}{2}x+2)^2$ depends on an attempt to integrate this form later
	$ (\pi) \left[\frac{\frac{2}{3} \left(\frac{1}{2} x + 2\right)^3 \text{ or } \frac{1}{12} x^3 + x^2 + 4x \right] $ $ (\pi) \left[\frac{(x-1)^{-3}}{-3} + \frac{4(x-1)^{-1}}{-1} + 4x \right] $	A1A1	Must have at least 2 terms correct for each integral
	$(\pi)\left\{18 - \frac{125}{12}or\frac{2}{3} + 4 + 8 - \left(\frac{1}{12} + 1 + 4\right)\right\} \left\{\frac{-1}{24} - 2 + 12 - \left(\frac{-1}{3} - 4 + 8\right)\right\}$	DM1	Apply limits to at least 1 integrated expansion
	Attempt to add 2 volume integrals (or 1 volume integral + frustum) $\pi \left\{ 7 \frac{7}{12} + 6 \frac{7}{24} \right\}$	DM1	
	$13\frac{7}{8}\pi$ or $\frac{111}{8}\pi$ or 13.9π or 43.6	A1	$\frac{2}{3} + 4 + 8 - \left(\frac{1}{12} + 1 + 4\right) \frac{-1}{24} - 2 + 12 - \left(\frac{-1}{3} - 4 + 8\right)$
		8	