

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $3^2(2x-1)^2 > (x+4)^2$, or corresponding quadratic equation, or pair of linear equations/inequalities $3(2x-1) = \pm(x+4)$	B1	$35x^2 - 44x - 7 = 0$
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1	Allow for reasonable attempt at factorising e.g. $(5x-7)(7x+1)$
	Obtain critical values $x = \frac{7}{5}$ and $x = -\frac{1}{7}$	A1	Accept 1.4 and -0.143 or better for penultimate A mark
	State final answer $x > \frac{7}{5}$, $x < -\frac{1}{7}$	A1	'and' is A0, $\frac{7}{5} < x < -\frac{1}{7}$ is A0. Must be exact values. Must be strict inequalities in final answer
	Alternative		
	Obtain critical value $x = \frac{7}{5}$ from a graphical method	B1	or by inspection, or by solving a linear equation or an inequality
	Obtain critical value $x = -\frac{1}{7}$ similarly	B2	
	State final answer $x > \frac{7}{5}$ or $x < -\frac{1}{7}$ or equivalent	B1	[Do not condone \geq for $>$, or \leq for $<$.]
	4		

Question	Answer	Marks	Guidance
2	Use trig formula and obtain an equation in $\sin \theta$ and $\cos \theta$	M1*	Condone sign error in expansion and/or omission of "+ $\cos \theta$ " $\sin \theta \cos 30^\circ - \cos \theta \sin 30^\circ + \cos \theta = 2 \sin \theta$
	Obtain an equation in $\tan \theta$	M1(dep*)	e.g. $\tan \theta = \frac{1 - \sin 30^\circ}{2 - \cos 30^\circ}$ Can be implied by correct answer following correct expansion. Otherwise need to see working
	Obtain $\tan \theta = 1 / (4 - \sqrt{3})$, or equivalent	A1	$\frac{4 + \sqrt{3}}{13}$, 0.4409.... (2 s.f or better)
	Obtain final answer $\theta = 23.8^\circ$ and no others in range	A1	At least 3 sf (23.7939....) ignore extra values outside range
		4	

Question	Answer	Marks	Guidance
3(i)	Integrate by parts and reach $a \frac{\ln x}{x^2} + b \int \frac{1}{x} \cdot \frac{1}{x^2} dx$	M1*	
	Obtain $\pm \frac{1}{2} \frac{\ln x}{x^2} \pm \int \frac{1}{x} \cdot \frac{1}{2x^2} dx$, or equivalent	A1	
	Complete integration correctly and obtain $-\frac{\ln x}{2x^2} - \frac{1}{4x^2}$, or equivalent	A1	Condone without '+ C' ISW
		3	

Question	Answer	Marks	Guidance
3(ii)	Substitute limits correctly in an expression of the form $a\frac{\ln x}{x^2} + \frac{b}{x^2}$ or equivalent	M1(dep*)	$-\frac{1}{8}\ln 2 - \frac{1}{16} + \frac{1}{4}$
	Obtain the given answer following full and exact working	A1	The step $\ln 2 = \frac{1}{2}\ln 4$ or $2\ln 2 = \ln 4$ needs to be clear.
		2	

Question	Answer	Marks	Guidance
4	Substitute and obtain 3-term quadratic $3u^2 + 4u - 1 = 0$, or equivalent	B1	e.g. $3(e^x)^2 + 4e^x - 1 = 0$
	Solve a 3 term quadratic for u	M1	Must be an equation with real roots
	Obtain root $(\sqrt{7} - 2)/3$, or decimal in [0.21, 0.22]	A1	Or equivalent. Ignore second root (even if incorrect)
	Use correct method for finding x from a positive value of e^x	M1	Must see some indication of method: use of $x = \ln u$
	Obtain answer $x = -1.536$ only	A1	CAO. Must be 3 dp
		5	

Question	Answer	Marks	Guidance
5(i)	Use product rule on a correct expression	M1	Condone with $+\frac{x}{8-x}$ unless there is clear evidence of incorrect product rule.
	Obtain correct derivative in any form	A1	$\frac{dy}{dx} = \ln(8-x) - \frac{x}{8-x}$
	Equate derivative to 1 and obtain $x = 8 - \frac{8}{\ln(8-x)}$	A1	Given answer: check carefully that it follows from correct working
			Condone the use of a for x throughout
		3	
5(ii)	Calculate values of a relevant expression or pair of relevant expressions at $x = 2.9$ and $x = 3.1$	M1	$8 - \frac{8}{\ln 5.1} = 3.09 > 2.9$, $8 - \frac{8}{\ln 4.9} = 2.97 < 3.1$ Clear linking of pairs needed for M1 by this method (0.19 and -0.13)
	Complete the argument correctly with correct calculated values	A1	Note: valid to consider gradient at 2.9 (1.06..) and 3.1 (0.95..) and comment on comparison with 1
		2	

Question	Answer	Marks	Guidance
5(iii)	Use the iterative process $x_{n+1} = 8 - \frac{8}{\ln(8-x_n)}$ correctly to find at least two successive values. SR: Clear successive use of 0, 1, 2, 3 etc., or equivalent, scores M0.	M1	3, 3.0293, 3.0111, 3.0225, 3.0154, (3.0198) 2.9, 3.0897, 2.9728, 3.0460, 3.0006, 3.290, 3.0113, 3.0223, 3.0155 3.1, 2.9661, 3.0501, 2.9980, 3.0305, 3.0103, 3.0229, 3.0151 Allow M1 if values given to fewer than 4 dp
	Obtain final answer 3.02	A1	
	Show sufficient iterations to at least 4 d.p. to justify 3.02 to 2 d.p., or show there is a sign change in the interval (3.015, 3.025)	A1	Must have two consecutive values rounding correctly to 3.02
		3	

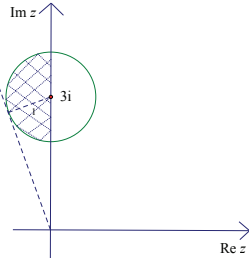
Question	Answer	Marks	Guidance
6	State equation $\frac{dy}{dx} = k \frac{y^2}{x}$, or equivalent	B1	SC: If $k = 1$ seen or implied give B0 and then allow B1B1B0M1, max 3/8.
	Separate variables correctly and integrate at least one side	B1	$\int \frac{k}{x} dx = \int \frac{1}{y^2} dy$ Allow with incorrect value substituted for k
	Obtain terms $-\frac{1}{y}$ and $k \ln x$	B1 + B1	Incorrect k used scores max. B1B0
	Use given coordinates correctly to find k and/or a constant of integration C in an equation containing terms $\frac{a}{y}$, $b \ln x$ and C	M1	SC: If an incorrect method is used to find k , M1 is allowable for a correct method to find C
	Obtain $k = \frac{1}{2}$ and $c = -1$, or equivalent	A1 + A1	$\frac{1}{2} \ln x = 1 - \frac{1}{y}$ A0 for fortuitous answers.
	Obtain answer $y = \frac{2}{2 - \ln x}$, or equivalent, and ISW	A1	$y = \frac{-1}{-1 + \ln \sqrt{x}}$
			SC: MR of the fraction. $\frac{dy}{dx} = k \frac{y^2}{x^2}$ B1 Separate variables and integrate B1 $\frac{-1}{y} = \frac{-k}{x} (+C)$ B1+B1 Substitute to find k and/or c M1 $k = \frac{e}{2(e-1)}$, $c = \frac{2-e}{2(e-1)}$ A1+A1 Answer A0
		8	

Question	Answer	Marks	Guidance
7(i)	Use correct quotient or product rule	M1	
	Obtain correct derivative in any form	A1	$\frac{dy}{dx} = \frac{-3\sin x(2 + \sin x) - 3\cos x \cos x}{(2 + \sin x)^2}$ Condone invisible brackets if recovery implied later.
	Equate numerator to zero	M1	
	Use $\cos^2 x + \sin^2 x = 1$ and solve for $\sin x$	M1	$-6\sin x - 3 = 0 \Rightarrow \sin x = \dots$
	Obtain coordinates $x = -\pi/6$ and $y = \sqrt{3}$ ISW	A1 + A1	From correct working. No others in range
			SR: A candidate who only states the numerator of the derivative, but justifies this, can have full marks. Otherwise they score M0A0M1M1A0A0
		6	
7(ii)	State indefinite integral of the form $k \ln(2 + \sin x)$	M1*	
	Substitute limits correctly, equate result to 1 and obtain $3 \ln(2 + \sin a) - 3 \ln 2 = 1$	A1	or equivalent
	Use correct method to solve for a	M1(dep*)	Allow for a correct method to solve an incorrect equation, so long as that equation has a solution. $1 + \frac{1}{2}\sin a = e^{1/3} \Rightarrow a = \sin^{-1}\left[2\left(e^{1/3} - 1\right)\right]$ Can be implied by 52.3°
	Obtain answer $a = 0.913$ or better	A1	Ignore additional solutions. Must be in radians.
		4	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $\frac{A}{1-2x} + \frac{B}{2-x} + \frac{C}{(2-x)^2}$	B1	
	Use a correct method for finding a constant M1 is available following a single slip in working from their form but no A marks (even if a constant is “correct”)	M1	$7 = A + 2B$ $-15 = -4A - 5B - 2C$ $8 = 4A + 2B + C$
	Obtain one of $A = 1, B = 3, C = -2$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	
	[Mark the form $\frac{A}{1-2x} + \frac{Dx+E}{(2-x)^2}$, where $A = 1, D = -3$ and $E = 4$, B1M1A1A1A1 as above.]		
		5	

Question	Answer	Marks	Guidance
8(ii)	Use a correct method to find the first two terms of the expansion of $(1-2x)^{-1}$, $(2-x)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $(2-x)^{-2}$ or $\left(1-\frac{1}{2}x\right)^{-2}$	M1	Symbolic coefficients are not sufficient for the M1
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A3ft	$1 + 2x + 4x^2$ The ft is on A, B, C . $\frac{3}{2} + \frac{3}{4}x + \frac{3}{8}x^2$ $-\frac{1}{2} - \frac{1}{2}x - \frac{3}{8}x^2$
	Obtain final answer $2 + \frac{9}{4}x + 4x^2$	A1	
	[For the A, D, E form of fractions give M1A2ft for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.]		[The ft is on A, D, E .]
		5	

Question	Answer	Marks	Guidance
9(a)(i)	Multiply numerator and denominator by $1 + 2i$, or equivalent	M1	Requires at least one of $2 + 10i + 12i^2$ and $1 - 4i^2$ together with use of $i^2 = -1$. Can be implied by $\frac{-10+10i}{5}$
	Obtain quotient $-2 + 2i$	A1	
	Alternative		
	Equate to $x + iy$, obtain two equations in x and y and solve for x or for y	M1	$x + 2y = 2, \quad y - 2x = 6$
	Obtain quotient $-2 + 2i$	A1	
		2	
9(a)(ii)	Use correct method to find either r or θ	M1	If only finding θ , need to be looking for θ in the correct quadrant
	Obtain $r = 2\sqrt{2}$, or exact equivalent	A1ft	ft their $x + iy$
	Obtain $\theta = \frac{3}{4}\pi$ from exact work	A1ft	ft on $k(-1 + i)$ for $k > 0$ Do not ISW
		3	

Question	Answer	Marks	Guidance
9(b)	Show a circle with centre $3i$	B1	
	Show a circle with radius 1	B1ft	Follow through their centre provided not at the origin For clearly unequal scales, should be an ellipse
	All correct with even scales and shade the correct region	B1	 <p>The diagram shows a complex plane with a vertical axis labeled 'Im z' and a horizontal axis labeled 'Re z'. A circle is drawn with its center at the point $3i$ on the imaginary axis. A dashed line is drawn from the origin to the upper-left part of the circle, representing the boundary of the region where the argument of z is greatest. The region between the circle and the dashed line is shaded with a cross-hatch pattern.</p>
	Carry out a correct method for calculating greatest value of $\arg z$	M1	e.g. $\arg z = \frac{\pi}{2} + \sin^{-1} \frac{1}{3}$
	Obtain answer 1.91	A1	
		5	

Question	Answer	Marks	Guidance
10(i)	Substitute for \mathbf{r} and expand the scalar product to obtain an equation in λ	M1*	e.g. $3(5 + \lambda) + (-3 - 2\lambda) + (-1 + \lambda) = 5$ ($2\lambda = 5 - 11$) or $3(4 + \lambda) + 1(-5 - 2\lambda) + (-1 + \lambda) = 0$ Must attempt to deal with $\mathbf{i} + 2\mathbf{j}$
	Solve a linear equation for λ	M1(dep*)	
	Obtain $\lambda = -3$ and position vector $\mathbf{r}_A = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ for A	A1	Accept coordinates
		3	
10(ii)	State or imply a normal vector of p is $3\mathbf{i} + \mathbf{j} + \mathbf{k}$, or equivalent	B1	
	Use correct method to evaluate a scalar product of relevant vectors e.g. $(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \cdot (3\mathbf{i} + \mathbf{j} + \mathbf{k})$	M1	
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse sine or cosine of the result	M1	$\cos \theta = \frac{2}{\sqrt{6}\sqrt{11}}$ Second M1 available if working with the wrong vectors
	Obtain answer 14.3° or 0.249 radians	A1	Or better

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10(ii)	Alternative 1		
	Use of a point on l and Cartesian equation $3x + y + z = 5$ to find distance of point from plane e.g. $B(5, -3, -1)$ $d = \frac{3 \times 5 - 3 - 1 - 5}{\sqrt{9+1+1}}$	M1	
	$= \frac{6}{\sqrt{11}}$ (= 1.809...)	A1	
	Complete method to find angle e.g. $\sin \theta = \frac{d}{AB}$	M1	
	$\theta = \sin^{-1} \left(\frac{6}{\sqrt{11}\sqrt{54}} \right) = 0.249$	A1	Or better
	Alternative 2		
	State or imply a normal vector of p is $3\mathbf{i} + \mathbf{j} + \mathbf{k}$, or equivalent	B1	
	Use correct method to evaluate a vector product of relevant vectors e.g. $(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \times (3\mathbf{i} + \mathbf{j} + \mathbf{k})$	M1	$3\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$
	Using the correct process for calculating the moduli, divide the vector product by the product of the moduli and evaluate the inverse sine or cosine of the result	M1	$\sin \theta = \frac{\sqrt{3^2 + 2^2 + 7^2}}{\sqrt{11}\sqrt{6}}$. Second M1 available if working with the wrong vectors
	Obtain answer 14.3° or 0.249 radians	A1	Or better
		4	

Question	Answer	Marks	Guidance
10(iii)	Taking the direction vector of the line to be $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, state a relevant equation in a, b, c , e.g. $3a + b + c = 0$	B1	
	State a second relevant equation, e.g. $a - 2b + c = 0$, and solve for one ratio, e.g. $a : b$	M1	
	Obtain $a : b : c = 3 : -2 : -7$, or equivalent	A1	
	State answer $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} - 7\mathbf{k})$	A1ft	Or equivalent. The f.t. is on \mathbf{r}_A Requires ‘ $\mathbf{r} = \dots$ ’
	Alternative		
	Attempt to calculate the vector product of relevant vectors, e.g. $(3\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	M1	
	Obtain two correct components of the product	A1	
	Obtain correct product, e.g. $3\mathbf{i} - 2\mathbf{j} - 7\mathbf{k}$	A1	
	State answer $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} - 7\mathbf{k})$	A1ft	Or equivalent. The f.t. is on \mathbf{r}_A Requires “ $\mathbf{r} = \dots$ ”
	4		