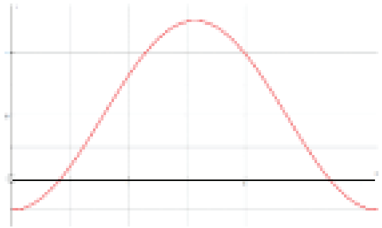
| Question | Answer | Marks | Guidance |
| --- | --- | --- | --- |
| 1 | For a correctly selected term in $\frac{1}{x^2}$ : $(3x)^4$ or $3^4$ | **B1** | Components of coefficient added together 0/4<br>B1 expect 81 |
| | $\times \left(\frac{2}{3x^2}\right)^3$ or $(2/3)^3$ | **B1** | B1 expect 8/27 |
| | $\times {}_7C_3$ or ${}_7C_4$ | **B1** | B1 expect 35 |
| | $\rightarrow$ **840 or** $\boldsymbol{\frac{840}{x^2}}$ | **B1** | All of the first three marks can be scored if the correct term is seen in an expansion **and it is selected** but then wrongly simplified. |
| | | | **SC:** A completely correct unsimplified term seen in an expansion but not correctly selected can be awarded B2. |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Integrate $\rightarrow \dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}}$ (+C) | **B1 B1** | B1 for each term correct – allow unsimplified. C not required. |
| | $\left[\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^4 \rightarrow \dfrac{40}{3} - \dfrac{14}{3}$ | **M1** | Evidence of 4 and 1 used correctly in their integrand ie at least one power increased by 1. |
| | $= \dfrac{26}{3}$ **oe** | **A1** | Allow 8.67 awrt. No integrand implies use of integration function on calculator 0/4. Beware a correct answer from wrong working. |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(i) | $P$ is $(t, 5t)$ $Q$ is $(t, t(9 - t^2)) \rightarrow 4t - t^3$ | **B1 B1** | B1 for both $y$ coordinates which can be implied by subsequent working. B1 for $PQ$ allow $\lvert 4t - t^3 \rvert$ or $\lvert t^3 - 4t \rvert$. **Note:** $4x - x^3$ from equating line and curve 0/2 even if $x$ then replaced by $t$. |
| | | **[2]** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(ii) | $\frac{d(PQ)}{dt} = 4 - 3t^2$ | **B1FT** | B1FT for differentiation of their *PQ*, which MUST be a cubic expression, but can be $\frac{d}{dx}f(x)$ from **(i)** but not the equation of the curve. |
| | $= 0 \rightarrow t = + \frac{2}{\sqrt{3}}$ | **M1** | Setting their differential of *PQ* to 0 and attempt to solve for t or *x*. |
| | **→ Maximum *PQ* =** $\frac{16}{3\sqrt{3}}$ **or** $\frac{16\sqrt{3}}{9}$ | **A1** | Allow 3.08 awrt. If answer comes from wrong method in **(i)** award A0.<br>Correct answer from correct expression by T&I scores 3/3. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | $fg(x) = 2 - 3\cos(\frac{1}{2}x)$ | **B1** | Correct *fg* |
| | $2 - 3\cos(\frac{1}{2}x) = 1 \rightarrow \cos(\frac{1}{2}x) = \frac{1}{3} \rightarrow \left(\frac{1}{2}x\right) = \cos^{-1}\left(their\ \frac{1}{3}\right)$ | **M1** | M1 for correct order of operations to solve their *fg*(x) = 1 as far as using inverse cos expect 1.23, ( or 70.5° ) condone *x* =. |
| | ***x* = 2.46 awrt or** $\frac{4.7\pi}{6}$ $(0.784\pi$ **awrt**$)$ | **A1** | One solution only in the given range, ignore answers outside the range.<br>Answer in degrees A0. |
| | | | Alternative:<br>Solve *f*(*y*) = 1 → y = 1.23→ $\frac{1}{2}x = 1.23$ **B1M1**<br>→ *x* = 2.46 **A1** |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(ii) |  | **B1** | One cycle of ± cos curve, evidence of turning at the ends not required at this stage. Can be a poor curve but not an inverted "V". If horizontal axis is not labelled mark everything to the right of the vertical axis. If axis is clearly labelled mark $0 \to 2\pi$. |
| | | **B1** | Start and finish at roughly the same negative $y$ value. Significantly more above the $x$ axis than below or correct range implied by labels . |
| | | **B1** | Fully correct. Curves not lines.<br>Must be a reasonable curve clearly turning at both ends.<br>Labels not required but must be appropriate if present. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | From the AP: $x-4=y-x$ | B1 | Or equivalent statement e.g. $y=2x-4$ or $x=\frac{y+4}{2}$. |
| | From the GP: $\frac{y}{x}=\frac{18}{y}$ | B1 | Or equivalent statement e.g. $y^2=18x$ or $x=\frac{y^2}{18}$. |
| | Simultaneous equations: $y^2-9y-36=0$ or $2x^2-17x+8=0$ | M1 | Elimination of either $x$ or $y$ to give a three term quadratic (= 0) |
| | **OR** | | |
| | $4+d=x$, $4+2d=y \rightarrow \frac{4+2d}{4+d}=r$ oe | B1 | |
| | $(4+d)\left(\frac{4+2d}{4+d}\right)^2=18 \rightarrow 2d^2-d-28=0$ | M1 | Uses $ar^2=18$ to give a three term quadratic (= 0) |
| | $d=4$ | B1 | Condone inclusion of $d=\frac{-7}{2}$ oe |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | **OR** | | |
| | From the GP $\frac{y}{x}=\frac{18}{y}$ | **B1** | |
| | $\rightarrow x=\frac{y^2}{18}\rightarrow 4+d=\frac{y^2}{18}\rightarrow d=\frac{y^2}{18}-4$ | **B1** | |
| | $4+2\left(\frac{y^2}{18}-4\right)=y \rightarrow y^2-9y-36=0$ | **M1** | |
| | $\boldsymbol{x=8, y=12.}$ | **A1** | Needs both $x$ and $y$. Condone $\left(\frac{1}{2},-3\right)$ included in final answer.<br>Fully correct answer www 4/4. |
| | | **4** | |
| 5(ii) | AP 4th term = **16** | **B1** | Condone inclusion of $\frac{-13}{2}$ oe |
| | GP 4th term = $8\times\left(\frac{12}{8}\right)^3$ | **M1** | A valid method using their $x$ and $y$ from **(i)**. |
| | = **27** | **A1** | Condone inclusion of **–**108 |
| | | | Note: Answers from fortuitous $x = 8$, $y = 12$ in **(i)** can only score M1.<br>Unidentified correct answer(s) with no working seen after valid $x = 8$, $y = 12$ to be credited with appropriate marks. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | In Δ $ABD$, $\tan\theta = \frac{9}{BD} \rightarrow BD = \frac{9}{\tan\theta}$ or $9\tan(90-\theta)$ or $9\cot\theta$ or $\sqrt{\left[(20\tan\theta)^2 - 9^2\right]}$ (Pythag) or $\frac{9\sin(90-\theta)}{\sin\theta}$ (Sine rule) | B1 | Both marks can be gained for correct equated expressions. |
| | In Δ $DBC$, $\sin\theta = \frac{BD}{20} \rightarrow BD = 20\sin\theta$ | B1 | |
| | $20\sin\theta = \frac{9}{\tan\theta}$ | M1 | Equates their expressions for BD and uses $\sin\theta/\cos\theta = \tan\theta$ or $\cos\theta/\sin\theta = \cot\theta$ if necessary. |
| | **$\rightarrow 20\sin^2\theta = 9\cos\theta$ AG** | A1 | Correct manipulation of their expression to arrive at given answer. |
| | | | **SC:**<br>In Δ $DBC$, $\sin\theta = \frac{BD}{20} \rightarrow BD = 20\sin\theta$ B1<br>In Δ $ABD$, $BA = \frac{9}{\sin\theta}$ and $\cos\theta = \frac{BD}{BA}$<br>$\cos\theta = \frac{20\sin\theta}{9/\sin\theta} \rightarrow \cos\theta = \frac{20\sin^2\theta}{9}$ M1<br>**$\rightarrow 20\sin^2\theta = 9\cos\theta$** A1 Scores 3/4 |
| | | **4** | |
| 6(ii) | Uses $s^2 + c^2 = 1 \rightarrow 20\cos^2\theta + 9\cos\theta - 20\ (= 0)$ | M1 | Uses $s^2 + c^2 = 1$ to form a three term quadratic in $\cos\theta$ |
| | $\rightarrow \cos\theta = 0.8$ | A1 | www |
| | **$\rightarrow \theta = 36.9^\circ$ awrt** | A1 | www. Allow $0.644^c$ awrt. Ignore $323.1^\circ$ or $2.50^c$.<br>Note: correct answer without working scores 0/3. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7 | $\overrightarrow{PN} = 8\mathbf{i} - 8\mathbf{k}$ | **B1** | |
| | $\overrightarrow{PM} = 4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$ | **B2,1,0** | Loses 1 mark for each component incorrect |
| | | | **SC:** $\overrightarrow{PN} = -8\mathbf{i} + 8\mathbf{k}$ **and** $\overrightarrow{PM} = -4\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$ scores 2/3. |
| | $\overrightarrow{PN}.\overrightarrow{PM} = 32 + 0 + 48 = 80$ | **M1** | Evaluates $x_1x_2 + y_1y_2 + z_1z_2$ for correct vectors or one or both reversed. |
| | $\lvert PN\rvert \times \lvert PM\rvert = \sqrt{128} \times \sqrt{68}$ $(= 16\sqrt{34})$ | **M1** | Product of their moduli – may be seen in cosine rule |
| | $\sqrt{128} \times \sqrt{68} \cos M\hat{P}N = 80$ | **M1** | All linked correctly. |
| | Angle $M\hat{P}N =$ **31.0° awrt** | **A1** | Answer must come directly from +ve cosine ratio.<br>Cosine rule not accepted as a complete method. Allow $0.540^c$ awrt.<br>**Note:** Correct answer from incorrect vectors scores A0 (XP) |
| | | **7** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(i) | $A\hat{B}C$ using cosine rule giving $\cos^{-1}(\frac{-1}{8})$ or $2\sin^{-1}(¾)$ or $2\cos^{-1}\left(\frac{\sqrt{7}}{2}\right)$ <br> **or** $B\hat{A}C = \cos^{-1}(¾)$ or $B\hat{A}C = \sin^{-1}\frac{\sqrt{7}}{4}$ or $B\hat{A}C = \tan^{-1}\frac{\sqrt{7}}{3}$ | **M1** | Correct method for $A\hat{B}C$, expect $1.696^c$ awrt <br> Or for $B\hat{A}C$, expect $0.723^c$ awrt |
| | $C\hat{B}Y = \pi - A\hat{B}C$ or $2 \times C\hat{A}B$ | **M1** | For attempt at $C\hat{B}Y = \pi - A\hat{B}C$ or $C\hat{B}Y = 2 \times C\hat{A}B$ |
| | **OR** | | |
| | Find $CY$ from $\Delta\, ACY$ using Pythagoras or similar $\Delta$s | **M1** | Expect $4\sqrt{7}$ |
| | $C\hat{B}Y = \cos^{-1}\left(\frac{8^2+8^2-(their\ CY)^2}{2\times8\times8}\right)$ | **M1** | Correct use of cosine rule |
| | $\boldsymbol{C\hat{B}Y = 1.445^c}$ AG | **A1** | Numerical values for angles in radians, if given, need to be correct to 3 decimal places. Method marks can be awarded for working in degrees. <br> Need 82.8° awrt converted to radians for A1. <br> Identification of angles must be consistent for A1. |
| | | **3** | |
| 8(ii) | Arc $CY = 8 \times 1.445$ | **B1** | Use of $s=8\theta$ for arc $CY$, Expect 11.56 |
| | $B\hat{A}C = ½(\pi - A\hat{B}C)$ or $\cos^{-1}(¾)$ | ***M1** | For a valid attempt at $B\hat{A}C$, may be from **(i)**. Expect $0.7227^c$ |
| | Arc $XC = 12 \times$ (their $B\hat{A}C$) | **DM1** | Expect 8.673 |
| | Perimeter = 11.56 + 8.673 + 4 = **24.2 cm awrt www** | **A1** | Omission of '+4' only penalised here. |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(i) | $2x^2 - 12x + 7 = 2(\boldsymbol{x}-3)^2 - 11$ | B1 B1 | Mark full expression if present: B1 for $2(x-3)^2$ and B1 for – 11.<br>If no clear expression award $a = -3$ and $b = -11$. |
| | | **2** | |
| 9(ii) | Range (of f or y) $\geqslant$ **'their – 11'** | **B1FT** | FT for their '$b$' or start again. Condone **>.**<br>Do **NOT** accept $x >$ or $\geqslant$ |
| | | **1** | |
| 9(iii) | ($k$ =) **–**"their a" also allow $\boldsymbol{x}$ **or** $\boldsymbol{k} \leqslant$ **3** | **B1FT** | FT for their "$a$" or start again using $\frac{dy}{dx} = 0$.<br>Do **NOT accept** $\boldsymbol{x}$ **= 3.** |
| | | **1** | |
| 9(iv) | $y = 2(x-3)^2 - 11 \rightarrow y + 11 = 2(x-3)^2$<br>$\frac{y+11}{2} = (x-3)^2$ | ***M1** | Isolating their $(x-3)^2$, condone – 11. |
| | $x = 3 + \sqrt{\left(\frac{y+11}{2}\right)}$ or $3 - \sqrt{\left(\frac{y+11}{2}\right)}$ | **DM1** | Other operations in correct order, allow ± at this stage. Condone – 3. |
| | ($g^{-1}(x)$ or y) = $3 - \sqrt{\left(\frac{x+11}{2}\right)}$ | **A1** | needs '–'. $x$ and $y$ could be interchanged at the start. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(i) | $2x+\frac{12}{x}=k-x$ or $y=2(k-y)+\frac{12}{k-y}$ → 3 term quadratic. | *M1 | Attempt to eliminate $y$ (or $x$) to form a 3 term quadratic. Expect $3x^2-kx+12$ or $3y^2-5ky+(2k^2+12)$ (= 0) |
| | Use of $b^2-4ac \rightarrow k^2-144<0$ | DM1 | Using the discriminant, allow ⩽ , = 0; expect 12 and −12 |
| | $\mathbf{-12<k<12}$ | A1 | Do **NOT accept ⩽ .** Separate statements OK. |
| | | 3 | |
| 10(ii) | Using $k$ = 15 in their 3 term quadratic | M1 | From (i) or restart. Expect $3x^2-15x+12$ or $3y^2-75y+462$ (= 0) |
| | $x$ = 1,4 or $y$ = 11, 14 | A1 | Either pair of $x$ or $y$ values correct.. |
| | **(1, 14) and (4, 11)** | A1 | Both pairs of coordinates |
| | | 3 | |
| 10(iii) | Gradient of $AB$ = −1 → Perpendicular gradient = +1 | B1FT | Use of $m_1m_2$=−1 to give +1 or ft from their $A$ and $B$. |
| | Finding their midpoint using their (1, 14) and (4, 11) | M1 | Expect (2½, 12½) |
| | Equation: $\mathbf{y-12½=(x-2½)}$ [$y=x+10$] | A1 | Accept correct unsimplified and isw |
| | | 3 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(i) | $\frac{dy}{dx} = \left[\frac{3}{2}\times(4x+1)^{-\frac{1}{2}}\right]$ [×4] [− 2] $\left(\frac{6}{\sqrt{4x+1}}-2\right)$ | B2,1,0 | Looking for 3 components |
| | $\int y\,dx = \left[3(4x+1)^{\frac{3}{2}} \div \frac{3}{2}\right]$ [ ÷ 4 ] [ $-\frac{2x^2}{2}$ ] (+ C) $\left(=\frac{(4x+1)^{\frac{3}{2}}}{2}-x^2\right)$ | B1 B1 B1 | B1 for $3(4x+1)^{\frac{3}{2}} \div \frac{3}{2}$ B1 for '÷4'. B1 for '$-\frac{2x^2}{2}$'. Ignore omission of + C. If included isw any attempt at evaluating. |
| | | 5 | |
| 11(ii) | At $M$, $\frac{dy}{dx}=0 \to \frac{6}{\sqrt{4x+1}}=2$ | M1 | Sets their 2 term $\frac{dy}{dx}$ to 0 and attempts to solve (as far as $x$ = k) |
| | $\boldsymbol{x = 2, y = 5}$ | A1 A1 | |
| | | 3 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(iii) | Area under the curve = $\left[\frac{1}{2}(4x+1)^{\frac{3}{2}} - x^2\right]_0^2$ | M1 | Uses their integral and their '2' and 0 correctly |
| | (13.5 – 4) – 0.5 or 9.5 – 0.5 = 9 | A1 | No working implies use of integration function on calculator M0A0. |
| | Area under the chord = trapezium = ½ × 2 × (3 + 5) = 8<br>Or $\left[\frac{x^2}{2}+3x\right]_0^2 = 8$ | M1 | Either using the area of a trapezium with their 2, 3 and 5 or $\int(their\ x+3)dx$ using their '2' and 0 correctly. |
| | (Shaded area = 9 – 8) **= 1** | A1 | Dependent on both method marks, |
| | OR Area between the chord and the curve is: | | |
| | $\int_0^2 3\sqrt{4x+1}-2x-(x+3)dx$<br>$=\int_0^2 3\sqrt{4x+1}-3x-3dx$ | M1 | Subtracts their line from given curve and uses their '2' and 0 correctly. |
| | $=3\left[\frac{1}{6}(4x+1)^{\frac{3}{2}} - \frac{x^2}{2}-x\right]_0^2$ | A1 | All integration correct and limits 2 and 0. |
| | $=3\left\{\left(\frac{27}{6}-2-2\right)-\left(\frac{1}{6}\right)\right\}$ | M1 | Evidence of substituting their '2' and 0 into their integral. |
| | $=3\left\{\frac{1}{2}-\frac{1}{6}\right\}=3\left\{\frac{1}{3}\right\}=1$ | A1 | No working implies use of a calculator M0A0. |
| | | **[4]** | |