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1	Use law of the logarithm of a quotient Remove logarithms and obtain a correct equation, e.g. $e^z = \frac{y+2}{y+1}$ Obtain answer $y = \frac{2-e^z}{e^z-1}$, or equivalent	M1 A1 A1	[3]
2	Use correct quotient or product rule Obtain correct derivative in any form Use Pythagoras to simplify the derivative to $\frac{1}{1+\cos x}$, or equivalent Justify the given statement, $-1 < \cos x < 1$ statement, or equivalent	M1 A1 A1 A1	[4]
3	Use the $\tan 2A$ formula to obtain an equation in $\tan \theta$ only Obtain a correct horizontal equation Rearrange equation as a quadratic in $\tan \theta$, e.g. $3 \tan^2 \theta + 2 \tan \theta - 1 = 0$ Solve for θ (usual requirements for solution of quadratic) Obtain answer, e.g. 18.4° Obtain second answer, e.g. 135° , and no others in the given interval	M1 A1 A1 M1 A1 A1	[6]
4 (i)	Commence division by $x^2 - x + 2$ and reach a partial quotient $4x^2 + kx$ Obtain quotient $4x^2 + 4x + a - 4$ or $4x^2 + 4x + b / 2$ Equate x or constant term to zero and solve for a or b Obtain $a = 1$ Obtain $b = -6$	M1 A1 M1 A1 A1	[5]
(ii)	Show that $x^2 - x + 2 = 0$ has no real roots Obtain roots $\frac{1}{2}$ and $-\frac{3}{2}$ from $4x^2 + 4x - 3 = 0$	B1 B1	[2]
5 (i)	State equation $\frac{dy}{dx} = \frac{1}{2}xy$	B1	[1]
(ii)	Separate variables correctly and attempts to integrate one side of equation Obtain terms of the form $a \ln y$ and bx^2 Use $x = 0$ and $y = 2$ to evaluate a constant, or as limits, in expression containing $a \ln y$ or bx^2 Obtain correct solution in any form, e.g. $\ln y = \frac{1}{4}x^2 + \ln 2$ Obtain correct expression for y , e.g. $y = 2e^{\frac{1}{4}x^2}$	M1 A1 M1 A1 A1	[5]
(iii)	Show correct sketch for $x \geq 0$. Needs through $(0, 2)$ and rapidly increasing positive gradient.	B1	[1]

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6	(i)	State or imply $du = \frac{1}{2\sqrt{x}} dx$ Substitute for x and dx throughout Justify the change in limits and obtain the given answer	B1 M1 A1	[3]
	(ii)	Convert integrand into the form $A + \frac{B}{u+1}$ Obtain integrand $A=1, B=-2$ Integrate and obtain $u - 2\ln(u+1)$ Substitute limits correctly in an integral containing terms au and $b\ln(u+1)$, where $ab \neq 0$ Obtain the given answer following full and correct working [The f.t. is on A and B .]	M1* A1 A1 [✓] + A1 [✓] DM1 A1	[6]
7	(i)	State modulus $2\sqrt{2}$, or equivalent State argument $-\frac{1}{3}\pi$ (or -60°)	B1 B1	[2]
	(ii) (a)	State answer $3\sqrt{2} + \sqrt{6}i$	B1	
	(b)	<i>EITHER:</i> Substitute for z and multiply numerator and denominator by conjugate of iz Simplify the numerator to $4\sqrt{3} + 4i$ or the denominator to 8 Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}i$ <i>OR:</i> Substitute for z , obtain two equations in x and y and solve for x or for y Obtain $x = \frac{1}{2}\sqrt{3}$ or $y = \frac{1}{2}$ Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}i$	M1 A1 A1 M1 A1 A1	[4]
	(iii)	Show points A and B in relatively correct positions Carry out a complete method for finding angle AOB , e.g. calculate the argument of $\frac{z^*}{iz}$ Obtain the given answer	B1 M1 A1	[3]
8	(i)	State or imply the form $\frac{A}{x+2} + \frac{Bx+C}{x^2+4}$ Use a correct method to determine a constant Obtain one of $A=2, B=1, C=-1$ Obtain a second value Obtain a third value	B1 M1 A1 A1 A1	[5]

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(ii)	<p>Use correct method to find the first two terms of the expansion of $(x + 2)^{-1}$, $(1 + \frac{1}{2}x)^{-1}$, $(4 + x^2)^{-1}$ or $(1 + \frac{1}{4}x^2)^{-1}$</p> <p>Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction</p> <p>Multiply out fully by $Bx + C$, where $BC \neq 0$</p> <p>Obtain final answer $\frac{3}{4} - \frac{1}{4}x + \frac{5}{16}x^2$, or equivalent</p> <p>[Symbolic binomial coefficients, e.g. $\binom{-1}{1}$ are not sufficient for the M1. The f.t. is on A, B, C.]</p> <p>[In the case of an attempt to expand $(3x^2 + x + 6)(x + 2)^{-1}(x^2 + 4)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]</p>	<p>M1</p> <p>A1✓ + A1✓</p> <p>M1</p> <p>A1</p>	[5]
9 (i)	<p>Differentiate both equations and equate derivatives</p> <p>Obtain equation $\cos a - a \sin a = -\frac{k}{a^2}$</p> <p>State $a \cos a = \frac{k}{a}$ and eliminate k</p> <p>Obtain the given answer showing sufficient working</p>	<p>M1*</p> <p>A1 + A1</p> <p>DM1</p> <p>A1</p>	[5]
(ii)	<p>Show clearly correct use of the iterative formula at least once</p> <p>Obtain answer 1.077</p> <p>Show sufficient iterations to 5 d.p. to justify 1.077 to 3 d.p., or show there is a sign change in the interval (1.0765, 1.0775)</p>	<p>M1</p> <p>A1</p> <p>A1</p>	[3]
(iii)	<p>Use a correct method to determine k</p> <p>Obtain answer $k = 0.55$</p>	<p>M1</p> <p>A1</p>	[2]
10 (i)	<p>Express general point of l in component form e.g. $(1 + 2\lambda, 2 - \lambda, 1 + \lambda)$</p> <p>Using the correct process for the modulus form an equation in λ</p> <p>Reduce the equation to a quadratic, e.g. $6\lambda^2 + 2\lambda - 4 = 0$</p> <p>Solve for λ (usual requirements for solution of a quadratic)</p> <p>Obtain final answers $-\mathbf{i} + 3\mathbf{j}$ and $\frac{7}{3}\mathbf{i} + \frac{4}{3}\mathbf{j} + \frac{5}{3}\mathbf{k}$</p>	<p>B1</p> <p>M1*</p> <p>A1</p> <p>DM1</p> <p>A1</p>	[5]
(ii)	<p>Using the correct process, find the scalar product of a direction vector for l and a normal for p</p> <p>Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\frac{2}{3}$</p> <p>State a correct equation in any form, e.g. $\frac{2a - 1 + 1}{\sqrt{(a^2 + 1 + 1)} \cdot \sqrt{(2^2 + (-1)^2 + 1)}} = \pm \frac{2}{3}$</p> <p>Solve for a^2</p> <p>Obtain answer $a = \pm 2$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	[5]