1	Draw curve with increasing gradient existing for negative and positive values of x	**M1**	
	Draw correct curve passing through the origin	**A1**	[2]
2	Either State correct unsimplified x^2 or x^3 term	**M1**	
	Obtain $a = -9$	**A1**	
	Obtain $b = 45$	**A1**	
	Or Use chain rule to differentiate twice to obtain form $k(1+9x)^{-\frac{5}{3}}$	**M1**	
	Obtain $f''(x) = -18(1+9x)^{-\frac{5}{3}}$ and hence $a = -9$	**A1**	
	Obtain $f'''(x) = 270(1+9x)^{-\frac{8}{3}}$ and hence $b = 45$	**A1**	[3]
3	Use correct quotient rule or equivalent to find first derivative	**M1***	
	Obtain $\dfrac{-(1+\tan x)\sec^2 x - \sec^2 x(2-\tan x)}{(1+\tan x)^2}$ or equivalent	**A1**	
	Substitute $x = \frac{1}{4}\pi$ to find gradient	**dep M1***	
	Obtain $-\frac{3}{2}$	**A1**	
	Form equation of tangent at $x = \frac{1}{4}\pi$	**M1**	
	Obtain $y = -\frac{3}{2}x + 1.68$ or equivalent	**A1**	[6]
4	**(i)** Use $\dfrac{dy}{dx} = \dfrac{\dot{y}}{\dot{x}}$ and equate $\dfrac{dy}{dx}$ to 4	**M1**	
	Obtain $\dfrac{4p^3}{2p+3} = 4$ or equivalent	**A1**	
	Confirm given result $p = \sqrt[3]{2p+3}$ correctly	**A1**	[3]
	(ii) Evaluate $p - \sqrt[3]{2p+3}$ or $p^3 - 2p - 3$ or equivalent at 1.8 and 2.0	**M1**	
	Justify result with correct calculations and argument (–0.076 and 0.087 or –0.77 and 1 respectively)	**A1**	[2]
	(iii) Use the iterative process correctly at least once with $1.8 \leqslant p_n \leqslant 2.0$	**M1**	
	Obtain final answer 1.89	**A1**	
	Show sufficient iterations to at least 4 d.p. to justify 1.89 or show sign change in interval (1.885, 1.895)	**A1**	[3]

5	State $\mathrm{d}u = 3\sin x\,\mathrm{d}x$ or equivalent	**B1**	
	Use identity $\sin 2x = 2\sin x \cos x$	**B1**	
	Carry out complete substitution, for x and $\mathrm{d}x$	**M1**	
	Obtain $\int \frac{8-2u}{\sqrt{u}}\,\mathrm{d}u$, or equivalent	**A1**	
	Integrate to obtain expression of form $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$, $ab \neq 0$	**M1***	
	Obtain correct $16u^{\frac{1}{2}} - \frac{4}{3}u^{\frac{3}{2}}$	**A1**	
	Apply correct limits correctly	**dep M1***	
	Obtain $\frac{20}{3}$ or exact equivalent	**A1**	[8]
6	State or imply $\sin A \times \cos 45 + \cos A \times \sin 45 = 2\sqrt{2}\cos A$	**B1**	
	Divide by $\cos A$ to find value of $\tan A$	**M1**	
	Obtain $\tan A = 3$	**A1**	
	Use identity $\sec^2 B = 1 + \tan^2 B$	**B1**	
	Solve three-term quadratic equation and find $\tan B$	**M1**	
	Obtain $\tan B = \frac{3}{2}$ only	**A1**	
	Substitute **numerical values** in $\frac{\tan A - \tan B}{1 + \tan A \tan B}$	**M1**	
	Obtain $\frac{3}{11}$	**A1**	[8]
7	**(i)** Either Substitute $x = -1$ and evaluate	**M1**	
	Obtain 0 and conclude $x+1$ is a factor	**A1**	
	Or Divide by $x+1$ and obtain a constant remainder	**M1**	
	Obtain remainder = 0 and conclude $x+1$ is a factor	**A1**	[2]
	(ii) Attempt division, or equivalent, at least as far as quotient $4x^2 + kx$	**M1**	
	Obtain complete quotient $4x^2 - 5x - 6$	**A1**	
	State form $\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{4x+3}$	**A1**	
	Use relevant method for finding at least one constant	**M1**	
	Obtain one of $A = -2, B = 1, C = 8$	**A1**	
	Obtain all three values	**A1**	
	Integrate to obtain three terms each involving natural logarithm of linear form	**M1**	
	Obtain $-2\ln(x+1) + \ln(x-2) + 2\ln(4x+3)$, condoning no use of modulus signs and absence of … + c	**A1**	[8]

8	**(i)**	Express a general point on the line in single component form, e.g. $(\lambda, 2-3\lambda, -8+4\lambda)$, substitute in equation of plane and solve for λ		**M1**	
		Obtain $\lambda = 3$		**A1**	
		Obtain $(3, -7, 4)$		**A1**	[3]
	(ii)	State or imply normal vector to plane is 4**i** – **j** + 5**k**		**B1**	
		Carry out process for evaluating scalar product of two relevant vectors		**M1**	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate $\sin^{-1}$ or $\cos^{-1}$ of the result.		**M1**	
		Obtain $54.8°$ or 0.956 radians		**A1**	[4]
	(iii)	<u>Either</u>	Find at least one position of *C* by translating by appropriate multiple of direction vector **i** – 3**j** + 4**k** from *A* or *B*	**M1**	
			Obtain $(-3, 11, -20)$	**A1**	
			Obtain $(9, -25, 28)$	**A1**	
		<u>Or</u>	Form quadratic equation in λ by considering $BC^2 = 4AB^2$	**M1**	
			Obtain $26\lambda^2 - 156\lambda - 702 = 0$ or equivalent and hence $\lambda = -3, \lambda = 9$	**A1**	
			Obtain $(-3, 11, -20)$ and $(9, -25, 28)$	**A1**	[3]
9	**(a)**	<u>Either</u>	Find *w* using conjugate of $1+3\text{i}$	**M1**	
			Obtain $\dfrac{7-\text{i}}{5}$ or equivalent	**A1**	
			Square $x + \text{i}y$ form to find w^2	**M1**	
			Obtain $w^2 = \dfrac{48-14\text{i}}{25}$ and confirm modulus is 2	**A1**	
			Use correct process for finding argument of w^2	**M1**	
			Obtain –0.284 radians or $-16.3°$	**A1**	
		<u>Or 1</u>	Find *w* using conjugate of $1+3\text{i}$	**M1**	
			Obtain $\dfrac{7-\text{i}}{5}$ or equivalent	**A1**	
			Find modulus of *w* and hence of w^2	**M1**	
			Confirm modulus is 2	**A1**	
			Find argument of *w* and hence of w^2	**M1**	
			Obtain –0.284 radians or $-16.3°$	**A1**	
		<u>Or 2</u>	Square both sides to obtain $(-8+6\text{i})w^2 = -12+16\text{i}$	**B1**	
			Find w^2 using relevant conjugate	**M1**	
			Use correct process for finding modulus of w^2	**M1**	
			Confirm modulus is 2	**A1**	
			Use correct process for finding argument of w^2	**M1**	
			Obtain –0.284 radians or $-16.3°$	**A1**	

Or 3	Find modulus of LHS and RHS	**M1**	
	Find argument of LHS and RHS	**M1**	
	Obtain $\sqrt{10}\,\mathrm{e}^{1.249\mathrm{i}}\ w = \sqrt{20}\,\mathrm{e}^{1.107\mathrm{i}}$ or equivalent	**A1**	
	Obtain $w = \sqrt{2}\,\mathrm{e}^{-0.1419\mathrm{i}}$ or equivalent	**A1**	
	Use correct process for finding w^2	**M1**	
	Obtain 2 and –0.284 radians or $-16.3°$	**A1**	
Or 4	Find moduli of $2+4\mathrm{i}$ and $1+3\mathrm{i}$	**M1**	
	Obtain $\sqrt{20}$ and $\sqrt{10}$	**A1**	
	Obtain $\left\|w^2\right\| = 2$ correctly	**A1**	
	Find $\arg(2+4\mathrm{i})$ and $\arg(1+3\mathrm{i})$	**M1**	
	Use correct process for $\arg(w^2)$	**A1**	
	Obtain –0.284 radians or $-16.3°$	**A1**	
Or 5	Let $w = a + \mathrm{i}b$, form and solve simultaneous equations in *a* and *b*	**M1**	
	$a = \frac{7}{5}$ and $b = -\frac{1}{5}$	**A1**	
	Find modulus of *w* and hence of w^2	**M1**	
	Confirm modulus is 2	**A1**	
	Find argument of *w* and hence of w^2	**M1**	
	Obtain –0.284 radians or $-16.3°$	**A1**	
Or 6	Find *w* using conjugate of $1+3\mathrm{i}$	**M1**	
	Obtain $\frac{7-\mathrm{i}}{5}$ or equivalent	**A1**	
	Use $\left\|w^2\right\| = w\overline{w}$	**M1**	
	Confirm modulus is 2	**A1**	
	Find argument of *w* and hence of w^2	**M1**	
	Obtain –0.284 radians or $-16.3°$	**A1**	[6]

(b)	Draw circle with centre the origin and radius 5	**B1**	
	Draw straight line parallel to imaginary axis in correct position	**B1**	
	Use relevant trigonometry on a correct diagram to find argument(s)	**M1**	
	Obtain $5\mathrm{e}^{\pm\frac{1}{3}\pi\mathrm{i}}$ or equivalents in required form	**A1**	[4]

10	**(i)**	State $\frac{dN}{dt} = k(N - 150)$	**B1**	[1]
	(ii)	Substitute $\frac{dN}{dt} = 60$ and $N = 900$ to find value of k	**M1**	
		Obtain $k = 0.08$	**A1**	
		Separate variables and obtain general solution involving $\ln(N - 150)$	**M1***	
		Obtain $\ln(N - 150) = 0.08t + c$ (following their k) or $\ln(N - 150) = kt + c$	**A1**$\sqrt{}$	
		Substitute $t = 0$ and $N = 650$ to find c	**dep M1***	
		Obtain $\ln(N - 150) = 0.08t + \ln 500$ or equivalent	**A1**	
		Obtain $N = 500e^{0.08t} + 150$	**A1**	[7]
	(iii)	Either Substitute $t = 15$ to find N or solve for t with $N = 2000$	**M1**	
		Obtain Either $N = 1810$ or $t = 16.4$ and conclude target not met	**A1**	[2]