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- 1 Use law of the logarithm of a power M1
 Obtain a correct linear equation in any form, e.g. $x = (x - 2) \ln 3$ A1
 Obtain answer $x = 22.281$ A1 [3]
- 2 (i) State or imply ordinates 2, 1.1547..., 1, 1.1547... B1
 Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates M1
 Obtain answer 1.95 A1 [3]
- (ii) Make recognisable sketch of $y = \operatorname{cosec} x$ for the given interval B1
 Justify a statement that the estimate will be an overestimate B1 [2]
- 3 Substitute $x = -\frac{1}{3}$, equate result to zero or divide by $3x + 1$ and equate the remainder to zero
 and obtain a correct equation, e.g. $-\frac{1}{27}a + \frac{1}{9}b - \frac{1}{3} + 3 = 0$ B1
 Substitute $x = 2$ and equate result to 21 or divide by $x - 2$ and equate constant remainder to 21 M1
 Obtain a correct equation, e.g. $8a + 4b + 5 = 21$ A1
 Solve for a or for b M1
 Obtain $a = 12$ and $b = -20$ A1 [5]
- 4 (i) Use chain rule correctly at least once M1
 Obtain either $\frac{dx}{dt} = \frac{3\sin t}{\cos^4 t}$ or $\frac{dy}{dt} = 3\tan^2 t \sec^2 t$, or equivalent A1
 Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1
 Obtain the given answer A1 [4]
- (ii) State a correct equation for the tangent in any form B1
 Use Pythagoras M1
 Obtain the given answer A1 [3]
- 5 (i) Substitute $z = 1 + i$ and obtain $w = \frac{1 + 2i}{1 + i}$ B1
 EITHER: Multiply numerator and denominator by the conjugate of the denominator, M1
 or equivalent A1
 Simplify numerator to $3 + i$ or denominator to 2 A1
 Obtain final answer $\frac{3}{2} + \frac{1}{2}i$, or equivalent A1
- OR: Obtain two equations in x and y , and solve for x or for y M1
 Obtain $x = \frac{3}{2}$ or $y = \frac{1}{2}$, or equivalent A1
 Obtain final answer $\frac{3}{2} + \frac{1}{2}i$, or equivalent A1 [4]

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- (ii) *EITHER*: Substitute $w = z$ and obtain a 3-term quadratic equation in z ,
 e.g. $iz^2 + z - i = 0$ B1
 Solve a 3-term quadratic for z or substitute $z = x + iy$ and use a correct
 method to solve for x and y M1
OR: Substitute $w = x + iy$ and obtain two correct equations in x and y by equating
 real and imaginary parts B1
 Solve for x and y M1
- Obtain a correct solution in any form, e.g. $z = \frac{-1 \pm \sqrt{3}i}{2i}$ A1
- Obtain final answer $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$ A1 [4]
- 6 (i) Integrate and reach $b \ln 2x - c \int x \cdot \frac{1}{x} dx$, or equivalent M1*
- Obtain $x \ln 2x - \int x \cdot \frac{1}{x} dx$, or equivalent A1
- Obtain integral $x \ln 2x - x$, or equivalent A1
- Substitute limits correctly and equate to 1, having integrated twice M1(dep*)
- Obtain a correct equation in any form, e.g. $a \ln 2a - a + 1 - \ln 2 = 1$ A1
- Obtain the given answer A1 [6]
- (ii) Use the iterative formula correctly at least once M1
- Obtain final answer 1.94 A1
- Show sufficient iterations to 4 d.p. to justify 1.94 to 2d.p. or show that there is a sign
 change in the interval (1.935, 1.945). A1 [3]
- 7 (i) Separate variables correctly and attempt to integrate at least one side B1
- Obtain term $\ln R$ B1
- Obtain $\ln x - 0.57x$ B1
- Evaluate a constant or use limits $x = 0.5$, $R = 16.8$, in a solution containing terms of the form
 $a \ln R$ and $b \ln x$ M1
- Obtain correct solution in any form A1
- Obtain a correct expression for R , e.g. $R = xe^{(3.80 - 0.57x)}$, $R = 44.7xe^{-0.57x}$ or
 $R = 33.6xe^{(0.285 - 0.57x)}$ A1 [6]
- (ii) Equate $\frac{dR}{dx}$ to zero and solve for x M1
- State or imply $x = 0.57^{-1}$, or equivalent, e.g. 1.75 A1
- Obtain $R = 28.8$ (allow 28.9) A1 [3]
- 8 (i) Use $\sin(A + B)$ formula to express $\sin 3\theta$ in terms of trig. functions of 2θ and θ M1
- Use correct double angle formulae and Pythagoras to express $\sin 3\theta$ in terms of $\sin \theta$ M1
- Obtain a correct expression in terms of $\sin \theta$ in any form A1
- Obtain the given identity A1 [4]
- [SR: Give M1 for using correct formulae to express RHS in terms of $\sin \theta$ and $\cos 2\theta$,
 then M1A1 for expressing in terms of $\sin \theta$ and $\sin 3\theta$ only, or in terms
 of $\cos \theta$, $\sin \theta$, $\cos 2\theta$ and $\sin 2\theta$, then A1 for obtaining the given identity.]

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- (ii) Substitute for x and obtain the given answer B1 [1]
- (iii) Carry out a correct method to find a value of x
Obtain answers 0.322, 0.799, -1.12 M1
[Solutions with more than 3 answers can only earn a maximum of A1 + A1.] A1 + A1 + A1 [4]
- 9 (i) State or imply the form $\frac{A}{1-x} + \frac{B}{2-x} + \frac{C}{(2-x)^2}$ B1
Use a correct method to determine a constant M1
Obtain one of $A = 2, B = -1, C = 3$ A1
Obtain a second value A1
Obtain a third value A1 [5]
- [The alternative form $\frac{A}{1-x} + \frac{Dx+E}{(2-x)^2}$, where $A = 2, D = 1, E = 1$ is marked B1M1A1A1A1 as above.]
- (ii) Use correct method to find the first two terms of the expansion of $(1-x)^{-1}, (2-x)^{-1}, (2-x)^{-2}, (1-\frac{1}{2}x)^{-1}$ or $(1-\frac{1}{2}x)^{-2}$ M1
Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction A1✓ + A1✓ + A1✓
Obtain final answer $\frac{9}{4} + \frac{5}{2}x + \frac{39}{16}x^2$, or equivalent A1 [5]
- [Symbolic binomial coefficients, e.g. $\binom{-1}{1}$ are not sufficient for M1. The ✓ is on A, B, C .]
- [For the A, D, E form of partial fractions, give M1 A1✓ A1✓ for the expansions then, if $D \neq 0$, M1 for multiplying out fully and A1 for the final answer.]
[In the case of an attempt to expand $(x^2 - 8x + 9)(1-x)^{-1}(2-x)^{-2}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]
- 10 (i) EITHER: Find \overrightarrow{AP} (or \overrightarrow{PA}) for a point P on l with parameter λ ,
e.g. $\mathbf{i} - 17\mathbf{j} + 4\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ B1
Calculate scalar product of \overrightarrow{AP} and a direction vector for l and equate to zero M1
Solve and obtain $\lambda = 3$ A1
Carry out a complete method for finding the length of AP M1
Obtain the given answer 15 correctly A1
- OR1: Calling $(4, -9, 9)$ B , state \overrightarrow{BA} (or \overrightarrow{AB}) in component form, e.g. $-\mathbf{i} + 17\mathbf{j} - 4\mathbf{k}$ B1
Calculate vector product of \overrightarrow{BA} and a direction vector for l ,
e.g. $(-\mathbf{i} + 17\mathbf{j} - 4\mathbf{k}) \times (-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ M1
Obtain correct answer, e.g. $-30\mathbf{i} + 6\mathbf{j} + 33\mathbf{k}$ A1
Divide the modulus of the product by that of the direction vector M1
Obtain the given answer correctly A1
- OR2: State \overrightarrow{BA} (or \overrightarrow{AB}) in component form B1
Use a scalar product to find the projection of BA (or AB) on l M1
Obtain correct answer in any form, e.g. $\frac{27}{\sqrt{9}}$ A1
Use Pythagoras to find the perpendicular M1

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| | Obtain the given answer correctly | A1 | |
| OR3: | State \overline{BA} (or \overline{AB}) in component form | B1 | |
| | Use a scalar product to find the cosine of ABP | M1 | |
| | Obtain correct answer in any form, e.g. $\frac{27}{\sqrt{9}\sqrt{306}}$ | A1 | |
| | Use trig. to find the perpendicular | M1 | |
| | Obtain the given answer correctly | A1 | |
| OR4: | State \overline{BA} (or \overline{AB}) in component form | B1 | |
| | Find a second point C on l and use the cosine rule in triangle ABC to find the cosine of angle A , B , or C , or use a vector product to find the area of ABC | M1 | |
| | Obtain correct answer in any form | A1 | |
| | Use trig. or area formula to find the perpendicular | M1 | |
| | Obtain the given answer correctly | A1 | |
| OR5: | State correct \overline{AP} (or \overline{PA}) for a point P on l with parameter λ in any form | B1 | |
| | Use correct method to express AP^2 (or AP) in terms of λ | M1 | |
| | Obtain a correct expression in any form, e.g. $(1 - 2\lambda)^2 + (-17 + \lambda)^2 + (4 - 2\lambda)^2$ | A1 | |
| | Carry out a method for finding its minimum (using calculus, algebra or Pythagoras) | M1 | |
| | Obtain the given answer correctly | A1 | [5] |
| (ii) EITHER: | Substitute coordinates of a general point of l in equation of plane and either equate constant terms or equate the coefficient of λ to zero, obtaining an equation in a and b | M1* | |
| | Obtain a correct equation, e.g. $4a - 9b - 27 + 1 = 0$ | A1 | |
| | Obtain a second correct equation, e.g. $-2a + b + 6 = 0$ | A1 | |
| | Solve for a or for b | M1(dep*) | |
| | Obtain $a = 2$ and $b = -2$ | A1 | |
| OR: | Substitute coordinates of a point of l and obtain a correct equation, e.g. $4a - 9b = 26$ | B1 | |
| | EITHER: Find a second point on l and obtain an equation in a and b | M1* | |
| | Obtain a correct equation | A1 | |
| | OR: Calculate scalar product of a direction vector for l and a vector normal to the plane and equate to zero | M1* | |
| | Obtain a correct equation, e.g. $-2a + b + 6 = 0$ | A1 | |
| | Solve for a or for b | M1(dep*) | |
| | Obtain $a = 2$ and $b = -2$ | A1 | [5] |