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	GCE A LEVEL – October/November 2013	9709	33

1 Apply at least one logarithm property correctly *M1

Obtain $\frac{(x+4)^2}{x} = x + a$ or equivalent **without logarithm** involved A1

Rearrange to express x in terms of a M1 d*M

Obtain $\frac{16}{a-8}$ or equivalent A1 [4]

2 Carry out complete substitution including the use of $\frac{du}{dx} = 3$ M1

Obtain $\int\left(\frac{1}{3} - \frac{1}{3u}\right)du$ A1

Integrate to obtain form $k_1u + k_2\ln u$ or $k_1u + k_2\ln 3u$ where $k_1k_2 \neq 0$ M1

Obtain $\frac{1}{3}(3x+1) - \frac{1}{3}\ln(3x+1)$ or equivalent, condoning absence of modulus signs and + c A1 [4]

3 **(i)** Substitute –2 and equate to zero or divide by $x + 2$ and equate remainder to zero or use –2 in synthetic division M1

Obtain $a = -1$ A1 [2]

(ii) Attempt to find quadratic factor by division reaching $x^2 + kx$, or inspection as far as $(x+2)(x^2 + Bx + c)$ and equations for one or both of B and C, or $(x+2)(Ax^2 + Bx + 7)$ and equations for one or both of A and B. M1

Obtain $x^2 - 3x + 7$ A1

Use discriminant to obtain –19, or equivalent, and **confirm one root** cwo A1 [3]

4 Differentiate y^3 to obtain $3y^2\frac{dy}{dx}$ B1

Use correct product rule at least once *M1

Obtain $6e^{2x}y + 3e^{2x}\frac{dy}{dx} + e^xy^3 + 3e^xy^2\frac{dy}{dx}$ as derivative of LHS A1

Equate derivative of LHS to zero, substitute $x = 0$ and $y = 2$ and find value of $\frac{dy}{dx}$ M1(d*M)

Obtain $-\frac{4}{3}$ or equivalent as **final answer** A1 [5]

5 **(i)** Use integration by parts to obtain $axe^{-\frac{1}{2}x} + \int be^{-\frac{1}{2}x}dx$ M1*

Obtain $-8xe^{-\frac{1}{2}x} + \int 8e^{-\frac{1}{2}x}dx$ or unsimplified equivalent A1

Obtain $-8xe^{-\frac{1}{2}x} - 16e^{-\frac{1}{2}x}$ A1

Use limits correctly and equate to 9 M1(d*M)

Obtain given answer $p = 2\ln\left(\frac{8p+16}{7}\right)$ correctly A1 [5]

(ii) Use correct iteration formula correctly at least once M1
Obtain final answer 3.77 A1
Show sufficient iterations to 5sf or better to justify accuracy 3.77 or show sign change in interval (3.765, 3.775) A1 [3]
[$3.5 \rightarrow 3.6766 \rightarrow 3.7398 \rightarrow 3.7619 \rightarrow 3.7696 \rightarrow 3.7723$]

6 (i) Find scalar product of the normals to the planes M1
Using the correct process for the moduli, divide the scalar product by the product of the moduli and find $\cos^{-1}$ of the result. M1
Obtain 67.8° (or 1.18 radians) A1 [3]

(ii) EITHER Carry out complete method for finding point on line M1
Obtain one such point, e.g. $(2,-3,0)$ or $\left(\frac{17}{7},0,\frac{6}{7}\right)$ or $(0,-17,-4)$ or … A1…

Either State $3a-b+2c=0$ and $a+b-4c=0$ or equivalent B1
Attempt to solve for one ratio, e.g. $a:b$ M1
Obtain $a:b:c=1:7:2$ or equivalent A1
State a correct final answer, e.g. r = [2, –3, 0] + λ[1, 7, 2] A1√

Or 1 Obtain a second point on the line A1
Subtract position vectors to obtain direction vector M1
Obtain [1, 7, 2] or equivalent A1
State a correct final answer, e.g. r = [2, –3, 0] + λ[1, 7, 2] A1√

Or 2 Use correct method to calculate vector product of two normals M1
Obtain two correct components A1
Obtain [2, 14, 4] or equivalent A1
State a correct final answer, e.g. r = [2, –3, 0] + λ[1, 7, 2] A1√
[√ is dependent on both M marks in all three cases]

OR 3 Express one variable in terms of a second variable M1
Obtain a correct simplified expression, e.g. $x=\frac{1}{2}(4+z)$ A1
Express the first variable in terms of third variable M1
Obtain a correct simplified expression, e.g. $x=\frac{1}{7}(17+y)$ A1
Form a vector equation for the line M1
State a correct final answer, e.g. r = [0, –17, –4] + λ[1, 7, 2] A1

OR 4 Express one variable in terms of a second variable M1
Obtain a correct simplified expression, e.g. $z=2x-4$ A1
Express third variable in terms of the second variable M1
Obtain a correct simplified expression, e.g. $y=7x-17$ A1
Form a vector equation for the line M1
State a correct final answer, e.g. r = [0, –17, –4] + λ[1, 7, 2] A1 [6]

7 **(i)** Use $\sec\theta = \frac{1}{\cos\theta}$ and $\text{cosec}\,\theta = \frac{1}{\sin\theta}$ B1

Use $\sin 2\theta = 2\sin\theta\cos\theta$ and to form a horizontal equation in $\sin\theta$ and $\cos\theta$ or fractions with common denominators M1

Obtain given equation $2\sin\theta + 4\cos\theta = 3$ correctly A1 [3]

(ii) State or imply $R = \sqrt{20}$ or 4.47 or equivalent B1

Use correct trigonometry to find α M1

Obtain 63.43 or 63.44 with no errors seen A1 [3]

(iii) Carry out a correct method to find one value in given range M1

Obtain 74.4° (or 338.7°) A1

Carry out a correct method to find second value in given range M1

Obtain 338.7° (or 74.4°) and no others between 0° and 360° A1 [4]

8 **(i)** Either State or imply form $\frac{A}{1+x} + \frac{B}{(1+x)^2} + \frac{C}{2-3x}$ B1

Use any relevant method to find at least one constant M1

Obtain $A = -1$ A1

Obtain $B = 3$ A1

Obtain $C = 4$ A1

Or State or imply form $\frac{A}{1+x} + \frac{Bx}{(1+x)^2} + \frac{C}{2-3x}$ B1

Use any relevant method to find at least one constant M1

Obtain $A = 2$ A1

Obtain $B = -3$ A1

Obtain $C = 4$ A1

Or State or imply form $\frac{Dx+E}{(1+x)^2} + \frac{F}{2-3x}$ B1

Use any relevant method to find at least one constant M1

Obtain $D = -1$ A1

Obtain $E = 2$ A1

Obtain $F = 4$ A1 [5]

(ii) Either Use correct method to find first two terms of expansion of $(1+x)^{-1}$ or $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$ M1

Obtain correct unsimplified expansion of first partial fraction up to x^2 term A1√

Obtain correct unsimplified expansion of second partial fraction up to x^2 term A1√

Obtain correct unsimplified expansion of third partial fraction up to x^2 term A1√

Obtain final answer $4-2x+\frac{25}{2}x^2$ A1

Or 1 Use correct method to find first two terms of expansion of $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$ M1

Obtain correct unsimplified expansion of first partial fraction up to x^2 term A1√

Obtain correct unsimplified expansion of second partial fraction up to x^2 term A1√

Expand and obtain sufficient terms to obtain three terms M1

Obtain final answer $4-2x+\frac{25}{2}x^2$ A1

Or 2 (expanding original expression)

Use correct method to find first two terms of expansion of $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$ M1

Obtain correct expansion $1-2x+3x^2$ or unsimplified equivalent A1

Obtain correct expansion $\frac{1}{2}\left(1+\frac{3}{2}x+\frac{9}{4}x^2\right)$ or unsimplified equivalent A1

Expand and obtain sufficient terms to obtain three terms M1

Obtain final answer $4-2x+\frac{25}{2}x^2$ A1

Or 3 (McLaurin expansion)

Obtain first derivative $f'(x)=(1+x)^{-2}-6(1+x)^{-3}+12(2-3x)^{-2}$ M1

Obtain $f'(0)=1-6+3$ or equivalent A1

Obtain $f''(0)=-2+18+9$ or equivalent A1

Use correct form for McLaurin expansion M1

Obtain final answer $4-2x+\frac{25}{2}x^2$ A1 [5]

9 **(a)** Solve using formula, including simplification under square root sign M1*

Obtain $\frac{-2\pm 4\mathrm{i}}{2(2-\mathrm{i})}$ or similarly simplified equivalents A1

Multiply by $\frac{2+\mathrm{i}}{2+\mathrm{i}}$ or equivalent in at least one case M1(d*M)

Obtain final answer $-\frac{4}{5}+\frac{3}{5}\mathrm{i}$ A1

Obtain final answer –i A1 [5]

(b) Show w in first quadrant with modulus and argument relatively correct — B1
Show w^3 in second quadrant with modulus and argument relatively correct — B1
Show w^* in fourth quadrant with modulus and argument relatively correct — B1
Use correct method for area of triangle — M1
Obtain 10 by calculation — A1 [5]

10 Use $2\cos^2 x = 1 + \cos 2x$ or equivalent — B1
Separate variables and integrate at least one side — M1
Obtain $\ln\left(y^3 + 1\right) = \ldots$ or equivalent — A1
Obtain $\ldots = 2x + \sin 2x$ or equivalent — A1
Use $x = 0$, $y = 2$ to find constant of integration (or as limits) in an expression containing at least two terms of the form $a\ln\left(y^3 + 1\right)$, bx or $c\sin 2x$ — M1*
Obtain $\ln\left(y^3 + 1\right) = 2x + \sin 2x + \ln 9$ or equivalent e.g. implied by correct constant — A1
Identify at least one of $\frac{1}{2}\pi$ and $\frac{3}{2}\pi$ as x-coordinate at stationary point — B1
Use correct process to find y-coordinate for at least one x-coordinate — M1(d*M)
Obtain 5.9 — A1
Obtain 48.1 — A1 [10]